**NAG Library Function Document**

**nag_rand_matrix_multi_normal (g05rzc)**

1 **Purpose**

`nag_rand_matrix_multi_normal (g05rzc)` sets up a reference vector and generates an array of pseudorandom numbers from a multivariate Normal distribution with mean vector \(a\) and covariance matrix \(C\).

2 **Specification**

```c
#include <nag.h>
#include <nagg05.h>
void nag_rand_matrix_multi_normal (Nag_OrderType order, Nag_ModeRNG mode,
                Integer n, Integer m, const double xmu[], const double c[],
                Integer pdc, double r[], Integer lr, Integer state[],
                double x[], Integer pdx, NagError *fail)
```

3 **Description**

When the covariance matrix is nonsingular (i.e., strictly positive definite), the distribution has probability density function

\[
    f(x) = \sqrt{\frac{|C^{-1}|}{(2\pi)^n}} \exp\left( -\frac{1}{2} (x-a)^T C^{-1} (x-a) \right)
\]

where \(m\) is the number of dimensions, \(C\) is the covariance matrix, \(a\) is the vector of means and \(x\) is the vector of positions.

Covariance matrices are symmetric and positive semidefinite. Given such a matrix \(C\), there exists a lower triangular matrix \(L\) such that \(LL^T = C\). \(L\) is not unique, if \(C\) is singular.

`nag_rand_matrix_multi_normal (g05rzc)` decomposes \(C\) to find such an \(L\). It then stores \(m\), \(a\) and \(L\) in the reference vector \(r\) which is used to generate a vector \(x\) of independent standard Normal pseudorandom numbers. It then returns the vector \(a + Lx\), which has the required multivariate Normal distribution.

It should be noted that this function will work with a singular covariance matrix \(C\), provided \(C\) is positive semidefinite, despite the fact that the above formula for the probability density function is not valid in that case. Wilkinson (1965) should be consulted if further information is required.

One of the initialization functions `nag_rand_init_repeatable (g05kfc)` (for a repeatable sequence if computed sequentially) or `nag_rand_init_nonrepeatable (g05kgc)` (for a non-repeatable sequence) must be called prior to the first call to `nag_rand_matrix_multi_normal (g05rzc)`.

4 **References**


5 **Arguments**

1. `order` – Nag_OrderType
   Input
   
   On entry: the `order` argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by

---

Mark 25
order = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: order = Nag_RowMajor or Nag_ColMajor.

2: mode – Nag_ModeRNG

On entry: a code for selecting the operation to be performed by the function.

mode = Nag.InitializeReference
Set up reference vector only.

mode = Nag.GenerateFromReference
Generate variates using reference vector set up in a prior call to nag_rand_matrix_multi_normal (g05rzc).

mode = Nag.InitializeAndGenerate
Set up reference vector and generate variates.


3: n – Integer

On entry: n, the number of random variates required.

Constraint: n ≥ 0.

4: m – Integer

On entry: m, the number of dimensions of the distribution.

Constraint: m > 0.

5: xmu[m] – const double

On entry: a, the vector of means of the distribution.

6: c[dim] – const double

Note: the dimension, dim, of the array c must be at least pdc × m.

The (i,j)th element of the matrix C is stored in

\[ c((j-1) \times pdc + i - 1) \]

when order = Nag_ColMajor;

\[ c((i-1) \times pdc + j - 1) \]

when order = Nag_RowMajor.

On entry: the covariance matrix of the distribution. Only the upper triangle need be set.

Constraint: C must be positive semidefinite to machine precision.

7: pdc – Integer

On entry: the stride separating row or column elements (depending on the value of order) in the array c.

Constraint: pdc ≥ m.

8: r[lr] – double

On entry: if mode = Nag.GenerateFromReference, the reference vector as set up by nag_rand_matrix_multi_normal (g05rzc) in a previous call with mode = Nag.InitializeReference or Nag.InitializeAndGenerate.

On exit: if mode = Nag.InitializeReference or Nag.InitializeAndGenerate, the reference vector that can be used in subsequent calls to nag_rand_matrix_multi_normal (g05rzc) with mode = Nag.GenerateFromReference.
9: \( l_r \) – Integer

Input

On entry: the dimension of the array \( r \). If \( \text{mode} = \text{Nag\_GenerateFromReference} \), it must be the same as the value of \( l_r \) specified in the prior call to \( \text{nag\_rand\_matrix\_multi\_normal} \) (g05rzg) with \( \text{mode} = \text{Nag\_InitializeReference} \) or \( \text{Nag\_InitializeAndGenerate} \).

Constraint: \( l_r \geq m \times (m + 1) + 1 \).

10: \( \text{state}[\text{dim}] \) – Integer

Communication Array

Note: the dimension, \( \text{dim} \), of this array is dictated by the requirements of associated functions that must have been previously called. This array MUST be the same array passed as argument \( \text{state} \) in the previous call to \( \text{nag\_rand\_init\_repeatable} \) (g05kfc) or \( \text{nag\_rand\_init\_nonrepeatable} \) (g05kgc).

