NAG Library Function Document

nag_dawson (s15afc)

1 Purpose

nag_dawson (s15afc) returns a value for Dawson’s Integral, $F(x)$.

2 Specification

```c
#include <nag.h>
#include <nags.h>
double nag_dawson (double x)
```

3 Description

nag_dawson (s15afc) evaluates an approximation for Dawson’s Integral

$$F(x) = e^{-x^2} \int_0^x e^{t^2} dt.$$ 

The function is based on two Chebyshev expansions:

For $0 < |x| \leq 4$,

$$F(x) = x \sum_{r=0}^{t} a_r T_r(t), \quad \text{where} \quad t = 2 \left( \frac{x}{4} \right)^2 - 1.$$ 

For $|x| > 4$,

$$F(x) = \frac{1}{x} \sum_{r=0}^{t} b_r T_r(t), \quad \text{where} \quad t = 2 \left( \frac{4}{x} \right)^2 - 1.$$ 

For $|x|$ near zero, $F(x) \simeq x$, and for $|x|$ large, $F(x) \simeq \frac{1}{2x}$. These approximations are used for those values of $x$ for which the result is correct to machine precision.

4 References


5 Arguments

1:  $x$ – double

   **Input**

   On entry: the argument $x$ of the function.

6 Error Indicators and Warnings

7 Accuracy

Let $\delta$ and $\epsilon$ be the relative errors in the argument and result respectively.

If $\delta$ is considerably greater than the machine precision (i.e., if $\delta$ is due to data errors etc.), then $\epsilon$ and $\delta$ are approximately related by:
\[ \epsilon \approx \left| \frac{x(1 - 2xF(x))}{F(x)} \right| \delta. \]

The following graph shows the behaviour of the error amplification factor \( \left| \frac{x(1 - 2xF(x))}{F(x)} \right| \):

![Graph](image)

However if \( \delta \) is of the same order as machine precision, then rounding errors could make \( \epsilon \) somewhat larger than the above relation indicates. In fact \( \epsilon \) will be largely independent of \( x \) or \( \delta \), but will be of the order of a few times the machine precision.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of the argument \( x \) from a file, evaluates the function at each value of \( x \) and prints the results.

10.1 Program Text

/* nag_dawson (s15afc) Example Program. *
 * Copyright 2014 Numerical Algorithms Group.
 * * Mark 7, 2002.
 */

#include <nag.h>
#include <stdio.h>
#include <nags.h>
int main(void)
{
    double x, y;
    Integer exit_status = 0;

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
    printf("nag_dawson (s15afc) Example Program Results\n");
    printf(" x y\n");
#ifdef _WIN32
    while (scanf_s("%lf", &x) != EOF)
#else
    while (scanf("%lf", &x) != EOF)
#endif
    {
        /* nag_dawson (s15afc).
         * Dawson’s integral
         */
        y = nag_dawson(x);
        printf("%12.3e %12.3e\n", x, y);
    }
    return exit_status;
}

10.2 Program Data

nag_dawson (s15afc) Example Program Data
-2.0
-0.5
1.0
1.5
2.0
5.0
10.0

10.3 Program Results

nag_dawson (s15afc) Example Program Results
 x       y
-2.000e+00 -3.013e-01
-5.000e-01 -4.244e-01
 1.000e+00  5.381e-01
 1.500e+00  4.282e-01
 2.000e+00  3.013e-01
 5.000e+00  1.021e-01
 1.000e+01  5.025e-02