NAG Library Function Document
nag_rand_matrix_multi_students_t (g05ryc)

1 Purpose

nag_rand_matrix_multi_students_t (g05ryc) sets up a reference vector and generates an array of pseudorandom numbers from a multivariate Student’s $t$ distribution with $\nu$ degrees of freedom, mean vector $\mu$ and covariance matrix $\frac{\nu}{\nu-2}C$.

2 Specification

```c
#include <nag.h>
#include <nagg05.h>
void nag_rand_matrix_multi_students_t (Nag_OrderType order,
           Nag_ModeRNG mode, Integer n, Integer df, Integer m, const double xmu[],
           const double c[], Integer pdc, double r[], Integer lr, Integer state[],
           double x[], Integer pdx, NagError *fail)
```

3 Description

When the covariance matrix is nonsingular (i.e., strictly positive definite), the distribution has probability density function

$$f(x) = \frac{\Gamma\left(\frac{\nu+m}{2}\right)}{(\pi\nu)^{m/2} \Gamma(\nu/2)|C|^{\frac{1}{2}}} \left[1 + \frac{(x-a)^T C^{-1} (x-a)}{\nu}\right]^{-\frac{\nu+m}{2}}$$

where $m$ is the number of dimensions, $\nu$ is the degrees of freedom, $\mu$ is the vector of means, $x$ is the vector of positions and $\frac{\nu}{\nu-2}C$ is the covariance matrix.

The function returns the value

$$x = \mu + \sqrt{\frac{\nu}{s}} z$$

where $z$ is generated by nag_rand_normal (g05skc) from a Normal distribution with mean zero and covariance matrix $C$ and $s$ is generated by nag_rand_chi_sq (g05sdc) from a $\chi^2$-distribution with $\nu$ degrees of freedom.

One of the initialization functions nag_rand_init_repeatable (g05kfc) (for a repeatable sequence if computed sequentially) or nag_rand_init_nonrepeatable (g05kgc) (for a non-repeatable sequence) must be called prior to the first call to nag_rand_matrix_multi_students_t (g05ryc).

4 References


5 Arguments

1: order – Nag_OrderType

   Input

   On entry: the order argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by
order = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: order = Nag_RowMajor or Nag_ColMajor.

2: mode – Nag_ModeRNG

On entry: a code for selecting the operation to be performed by the function.

mode = Nag.InitializeReference
Set up reference vector only.

mode = Nag.GenerateFromReference
Generate variates using reference vector set up in a prior call to
nag_rand_matrix_multi_students_t (g05ryc).

mode = Nag.InitializeAndGenerate
Set up reference vector and generate variates.

Constraint: mode = Nag.InitializeReference, Nag.GenerateFromReference or
Nag.InitializeAndGenerate.

3: n – Integer

On entry: n, the number of random variates required.

Constraint: n ≥ 0.

4: df – Integer

On entry: ν, the number of degrees of freedom of the distribution.

Constraint: df ≥ 3.

5: m – Integer

On entry: m, the number of dimensions of the distribution.

Constraint: m > 0.

6: xmu[m] – const double

On entry: a, the vector of means of the distribution.

7: c[dim] – const double

Note: the dimension, dim, of the array c must be at least pdc × m.

The (i, j)th element of the matrix C is stored in

c[(j - 1) × pdc + i - 1] when order = Nag_ColMajor;
c[(i - 1) × pdc + j - 1] when order = Nag_RowMajor.

On entry: matrix which, along with df, defines the covariance of the distribution. Only the upper triangle need be set.

Constraint: c must be positive semidefinite to machine precision.

8: pdc – Integer

On entry: the stride separating row or column elements (depending on the value of order) in the
array c.

Constraint: pdc ≥ m.
On entry: if \texttt{mode} = \texttt{Nag\_GenerateFromReference}, the reference vector as set up by \texttt{nag\_rand\_matrix\_multi\_students\_t (g05ryc)} in a previous call with \texttt{mode} = \texttt{Nag\_InitializeReference} or \texttt{Nag\_InitializeAndGenerate}.

