NAG Library Function Document
nag_rand_copula_gumbel (g05rkc)

1 Purpose

nag_rand_copula_gumbel (g05rkc) generates pseudorandom uniform variates with joint distribution of a
Gumbel–Hougaard Archimedean copula.

2 Specification

#include <nag.h>
#include <nagg05.h>

void nag_rand_copula_gumbel (Nag_OrderType order, Integer state[],
double theta, Integer n, Integer m, double x[], Integer pdx,
Integer sdx, NagError *fail)

3 Description

Generates \( n \) pseudorandom uniform \( m \)-variates whose joint distribution is the Gumbel–Hougaard
Archimedean copula \( C_\theta \), given by

\[
C_\theta = \exp\left\{ - \left[ (-\ln u_1)^\theta + (-\ln u_2)^\theta + \cdots + (-\ln u_m)^\theta \right] \right\}, \quad \begin{cases} \theta \in (1, \infty), \\ u_j \in (0, 1], \quad j = 1, 2, \ldots m; \end{cases}
\]

with the special cases:

\( C_1 = u_1 u_2 \cdots u_m \), the product copula;
\( C_\infty = \min(u_1, u_2, \ldots, u_m) \), the Fréchet–Hoeffding upper bound.

The generation method uses mixture of powers.

One of the initialization functions nag_rand_init_repeatable (g05kfc) (for a repeatable sequence if
computed sequentially) or nag_rand_init_nonrepeatable (g05kgc) (for a non-repeatable sequence) must
be called prior to the first call to nag_rand_copula_gumbel (g05rkc).

4 References

Statistical Association 83 403


5 Arguments

1: order – Nag_OrderType

Input

*On entry* the order argument specifies the two-dimensional storage scheme being used, i.e., row-
major ordering or column-major ordering. C language defined storage is specified by
order = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed
explanation of the use of this argument.

*Constraint:* order = Nag_RowMajor or Nag_ColMajor.

2: state[\( \text{dim} \)] – Integer

Communication Array

*Note:* the dimension, \( \text{dim} \), of this array is dictated by the requirements of associated functions that
must have been previously called. This array MUST be the same array passed as argument state in
the previous call to nag_rand_init_repeatable (g05kfc) or nag_rand_init_nonrepeatable (g05kgc).
On entry: contains information on the selected base generator and its current state.

On exit: contains updated information on the state of the generator.

3: theta – double

On entry: \( \theta \), the copula parameter.

Constraint: \( \theta \geq 1.0 \).

4: n – Integer

On entry: \( n \), the number of pseudorandom uniform variates to generate.

Constraint: \( n \geq 0 \).

5: m – Integer

On entry: \( m \), the number of dimensions.

Constraint: \( m \geq 2 \).

6: \( x[pdx \times sdx] \) – double

Output

Note: where \( X(i, j) \) appears in this document, it refers to the array element

\[
x[(j-1) \times pdx + i - 1] \text{ when } order = \text{Nag\_ColMajor};
\]

\[
x[(i-1) \times pdx + j - 1] \text{ when } order = \text{Nag\_RowMajor}.
\]

On exit: the pseudorandom uniform variates with joint distribution described by \( C_\theta \), with \( X(i, j) \) holding the \( i \)th value for the \( j \)th dimension if \( order = \text{Nag\_ColMajor} \) and the \( j \)th value for the \( i \)th dimension of \( order = \text{Nag\_RowMajor} \).

7: pdx – Integer

Input

On entry: the stride separating row or column elements (depending on the value of \( order \)) in the array \( x \).

Constraints:

if \( order = \text{Nag\_ColMajor} \), \( pdx \geq n \);

if \( order = \text{Nag\_RowMajor} \), \( pdx \geq m \).

8: sdx – Integer

Input

On entry: the secondary dimension of \( X \).

Constraints:

if \( order = \text{Nag\_ColMajor} \), \( sdx \geq m \);

if \( order = \text{Nag\_RowMajor} \), \( sdx \geq n \).

9: fail – NagError*

Input/Output

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM

On entry, argument \( \langle \text{value} \rangle \) had an illegal value.
**NE_INT**

On entry, \( m \equiv \langle \text{value} \rangle \).
Constraint: \( m \geq 2 \).

On entry, \( n \equiv \langle \text{value} \rangle \).
Constraint: \( n \geq 0 \).

**NE_INT_2**

On entry, \( \text{pdx} \) must be at least \( \langle \text{value} \rangle \): \( \text{pdx} = \langle \text{value} \rangle \).

On entry, \( \text{sdx} \) must be at least \( \langle \text{value} \rangle \): \( \text{sdx} = \langle \text{value} \rangle \).

**NE_INTERNAL_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

**NE_INVALID_STATE**

On entry, corrupt \text{state} argument.

**NE_NO_LICENCE**

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

**NE_REAL**

On entry, invalid \( \text{theta} \): \( \text{theta} = \langle \text{value} \rangle \).
Constraint: \( \text{theta} \geq 1.0 \).

