1 Purpose

nag_bessel_j0_vector (s17asc) returns an array of values of the Bessel function $J_0(x)$.

2 Specification

```c
#include <nag.h>
#include <nags.h>
void nag_bessel_j0_vector (Integer n, const double x[], double f[],
                        Integer ivalid[], NagError *fail)
```

3 Description

nag_bessel_j0_vector (s17asc) evaluates an approximation to the Bessel function of the first kind $J_0(x_i)$ for an array of arguments $x_i$, for $i = 1, 2, \ldots, n$.

**Note:** $J_0(-x) = J_0(x)$, so the approximation need only consider $x \geq 0$.

The function is based on three Chebyshev expansions:

For $0 < x \leq 8$,

$$J_0(x) = \sum_{r=0} a_r T_r(t), \quad \text{with} \quad t = 2 \left( \frac{x}{8} \right)^2 - 1.$$ 

For $x > 8$,

$$J_0(x) = \sqrt{\frac{2}{\pi x}} \left( P_0(x) \cos \left( x - \frac{\pi}{4} \right) - Q_0(x) \sin \left( x - \frac{\pi}{4} \right) \right),$$

where $P_0(x) = \sum_{r=0} b_r T_r(t)$,

and $Q_0(x) = \frac{8}{\pi} \sum_{x=0} c_r T_r(t)$,

with $t = 2 \left( \frac{x}{8} \right)^2 - 1$.

For $x$ near zero, $J_0(x) \approx 1$. This approximation is used when $x$ is sufficiently small for the result to be correct to *machine precision*.

For very large $x$, it becomes impossible to provide results with any reasonable accuracy (see Section 7), hence the function fails. Such arguments contain insufficient information to determine the phase of oscillation of $J_0(x)$; only the amplitude, $\sqrt{\frac{2}{\pi x}}$, can be determined and this is returned on failure. The range for which this occurs is roughly related to *machine precision*; the function will fail if $|x| \approx 1$/*machine precision* (see the Users’ Note for your implementation for details).

4 References


Clenshaw C W (1962) Chebyshev Series for Mathematical Functions *Mathematical tables* HMSO
5 Arguments

1: \( n \) – Integer \hspace{1cm} \text{Input}

\textit{On entry:} \( n \), the number of points.

\textit{Constraint:} \( n \geq 0 \).

2: \( x[n] \) – const double \hspace{1cm} \text{Input}

\textit{On entry:} the argument \( x_i \) of the function, for \( i = 1, 2, \ldots, n \).

3: \( f[n] \) – double \hspace{1cm} \text{Output}

\textit{On exit:} \( J_0(x_i) \), the function values.

4: \( \text{ivalid}[n] \) – Integer \hspace{1cm} \text{Output}

\textit{On exit:} \( \text{ivalid}[i - 1] \) contains the error code for \( x_i \), for \( i = 1, 2, \ldots, n \).

\( \text{ivalid}[i - 1] = 0 \)

No error.

\( \text{ivalid}[i - 1] = 1 \)

On entry, \( x_i \) is too large. \( f[i - 1] \) contains the amplitude of the \( J_0 \) oscillation, \( \sqrt{\frac{2}{\pi|x_i|}} \).

5: \( \text{fail} \) – NagError \* \hspace{1cm} \text{Input/Output}

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

\textbf{NE_ALLOC_FAIL}

Dynamic memory allocation failed.

See Section 3.2.1.2 in the Essential Introduction for further information.

\textbf{NE_BAD_PARAM}

On entry, argument \langle value \rangle had an illegal value.

\textbf{NE_INT}

On entry, \( n = \langle value \rangle \).

\textit{Constraint:} \( n \geq 0 \).

\textbf{NE_INTERNAL_ERROR}

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 3.6.6 in the Essential Introduction for further information.

\textbf{NE_NO_LICENCE}

Your licence key may have expired or may not have been installed correctly.

See Section 3.6.5 in the Essential Introduction for further information.

\textbf{NW_IVALID}

On entry, at least one value of \( x \) was invalid.

Check \texttt{ivalid} for more information.
7 Accuracy

Let $\delta$ be the relative error in the argument and $E$ be the absolute error in the result. (Since $J_0(x)$ oscillates about zero, absolute error and not relative error is significant.)

If $\delta$ is somewhat larger than the machine precision (e.g., if $\delta$ is due to data errors etc.), then $E$ and $\delta$ are approximately related by:

$$E \approx |xJ_1(x)|\delta$$

(provided $E$ is also within machine bounds). Figure 1 displays the behaviour of the amplification factor $|xJ_1(x)|$.