On entry: contains information on the selected base generator and its current state.

On exit: contains updated information on the state of the generator.

11: \( x[\text{dim}] \) – double

Output

Note: the dimension, \( \text{dim} \), of the array \( x \) must be at least

\[
\max(1, \text{pdx} \times m) \quad \text{when} \quad \text{order} = \text{Nag\_ColMajor};
\]

\[
\max(1, n \times \text{pdx}) \quad \text{when} \quad \text{order} = \text{Nag\_RowMajor}.
\]

Where \( X(i, j) \) appears in this document, it refers to the array element \( x[(j - 1) \times \text{pdx} + i - 1] \) when \( \text{order} = \text{Nag\_ColMajor} \);

\( x[(i - 1) \times \text{pdx} + j - 1] \) when \( \text{order} = \text{Nag\_RowMajor} \).

On exit: the array of pseudorandom multivariate Normal vectors generated by the function, with \( X(i, j) \) holding the \( j \)th dimension for the \( i \)th variate.

12: \( \text{pdx} \) – Integer

Input

On entry: the stride used in the array \( x \).

Constraints:

if \( \text{order} = \text{Nag\_ColMajor} \), \( \text{pdx} \geq n \);

if \( \text{order} = \text{Nag\_RowMajor} \), \( \text{pdx} \geq m \).

13: \( \text{fail} \) – \text{NagError*}

Input/Output

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

\textbf{NE\_ALLOC\_FAIL}

Dynamic memory allocation failed.

See Section 3.2.1.2 in the Essential Introduction for further information.

\textbf{NE\_BAD\_PARAM}

On entry, argument \( \langle \text{value} \rangle \) had an illegal value.

\textbf{NE\_INT}

On entry, \( l_r \) is not large enough, \( l_r = \langle \text{value} \rangle \): minimum length required = \( \langle \text{value} \rangle \).

On entry, \( m = \langle \text{value} \rangle \).

Constraint: \( m > 0 \).

On entry, \( n = \langle \text{value} \rangle \).

Constraint: \( n \geq 0 \).

Mark 25
On entry, \( \text{pdc} = \langle \text{value} \rangle \) and \( \text{m} = \langle \text{value} \rangle \).
Constraint: \( \text{pdc} \geq \text{m} \).

On entry, \( \text{pdx} = \langle \text{value} \rangle \) and \( \text{m} = \langle \text{value} \rangle \).
Constraint: \( \text{pdx} \geq \text{m} \).

On entry, \( \text{pdx} = \langle \text{value} \rangle \) and \( \text{n} = \langle \text{value} \rangle \).
Constraint: \( \text{pdx} \geq \text{n} \).

**NE_INTERNAL_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

**NE_INVALID_STATE**

On entry, \( \text{state} \) vector has been corrupted or not initialized.

**NE_NO_LICENCE**

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

**NE_POS_DEF**

On entry, the covariance matrix \( C \) is not positive semidefinite to *machine precision*.

**NE_PREV_CALL**

\( \text{m} \) is not the same as when \( \text{r} \) was set up in a previous call.
Previous value of \( \text{m} = \langle \text{value} \rangle \) and \( \text{m} = \langle \text{value} \rangle \).

7 **Accuracy**

Not applicable.

8 **Parallelism and Performance**

\text{nag_rand_matrix_multi_normal} (g05rzc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

\text{nag_rand_matrix_multi_normal} (g05rzc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users’ Note for your implementation for any additional implementation-specific information.

9 **Further Comments**

The time taken by \text{nag_rand_matrix_multi_normal} (g05rzc) is of order \( nm^3 \).

It is recommended that the diagonal elements of \( C \) should not differ too widely in order of magnitude. This may be achieved by scaling the variables if necessary. The actual matrix decomposed is \( C + E = LL^T \), where \( E \) is a diagonal matrix with small positive diagonal elements. This ensures that, even when \( C \) is singular, or nearly singular, the Cholesky factor \( L \) corresponds to a positive definite covariance matrix that agrees with \( C \) within *machine precision*. 
This example prints ten pseudorandom observations from a multivariate Normal distribution with means vector

\[
\begin{bmatrix}
1.0 \\
2.0 \\
-3.0 \\
0.0
\end{bmatrix}
\]

and covariance matrix

\[
\begin{bmatrix}
1.69 & 0.39 & -1.86 & 0.07 \\
0.39 & 98.01 & -7.07 & -0.71 \\
-1.86 & -7.07 & 11.56 & 0.03 \\
0.07 & -0.71 & 0.03 & 0.01
\end{bmatrix}
\]

generated by nag_rand_matrix_multi_normal (g05rzc). All ten observations are generated by a single call to nag_rand_matrix_multi_normal (g05rzc) with \texttt{mode = Nag\_InitializeAndGenerate}. The random number generator is initialized by nag_rand_init_repeatable (g05kfc).

