NAG Library Function Document

nag_bessel_k1_vector (s18arc)

1 Purpose

nag_bessel_k1_vector (s18arc) returns an array of values of the modified Bessel function $K_1(x)$.

2 Specification

```c
#include <nag.h>
#include <nags.h>

void nag_bessel_k1_vector(Integer n, const double x[], double f[],
                        Integer invalid[], NagError *fail)
```

3 Description

nag_bessel_k1_vector (s18arc) evaluates an approximation to the modified Bessel function of the second kind $K_1(x_i)$ for an array of arguments $x_i$, for $i = 1, 2, \ldots, n$.

Note: $K_1(x)$ is undefined for $x \leq 0$ and the function will fail for such arguments.

The function is based on five Chebyshev expansions:

For $0 < x \leq 1$,

$$K_1(x) = \frac{1}{x} + x \ln x \sum_{r=0}^\infty a_r T_r(t) - x \sum_{r=0}^\infty b_r T_r(t), \quad \text{where } t = 2x^2 - 1.$$  

For $1 < x \leq 2$,

$$K_1(x) = e^{-x} \sum_{r=0}^\infty c_r T_r(t), \quad \text{where } t = 2x - 3.$$  

For $2 < x \leq 4$,

$$K_1(x) = e^{-x} \sum_{r=0}^\infty d_r T_r(t), \quad \text{where } t = x - 3.$$  

For $x > 4$,

$$K_1(x) = e^{-x} \sqrt{\frac{2}{x}} \sum_{r=0}^\infty e_r T_r(t), \quad \text{where } t = \frac{9 - x}{1 + x}.$$  

For $x$ near zero, $K_1(x) \approx \frac{1}{x}$. This approximation is used when $x$ is sufficiently small for the result to be correct to machine precision. For very small $x$ it is impossible to calculate $\frac{1}{x}$ without overflow and the function must fail.

For large $x$, where there is a danger of underflow due to the smallness of $K_1$, the result is set exactly to zero.

4 References


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5 Arguments

1: n – Integer  
Input
On entry: n, the number of points.
Constraint: n ≥ 0.

2: x[n] – const double  
Input
On entry: the argument x_i of the function, for i = 1, 2, ..., n.
Constraint: x[i - 1] > 0.0, for i = 1, 2, ..., n.

3: f[n] – double  
Output
On exit: K_1(x_i), the function values.

4: invalid[n] – Integer  
Output
On exit: invalid[i - 1] contains the error code for x_i, for i = 1, 2, ..., n.
invalid[i - 1] = 0
No error.
invalid[i - 1] = 1
x_i ≤ 0.0, K_1(x_i) is undefined. f[i - 1] contains 0.0.
invalid[i - 1] = 2
x_i is too small, there is a danger of overflow. f[i - 1] contains zero. The threshold value is the same as for fail.code = NE_REAL_ARG_TOO_SMALL in nag_bessel_k1 (s18adc), as defined in the Users’ Note for your implementation.

5: fail – NagError*  
Input/Output
The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL
Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM
On entry, argument ⟨value⟩ had an illegal value.

NE_INT
On entry, n = ⟨value⟩.
Constraint: n ≥ 0.

NE_INTERNAL_ERROR
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.
An unexpected error has been triggered by this function. Please contact NAG. See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE
Your licence key may have expired or may not have been installed correctly. See Section 3.6.5 in the Essential Introduction for further information.
NW_IVALID

On entry, at least one value of x was invalid.
Check ivalid for more information.

7 Accuracy

Let $\delta$ and $\epsilon$ be the relative errors in the argument and result respectively.

If $\delta$ is somewhat larger than the machine precision (i.e., if $\delta$ is due to data errors etc.), then $\epsilon$ and $\delta$ are approximately related by:

$$\epsilon \approx \frac{|xK_0(x) - K_1(x)|}{K_1(x)} \delta.$$ 

Figure 1 shows the behaviour of the error amplification factor

$$\frac{|xK_0(x) - K_1(x)|}{K_1(x)}.$$ 

However if $\delta$ is of the same order as the machine precision, then rounding errors could make $\epsilon$ slightly larger than the above relation predicts.