On exit: if \texttt{mode} = \texttt{Nag\_InitializeReference} or \texttt{Nag\_InitializeAndGenerate}, the reference vector that can be used in subsequent calls to \texttt{nag\_rand\_matrix\_multi\_students\_t (g05ryc)} with \texttt{mode} = \texttt{Nag\_GenerateFromReference}.

\textbf{lr} – Integer

On entry: the dimension of the array \texttt{r}. If \texttt{mode} = \texttt{Nag\_GenerateFromReference}, it must be the same as the value of \texttt{lr} specified in the prior call to \texttt{nag\_rand\_matrix\_multi\_students\_t (g05ryc)} with \texttt{mode} = \texttt{Nag\_InitializeReference} or \texttt{Nag\_InitializeAndGenerate}.

\text{Constraint: } \texttt{lr} \geq \texttt{m} \times (\texttt{m} + 1) + 2.

\textbf{state[\texttt{dim}]} – Integer

\textit{Communication Array}

Note: the dimension, \texttt{dim}, of this array is dictated by the requirements of associated functions that must have been previously called. This array MUST be the same array passed as argument \texttt{state} in the previous call to \texttt{nag\_rand\_init\_repeatable (g05kfc)} or \texttt{nag\_rand\_init\_nonrepeatable (g05kgc)}.

On entry: contains information on the selected base generator and its current state.

On exit: contains updated information on the state of the generator.

\textbf{x[\texttt{dim}]} – double

\textit{Output}

Note: the dimension, \texttt{dim}, of the array \texttt{x} must be at least

\begin{align*}
\max(1, \texttt{pdx} \times \texttt{m}) & \text{ when } \texttt{order} = \texttt{Nag\_ColMajor}; \\
\max(1, \texttt{n} \times \texttt{pdx}) & \text{ when } \texttt{order} = \texttt{Nag\_RowMajor}.
\end{align*}

Where \texttt{X(i, j)} appears in this document, it refers to the array element

\begin{align*}
\texttt{x[(j - 1) \times \texttt{pdx} + i - 1]} & \text{ when } \texttt{order} = \texttt{Nag\_ColMajor}; \\
\texttt{x[(i - 1) \times \texttt{pdx} + j - 1]} & \text{ when } \texttt{order} = \texttt{Nag\_RowMajor}.
\end{align*}

On exit: the array of pseudorandom multivariate Student’s \(t\) vectors generated by the function, with \texttt{X(i, j)} holding the \(j\)th dimension for the \(i\)th variate.

\textbf{pdx} – Integer

\textit{Input}

On entry: the stride used in the array \texttt{x}.

\textbf{fail} – NagError*

\textit{Input/Output}

The NAG error argument (see Section 3.6 in the Essential Introduction).

\section{Error Indicators and Warnings}

\textbf{NE\_ALLOC\_FAIL}

Dynamic memory allocation failed.

See Section 3.2.1.2 in the Essential Introduction for further information.

\textbf{NE\_BAD\_PARAM}

On entry, argument \texttt{\langle value\rangle} had an illegal value.
NE_INT
On entry, \(df = \langle value\rangle\).
Constraint: \(df \geq 3\).

On entry, \(lr\) is not large enough, \(lr = \langle value\rangle\): minimum length required = \(\langle value\rangle\).

On entry, \(m = \langle value\rangle\).
Constraint: \(m > 0\).

On entry, \(n = \langle value\rangle\).
Constraint: \(n \geq 0\).

NE_INT_2
On entry, \(pdc = \langle value\rangle\) and \(m = \langle value\rangle\).
Constraint: \(pdc \geq m\).

On entry, \(pdx = \langle value\rangle\) and \(m = \langle value\rangle\).
Constraint: \(pdx \geq m\).