### 10.1 Program Text

```c
/* nag_rand_matrix_multi_normal (g05rzc) Example Program. */
/* Copyright 2014 Numerical Algorithms Group. */
/* Mark 9, 2009. */
/* Pre-processor includes */
#include <stdio.h>
#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg05.h>

#define X(I, J) x[(order == Nag_ColMajor)?(J*pdx + I):(I*pdx + J)]
#define C(I, J) c[(order == Nag_ColMajor)?(J*pdc + I):(I*pdc + J)]

int main(void)
{

    /* Integer scalar and array declarations */
    Integer exit_status = 0;
    Integer i, j, lstate, lr, x_size;
    Integer *state = 0;
    Integer pdx;

    /* NAG structures */
    NagError fail;
    Nag_ModeRNG mode;

    /* Double scalar and array declarations */
    double *r = 0, *x = 0;

    /* Use column major order */
    Nag_OrderType order = Nag_RowMajor;

    /* Set the number of variables and variates */
    Integer m = 4;
    Integer n = 10;

    /* Input the covariance matrix */
    double c[] = {
        1.69e0, 0.39e0, -1.86e0, 0.07e0, 
        0.39e0, 98.01e0, -7.07e0, -0.71e0, 
        -1.86e0, -7.07e0, 11.56e0, 0.03e0, 
        0.07e0, -0.71e0, 0.03e0, 0.01e0
    };
    Integer pdc = 4;

    
```

Mark 25
/* Input the means */
double xmu[] = { 1.0e0, 2.0e0, -3.0e0, 0.0e0 };

/* Choose the base generator */
Nag_BaseRNG genid = Nag_Basic;
Integer subid = 0;

/* Set the seed */
Integer seed[] = { 1762543 };
Integer lseed = 1;

/* Initialise the error structure */
INIT_FAIL(fail);

printf("nag_rand_matrix_multi_normal (g05rzc)"
" Example Program Results\n\n");

/* Get the length of the state array */
lstate = -1;
nag_rand_init_repeatable(genid, subid, seed, lseed, state, &lstate, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_rand_init_repeatable (g05kfc).
    fail.message);
    exit_status = 1;
    goto END;
}

pdx = (order == Nag_ColMajor)?n*m:
    x_size = (order == Nag_ColMajor)?pdx * m:
    lstate = m*m+m+1;

/* Allocate arrays */
if (!(r = NAG_ALLOC(lr, double)) ||
    !(x = NAG_ALLOC(x_size, double)) ||
    !(state = NAG_ALLOC(lstate, Integer)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Initialise the generator to a repeatable sequence */
nag_rand_init_repeatable(genid, subid, seed, lseed, state, &lstate, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_rand_init_repeatable (g05kfc).
    fail.message);
    exit_status = 1;
    goto END;
}

/* Set up reference vector and generate n numbers */
mode = Nag_InitializeAndGenerate;
nag_rand_matrix_multi_normal(order, mode, n, m, xmu,
    c, pdc, r, lr, state, x, pdx, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_rand_matrix_multi_normal (g05rzc).
    fail.message);
    exit_status = 1;
    goto END;
}

/* Display the variates */
for (i = 0; i < n; i++)
{
    printf(" ");
for (j = 0; j < m; j++)
    printf("%9.4f\n", X(i, j), (j+1)%10? "\n":"");
if (m%10) printf("\n");

END:
NAG_FREE(r);
NAG_FREE(x);
NAG_FREE(state);

return exit_status;
}

10.2 Program Data
None.

10.3 Program Results

nag_rand_matrix_multi_normal (g05rzc) Example Program Results

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4534</td>
<td>-14.1206</td>
<td>-3.7410</td>
<td>0.1184</td>
<td></td>
</tr>
<tr>
<td>-0.6191</td>
<td>-4.8000</td>
<td>-0.1473</td>
<td>-0.0304</td>
<td></td>
</tr>
<tr>
<td>1.8607</td>
<td>5.3206</td>
<td>-5.0753</td>
<td>0.0106</td>
<td></td>
</tr>
<tr>
<td>2.0861</td>
<td>-13.6996</td>
<td>-1.3451</td>
<td>0.1428</td>
<td></td>
</tr>
<tr>
<td>-0.6326</td>
<td>3.9729</td>
<td>0.5721</td>
<td>-0.0770</td>
<td></td>
</tr>
<tr>
<td>0.9754</td>
<td>-3.8162</td>
<td>-4.2978</td>
<td>0.0040</td>
<td></td>
</tr>
<tr>
<td>0.6174</td>
<td>-5.1573</td>
<td>2.5037</td>
<td>0.0772</td>
<td></td>
</tr>
<tr>
<td>2.0352</td>
<td>26.9359</td>
<td>2.2939</td>
<td>-0.0826</td>
<td></td>
</tr>
<tr>
<td>0.9941</td>
<td>14.7700</td>
<td>-1.0421</td>
<td>-0.0549</td>
<td></td>
</tr>
<tr>
<td>1.5780</td>
<td>2.8916</td>
<td>-2.1725</td>
<td>-0.0129</td>
<td></td>
</tr>
</tbody>
</table>