NAG Library Function Document
nag_tanh (s10aac)

1 Purpose
nag_tanh (s10aac) returns a value for the hyperbolic tangent, \( \tanh x \).

2 Specification
#include <nag.h>
#include <nags.h>
double nag_tanh (double x)

3 Description
nag_tanh (s10aac) calculates an approximate value for the hyperbolic tangent of its argument, \( \tanh x \).
For \( |x| \leq 1 \) it is based on the Chebyshev expansion
\[
\tanh x = x \times y(t) = x \sum_{r=0} a_r T_r(t)
\]
where \(-1 \leq x \leq 1\), \(-1 \leq t \leq 1\), and \( t = 2x^2 - 1 \).
For \( 1 < |x| < E_1 \) (see the Users’ Note for your implementation for value of \( E_1 \))
\[
\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}
\]
For \( |x| \geq E_1 \), \( \tanh x = \text{sign} x \) to within the representation accuracy of the machine and so this approximation is used.

4 References

5 Arguments
1:  \( x \) – double
\hspace{1cm} Input
\hspace{1cm} On entry: the argument \( x \) of the function.

6 Error Indicators and Warnings
None.

7 Accuracy
If \( \delta \) and \( \epsilon \) are the relative errors in the argument and the result respectively, then in principle,
\[
|\epsilon| \approx \left| \frac{2x}{\sinh 2x} \delta \right|
\]
That is, a relative error in the argument, \( x \), is amplified by a factor approximately \( \frac{2x}{\sinh 2x} \), in the result.
The equality should hold if \( \delta \) is greater than the *machine precision* \( (\delta \text{ due to data errors etc.}) \) but if \( \delta \) is due simply to the round-off in the machine representation it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the amplification factor is shown in the following graph:

![Graph showing the behaviour of the amplification factor](image)

*Figure 1*

It should be noted that this factor is always less than or equal to 1.0 and away from \( x = 0 \) the accuracy will eventually be limited entirely by the precision of machine representation.

8 Parallelism and Performance
Not applicable.

9 Further Comments
None.

10 Example
This example reads values of the argument \( x \) from a file, evaluates the function at each value of \( x \) and prints the results.

10.1 Program Text
/* nag_tanh (s10aac) Example Program. */
* Copyright 2014 Numerical Algorithms Group.
* Mark 2 revised, 1992.
*/

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer exit_status = 0;
    double x, y;
/* Skip heading in data file */
#endif

while (scanf_s("%lf", &x) != EOF)
{
    /* nag_tanh (s10aac).
    * Hyperbolic tangent, tanh x
    */
    y = nag_tanh(x);
    printf("%12.1f%12.5f\n", x, y);
}

return exit_status;

10.2 Program Data

nag_tanh (s10aac) Example Program Data
-20.0
-5.0
0.5
5.0

10.3 Program Results

nag_tanh (s10aac) Example Program Results

\begin{tabular}{ll}
\hline
x & y \\
\hline
-20.0 & -1.00000 \\
-5.0 & -0.99991 \\
0.5 & 0.46212 \\
5.0 & 0.99991 \\
\hline
\end{tabular}