On entry, \(pdx = \langle value\rangle\) and \(n = \langle value\rangle\).
Constraint: \(pdx \geq n\).

NE_INTERNAL_ERROR
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG. See Section 3.6.6 in the Essential Introduction for further information.

NE_INVALID_STATE
On entry, state vector has been corrupted or not initialized.

NE_NO_LICENCE
Your licence key may have expired or may not have been installed correctly. See Section 3.6.5 in the Essential Introduction for further information.

NE_POS_DEF
On entry, the covariance matrix \(C\) is not positive semidefinite to machine precision.

NE_PREV_CALL
\(m\) is not the same as when \(r\) was set up in a previous call.
Previous value of \(m = \langle value\rangle\) and \(m = \langle value\rangle\).

7 Accuracy
Not applicable.

8 Parallelism and Performance
\texttt{nag\_rand\_matrix\_multi\_students\_t} \((\texttt{g05ryc})\) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

\texttt{nag\_rand\_matrix\_multi\_students\_t} \((\texttt{g05ryc})\) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users’ Note for your implementation for any additional implementation-specific information.
9 Further Comments

The time taken by nag_rand_matrix_multi_students_t (g05ryc) is of order $nm^3$.

It is recommended that the diagonal elements of $C$ should not differ too widely in order of magnitude. This may be achieved by scaling the variables if necessary. The actual matrix decomposed is $C + E = LL^T$, where $E$ is a diagonal matrix with small positive diagonal elements. This ensures that, even when $C$ is singular, or nearly singular, the Cholesky factor $L$ corresponds to a positive definite covariance matrix that agrees with $C$ within machine precision.

10 Example

This example prints ten pseudorandom observations from a multivariate Student’s $t$-distribution with ten degrees of freedom, means vector

$$
\begin{pmatrix}
1.0 \\
2.0 \\
-3.0 \\
0.0
\end{pmatrix}
$$

and $c$ matrix

$$
\begin{pmatrix}
1.69 & 0.39 & -1.86 & 0.07 \\
0.39 & 98.01 & -7.07 & -0.71 \\
-1.86 & -7.07 & 11.56 & 0.03 \\
0.07 & -0.71 & 0.03 & 0.01
\end{pmatrix}
$$

generated by nag_rand_matrix_multi_students_t (g05ryc). All ten observations are generated by a single call to nag_rand_matrix_multi_students_t (g05ryc) with $\text{mode} = \text{Nag\_InitializeAndGenerate}$. The random number generator is initialized by nag_rand_init_repeatable (g05kfc).

10.1 Program Text

/* nag_rand_matrix_multi_students_t (g05ryc) Example Program. */
/* Copyright 2014 Numerical algorithms Group. */
/* Mark 9, 2009. */
/* Pre-processor includes */
#include <stdio.h>
#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg05.h>

#define X(I, J) x[(order == Nag_ColMajor)?(J*pdx + I):(I*pdx + J)]
#define C(I, J) c[(order == Nag_ColMajor)?(J*pdc + I):(I*pdc + J)]

int main(void)
{
    /* Integer scalar and array declarations */
    Integer exit_status = 0;
    Integer i, j, lstate, lr, x_size;
    Integer *state = 0;
    Integer pdx;

    /* NAG structures */
    NagError fail;
    Nag_ModeRNG mode;

    /* Double scalar and array declarations */
    double *r = 0, *x = 0;

    /* Use column major order */

Nag_OrderType order = Nag_RowMajor;

/* Set the number of variables and variates */
Integer m = 4;
Integer n = 10;

/* Input the covariance matrix */
double c[] = { 1.69e0, 0.39e0, -1.86e0, 0.07e0,
0.39e0, 98.01e0, -7.07e0, -0.71e0,
-1.86e0, -7.07e0, 11.56e0, 0.03e0,
0.07e0, -0.71e0, 0.03e0, 0.01e0};

