NAG Library Function Document

nag_erf (s15aec)

1 Purpose

nag_erf (s15aec) returns the value of the error function \( \text{erf}(x) \).

2 Specification

```c
#include <nag.h>
#include <nags.h>
double nag_erf (double x)
```

3 Description

nag_erf (s15aec) calculates an approximate value for the error function

\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt = 1 - \text{erfc}(x).
\]

Let \( \hat{x} \) be the root of the equation \( \text{erfc}(x) - \text{erf}(x) = 0 \) (then \( \hat{x} \approx 0.46875 \)). For \( |x| \leq \hat{x} \) the value of \( \text{erf}(x) \) is based on the following rational Chebyshev expansion for \( \text{erf}(x) \):

\[
\text{erf}(x) \approx x R_{\ell,m}(x^2),
\]

where \( R_{\ell,m} \) denotes a rational function of degree \( \ell \) in the numerator and \( m \) in the denominator.

For \( |x| > \hat{x} \) the value of \( \text{erf}(x) \) is based on a rational Chebyshev expansion for \( \text{erfc}(x) \): for \( \hat{x} < |x| \leq 4 \) the value is based on the expansion

\[
\text{erfc}(x) \approx e^{x^2} R_{\ell,m}(x);
\]

and for \( |x| > 4 \) it is based on the expansion

\[
\text{erfc}(x) \approx \frac{e^{x^2}}{x} \left( \frac{1}{\sqrt{\pi}} + \frac{1}{x^2} R_{\ell,m}(1/x^2) \right).
\]

For each expansion, the specific values of \( \ell \) and \( m \) are selected to be minimal such that the maximum relative error in the expansion is of the order \( 10^{-d} \), where \( d \) is the maximum number of decimal digits that can be accurately represented for the particular implementation (see nag_decimal_digits (X02BEC)).

For \( |x| \geq x_{\text{hi}} \) there is a danger of setting underflow in \( \text{erfc}(x) \) (the value of \( x_{\text{hi}} \) is given in the Users’ Note for your implementation). For \( x \geq x_{\text{hi}} \), nag_erf (s15aec) returns \( \text{erf}(x) = 1 \); for \( x \leq -x_{\text{hi}} \) it returns \( \text{erf}(x) = -1 \).

4 References


5 Arguments

1: \( x \) – double

\( \text{Input} \)

\( \text{On entry:} \) the argument \( x \) of the function.
6 Error Indicators and Warnings

None.

7 Accuracy

See Section 7 in nag_erfc (s15adc).

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of the argument $x$ from a file, evaluates the function at each value of $x$ and prints the results.

10.1 Program Text

```c
/* nag_erf (s15aec) Example Program.
   * Copyright 2014 Numerical Algorithms Group.
   * Mark 1, 1990.
   */
#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer exit_status = 0;
    double x, y;

    /* Skip heading in data file */
#ifndef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
    printf("nag_erf (s15aec) Example Program Results\n");
    printf(" %12.3e%12.3e\n", x, y);
#endif
    return exit_status;
}
```

10.2 Program Data

nag_erf (s15aec) Example Program Data
-6.0
-4.5
-1.0
1.0
4.5
6.0

10.3 Program Results

nag_erf (s15aec) Example Program Results

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6.000e+00</td>
<td>-1.000e+00</td>
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<td>-4.500e+00</td>
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<td>-8.427e-01</td>
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<tr>
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<td>8.427e-01</td>
</tr>
<tr>
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<td>1.000e-00</td>
</tr>
<tr>
<td>6.000e+00</td>
<td>1.000e+00</td>
</tr>
</tbody>
</table>