7 **Accuracy**

Not applicable.

8 **Parallelism and Performance**

nag_rand_copula_gumbel (g05rkc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users’ Note for your implementation for any additional implementation-specific information.

9 **Further Comments**

In practice, the need for numerical stability restricts the range of \( \theta \) such that:

- if \( (\theta - 1) < 1.0 \times 10^{-6} \), the function returns pseudorandom uniform variates with \( C_1 \) joint distribution;
- if \( \theta > \max(80.0, -0.5 \ln \epsilon_s) \), the function returns pseudorandom uniform variates with \( C_\infty \) joint distribution;

where \( \epsilon_s \) is the safe-range parameter, the value of which is returned by nag_real_safe_small_number (X02AMC).
10 Example

This example generates thirteen four-dimensional variates for copula $C_{2,4}$.

10.1 Program Text

```c
/* nag_rand_copula_gumbel (g05rkc) Example Program. */
/* Copyright 2014 Numerical Algorithms Group. */
/* Mark 9, 2009. */
/* Pre-processor includes */
#include <stdio.h>
#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nag_g05.h>

#define X(I, J) x[order == Nag_ColMajor?((J-1)*pdx + I-1):((I-1)*pdx + J-1)]

int main(void)
{
  /* Integer scalar and array declarations */
  Integer exit_status = 0;
  Integer i, j, lstate, pdx, sdx;
  Integer *state = 0;
  /* Double scalar and array declarations */
  double *x = 0;
  /* NAG structures */
  NagError fail;

  /* Use row major order */
  Nag_OrderType order = Nag_RowMajor;

  /* Set the number of variables and variates */
  Integer n = 13, m = 4;

  /* Choose the base generator */
  Nag_BaseRNG genid = Nag_Basic;
  Integer subid = 0;

  /* Set the seed */
  Integer seed[] = { 1762543 };  
  Integer lseed = 1;

  /* Set the theta parameter value */
  double theta = 2.4e0;

  /* Initialise the error structure */
  INIT_FAIL(fail);

  printf(
    "nag_rand_copula_gumbel (g05rkc) Example Program Results\\n\\n"
    );

  /* Get the length of the state array */
  lstate = -1;
  nag_rand_init_repeatable(genid, subid, seed, lseed, state, &lstate, &fail);
  if (fail.code != NE_NOERROR)
  {
    printf("Error from nag_rand_init_repeatable (g05kf).
    ", fail.message);
    exit_status = 1;
    goto END;
  }

  /* Set matrix size and principal dimension according to storage order */
```
pdx = (order == Nag_ColMajor)?n:m;
sdx = (order == Nag_ColMajor)?m:n;

/* Allocate arrays */
if (!(x = NAG_ALLOC((pdx*sdx), double)) ||
    !(state = NAG_ALLOC(lstate, Integer)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Initialise the generator to a repeatable sequence */
nag_rand_init_repeatable(genid, subid, seed, lseed, state, &lstate, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_rand_init_repeatable (g05kfc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Generate variates */
nag_rand_copula_gumbel(order, state, theta, n, m, x, pdx, sdx, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_rand_copula_gumbel (g05rkc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Display the results */
printf("Uniform variates with copula joint distribution\n");
for (i = 1; i <= n; i++)
{
    printf(" ");
    for (j = 1; j <= m; j++)
    {
        printf("%9.6f%s", X(i, j), j < m?" ":"\n");
    }
}

END:
NAG_FREE(x);
NAG_FREE(state);

    return exit_status;
}

10.2 Program Data

None.

10.3 Program Results

nag_rand_copula_gumbel (g05rkc) Example Program Results

Uniform variates with copula joint distribution
0.936870 0.867579 0.971315 0.885436
0.113937 0.306325 0.862550 0.274299
0.441848 0.221141 0.504169 0.498465
0.790226 0.600734 0.749311 0.647411
0.836246 0.984692 0.880681 0.907863
0.178145 0.460986 0.128349 0.132867
0.127161 0.175970 0.180519 0.038261
0.447304 0.217106 0.166239 0.130033
<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.889915</td>
<td>0.900532</td>
<td>0.884357</td>
<td>0.887901</td>
<td></td>
</tr>
<tr>
<td>0.906913</td>
<td>0.868125</td>
<td>0.844986</td>
<td>0.880437</td>
<td></td>
</tr>
<tr>
<td>0.222224</td>
<td>0.549883</td>
<td>0.496503</td>
<td>0.648758</td>
<td></td>
</tr>
<tr>
<td>0.380733</td>
<td>0.596745</td>
<td>0.509628</td>
<td>0.357674</td>
<td></td>
</tr>
<tr>
<td>0.844530</td>
<td>0.775454</td>
<td>0.866096</td>
<td>0.894776</td>
<td></td>
</tr>
</tbody>
</table>