Integer pdc = 4;

/* Input the means */
double xmu[] = { 1.0e0, 2.0e0, -3.0e0, 0.0e0};

/* Set the degrees of freedom*/
Integer df = 10;

/* Choose the base generator */
Nag_BaseRNG genid = Nag_Basic;
Integer subid = 0;

/* Set the seed */
Integer seed[] = { 1762543};

/* Set the seed */
Integer lseed = 1;

/* Initialise the error structure */
INIT_FAIL(fail);

printf("nag_rand_matrix_multi_students_t (g05ryc) "
"Example Program Results\n\n");

/* Get the length of the state array */
lstate = -1;
NAG_RAND_INIT_REPEATABLE(genid, subid, seed, lseed, state, &lstate, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_rand_init_repeatable (g05kfc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
pdx = (order == Nag_ColMajor)?n:m;
x_size = (order == Nag_ColMajor)?pdx * m:pdx * n;

/* Calculate the size of the reference vector */
lr = m*m+m+2;

/* Allocate arrays */
if (!((r = NAG_ALLOC(lr, double)) ||
    (x = NAG_ALLOC(x_size, double)) ||
    (state = NAG_ALLOC(lstate, Integer)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Initialise the generator to a repeatable sequence */
NAG_RAND_INIT_REPEATABLE(genid, subid, seed, lseed, state, &lstate, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_rand_init_repeatable (g05kfc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Set up reference vector and generate N numbers */
mode = Nag_InitializeAndGenerate;
nag_rand_matrix_multi_students_t(order, mode, n, df, m, xmu, c, pdc, r,
   lr, state, x, pdx, &fail);
if (fail.code != NE_NOERROR)
{
   printf("Error from nag_rand_matrix_multi_students_t (g05ryc).
   fail.message); 
   exit_status = 1;
   goto END;
}
/* Display the variates */
for (i = 0; i < n; i++)
{
   printf(" ");
   for (j = 0; j < m; j++)
      printf("%9.4f%s", X(i, j), (j+1)%10?" ":"\n");
   if (m%10) printf("\n");
}
END:
NAG_FREE(r);
NAG_FREE(x);
NAG_FREE(state);

return exit_status;

10.2 Program Data
None.

10.3 Program Results
nag_rand_matrix_multi_students_t (g05ryc) Example Program Results

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4957</td>
<td>-15.6226</td>
<td>-3.8101</td>
<td>0.1294</td>
<td></td>
</tr>
<tr>
<td>-1.0827</td>
<td>-6.7473</td>
<td>0.6696</td>
<td>-0.0391</td>
<td></td>
</tr>
<tr>
<td>2.1369</td>
<td>6.3861</td>
<td>-5.7413</td>
<td>0.0140</td>
<td></td>
</tr>
<tr>
<td>2.2481</td>
<td>-16.0417</td>
<td>-1.0982</td>
<td>0.1641</td>
<td></td>
</tr>
<tr>
<td>-0.2550</td>
<td>3.5166</td>
<td>-0.2541</td>
<td>-0.0592</td>
<td></td>
</tr>
<tr>
<td>0.9731</td>
<td>-4.3553</td>
<td>-4.4181</td>
<td>0.0043</td>
<td></td>
</tr>
<tr>
<td>0.7098</td>
<td>-3.4281</td>
<td>1.1741</td>
<td>0.0586</td>
<td></td>
</tr>
<tr>
<td>1.8827</td>
<td>23.2619</td>
<td>1.5140</td>
<td>-0.0704</td>
<td></td>
</tr>
<tr>
<td>0.9904</td>
<td>22.7479</td>
<td>0.1811</td>
<td>-0.0893</td>
<td></td>
</tr>
<tr>
<td>1.5026</td>
<td>2.7753</td>
<td>-2.2805</td>
<td>-0.0112</td>
<td></td>
</tr>
</tbody>
</table>