For small $x$, $\epsilon \approx \delta$ and there is no amplification of errors.

For large $x$, $\epsilon \approx x\delta$ and we have strong amplification of the relative error. Eventually $K_1$, which is asymptotically given by $\frac{e^{-x}}{\sqrt{x}}$, becomes so small that it cannot be calculated without underflow and hence the function will return zero. Note that for large $x$ the errors will be dominated by those of the standard function exp.

![Figure 1](image)

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.
10 Example

This example reads values of x from a file, evaluates the function at each value of xi and prints the results.

10.1 Program Text

/* nag_bessel_k1_vector (s18arc) Example Program.
 * * Copyright 2014 Numerical Algorithms Group.
 * * Mark 23, 2011.
 * */
#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer exit_status = 0;
    Integer i, n;
    double *f = 0, *x = 0;
    Integer *ivalid = 0;
    NagError fail;

    INIT_FAIL(fail);

    /* Skip heading in data file */
    #ifdef _WIN32
    scanf_s("%*[\n]");
    #else
    scanf("%*[\n]");
    #endif
    printf("nag_bessel_k1_vector (s18arc) Example Program Results\n");
    printf("\n");
    printf(" x f ivalid\n");
    printf("\n");
    #ifdef _WIN32
    scanf_s("%lf", &x[i]);
    #else
    scanf("%lf", &x[i]);
    #endif
    #ifdef _WIN32
    scanf_s("%*[\n]");
    #else
    scanf("%*[\n]");
    #endif

    /* Allocate memory */
    if (!(x = NAG_ALLOC(n, double)) ||
        !(f = NAG_ALLOC(n, double)) ||
        !(ivalid = NAG_ALLOC(n, Integer)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
    for (i=0; i<n; i++)
    #ifdef _WIN32
    scanf_s("%lf", &x[i]);
    #else
    scanf("%lf", &x[i]);
    #endif
    #ifdef _WIN32
    scanf_s("%*[\n]");
    #else

    */
```c
scanf("%*[\n");
#endif

/* nag_bessel_k1_vector (s18arc).
 * modified Bessel Function K1(x)
 */

nag_bessel_k1_vector(n, x, f, ivalid, &fail);
if (fail.code!=NE_NOERROR && fail.code!=NW_IVALID)
{
    printf("Error from nag_bessel_k1_vector (s18arc).
    %s
", fail.message);
    exit_status = 1;
    goto END;
}

for (i=0; i<n; i++)
    printf(" %11.3e %11.3e %4"NAG_IFMT"
", x[i], f[i], ivalid[i]);

END:
    NAG_FREE(f);
    NAG_FREE(x);
    NAG_FREE(ivalid);
    return exit_status;
}

10.2 Program Data

nag_bessel_k1_vector (s18arc) Example Program Data

10
0.4 0.6 1.4 1.6 2.5 3.5 6.0 8.0 10.0 1000.0

10.3 Program Results

nag_bessel_k1_vector (s18arc) Example Program Results

<table>
<thead>
<tr>
<th>x</th>
<th>f</th>
<th>ivalid</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.000e-01</td>
<td>2.184e+00</td>
<td>0</td>
</tr>
<tr>
<td>6.000e-01</td>
<td>1.303e+00</td>
<td>0</td>
</tr>
<tr>
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<td>3.208e-01</td>
<td>0</td>
</tr>
<tr>
<td>1.600e+00</td>
<td>2.406e-01</td>
<td>0</td>
</tr>
<tr>
<td>2.500e+00</td>
<td>7.389e-02</td>
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</tr>
<tr>
<td>3.500e+00</td>
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</tr>
<tr>
<td>6.000e+00</td>
<td>1.344e-03</td>
<td>0</td>
</tr>
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<td>1.554e-04</td>
<td>0</td>
</tr>
<tr>
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<td>1.865e-05</td>
<td>0</td>
</tr>
<tr>
<td>1.00e+03</td>
<td>0.000e+00</td>
<td>0</td>
</tr>
</tbody>
</table>
```