However, if $\delta$ is of the same order as machine precision, then rounding errors could make $E$ slightly larger than the above relation predicts.

For very large $x$, the above relation ceases to apply. In this region, $J_0(x) \simeq \sqrt{\frac{2}{\pi|x|}} \cos\left(x - \frac{\pi}{4}\right)$. The amplitude $\sqrt{\frac{2}{\pi|x|}}$ can be calculated with reasonable accuracy for all $x$, but $\cos\left(x - \frac{\pi}{4}\right)$ cannot. If $x - \frac{\pi}{4}$ is written as $2N\pi + \theta$ where $N$ is an integer and $0 \leq \theta < 2\pi$, then $\cos\left(x - \frac{\pi}{4}\right)$ is determined by $\theta$ only.

If $x \geq \delta^{-1}$, $\theta$ cannot be determined with any accuracy at all. Thus if $x$ is greater than, or of the order of, the inverse of the machine precision, it is impossible to calculate the phase of $J_0(x)$ and the function must fail.

![Figure 1](image-url)

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.
10 Example

This example reads values of x from a file, evaluates the function at each value of \( x_i \) and prints the results.

10.1 Program Text

```c
/* nag_bessel_j0_vector (s17asc) Example Program. */
/* Copyright 2014 Numerical Algorithms Group. */
/* Mark 23, 2011. */
#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer exit_status = 0;
    Integer i, n;
    double *f = 0, *x = 0;
    Integer *ivalid = 0;
    NagError fail;

    INIT_FAIL(fail);

    /* Skip heading in data file */
    #ifdef _WIN32
        scanf_s("%*[^\n]");
    #else
        scanf("%*[^\n]");
    #endif
    printf("nag_bessel_j0_vector (s17asc) Example Program Results\n");
    printf("\n");
    printf(" x f ivalid\n");
    printf("\n");
    #ifdef _WIN32
        scanf_s("%"NAG_IFMT", &n);
    #else
        scanf("%"NAG_IFMT", &n);
    #endif
    #ifdef _WIN32
        scanf_s("%*[^\n]");
    #else
        scanf("%*[^\n]");
    #endif

    /* Allocate memory */
    if (!(x = NAG_ALLOC(n, double)) ||
        !(f = NAG_ALLOC(n, double)) ||
        !(ivalid = NAG_ALLOC(n, Integer)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
    for (i=0; i<n; i++)
    {
        #ifdef _WIN32
            scanf_s("%lf", &x[i]);
        #else
            scanf("%lf", &x[i]);
        #endif
        #ifdef _WIN32
            scanf_s("%*
");
        #else
            scanf("%*
");
        #endif
    }
```

---

s17asc  
NAG Library Manual

s17asc.4  
Mark 25
```c
scanf("%*[\n"]);
#endif

/* nag_bessel_j0_vector (s17asc).
 * Bessel function J_0(x)
 */

if (fail.code!=NE_NOERROR && fail.code!=NW_IVALID)
{
    printf("Error from nag_bessel_j0_vector (s17asc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

for (i=0; i<n; i++)
    printf(" %11.3e %11.3e %4"NAG_IFMT"
", x[i], f[i], ivalid[i]);

END:
NAG_FREE(f);
NAG_FREE(x);
NAG_FREE(ivalid);
return exit_status;
}

10.2 Program Data

nag_bessel_j0_vector (s17asc) Example Program Data

9
0.0 0.5 1.0 3.0 6.0 8.0 10.0 -1.0 1000.0

10.3 Program Results

nag_bessel_j0_vector (s17asc) Example Program Results

<table>
<thead>
<tr>
<th>x</th>
<th>f</th>
<th>ivalid</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000e+00</td>
<td>1.000e+00</td>
<td>0</td>
</tr>
<tr>
<td>5.000e-01</td>
<td>9.385e-01</td>
<td>0</td>
</tr>
<tr>
<td>1.000e+00</td>
<td>7.652e-01</td>
<td>0</td>
</tr>
<tr>
<td>3.000e+00</td>
<td>-2.601e-01</td>
<td>0</td>
</tr>
<tr>
<td>6.000e+00</td>
<td>1.506e-01</td>
<td>0</td>
</tr>
<tr>
<td>8.000e+00</td>
<td>1.717e-01</td>
<td>0</td>
</tr>
<tr>
<td>1.000e+01</td>
<td>-2.459e-01</td>
<td>0</td>
</tr>
<tr>
<td>-1.000e+00</td>
<td>7.652e-01</td>
<td>0</td>
</tr>
<tr>
<td>1.000e+03</td>
<td>2.479e-02</td>
<td>0</td>
</tr>
</tbody>
</table>
```