1 Purpose

nag_sin_integral (s13adc) returns the value of the sine integral

\[ Si(x) = \int_0^x \frac{\sin u}{u} du, \]

2 Specification

```c
#include <nag.h>
#include <nags.h>

double nag_sin_integral (double x)
```

3 Description

nag_sin_integral (s13adc) calculates an approximate value for \( Si(x) \).

For \( |x| \leq 16.0 \) it is based on the Chebyshev expansion

\[ Si(x) = x \sum_{r=0}^{\prime} a_r T_r(t), t = 2 \left( \frac{x}{16} \right)^2 - 1. \]

For \( 16 < |x| < x_{hi} \), where \( x_{hi} \) is an implementation-dependent number,

\[ Si(x) = \text{sign}(x) \left\{ \frac{\pi}{2} - \frac{f(x) \cos x}{x} - \frac{g(x) \sin x}{x^2} \right\} \]

where \( f(x) = \sum_{r=0} T_r(x) \) and \( g(x) = \sum_{r=0} g_r T_r(x) \), \( t = 2 \left( \frac{16}{x} \right)^2 - 1. \)

For \( |x| \geq x_{hi} \), \( Si(x) = \frac{1}{2} \pi \text{sign} x \) to within machine precision.

4 References


5 Arguments

1: \( x \) – double

Input

On entry: the argument \( x \) of the function.

6 Error Indicators and Warnings

None.
7 Accuracy

If $\delta$ and $\epsilon$ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \approx \left| \frac{\delta \sin x}{Si(x)} \right|.$$ 

The equality may hold if $\delta$ is greater than the machine precision ($\delta$ due to data errors etc.) but if $\delta$ is simply due to round-off in the machine representation, then since the factor relating $\delta$ to $\epsilon$ is always less than one, the accuracy will be limited by machine precision.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of the argument $x$ from a file, evaluates the function at each value of $x$ and prints the results.

10.1 Program Text

```c
/* nag_sin_integral (s13adc) Example Program.
 * Copyright 2014 Numerical Algorithms Group.
 * Mark 1, 1990.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer exit_status = 0;
    double x, y;

    /* nag_sin_integral (s13adc).
        Sine integral Si(x) */
    y = nag_sin_integral(x);
    /* nag_sin_integral (s13adc).
        Sine integral Si(x) */
    return exit_status;
}
```

s13adc
printf("%12.3e%12.3e\n", x, y);
}

return exit_status;
}

10.2 Program Data

nag_sin_integral (s13adc) Example Program Data

0.0
0.2
0.4
0.6
0.8
1.0

10.3 Program Results

nag_sin_integral (s13adc) Example Program Results

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000e+00</td>
<td>0.000e+00</td>
</tr>
<tr>
<td>2.000e-01</td>
<td>1.996e-01</td>
</tr>
<tr>
<td>4.000e-01</td>
<td>3.965e-01</td>
</tr>
<tr>
<td>6.000e-01</td>
<td>5.881e-01</td>
</tr>
<tr>
<td>8.000e-01</td>
<td>7.721e-01</td>
</tr>
<tr>
<td>1.000e+00</td>
<td>9.461e-01</td>
</tr>
</tbody>
</table>

Example Program

Returned Values for the Sine Integral $Si(x)$