NAG Library Function Document
nag_rand_bivariate_copula_plackett (g05rgc)

1 Purpose
nag_rand_bivariate_copula_plackett (g05rgc) generates pseudorandom uniform bivariates with joint distribution of a Plackett copula.

2 Specification
#include <nag.h>
#include <nagg05.h>
void nag_rand_bivariate_copula_plackett (Nag_OrderType order,
        Integer state[], double theta, Integer n, double x[], Integer pdx,
        Integer sdx, NagError *fail)

3 Description
Generates pseudorandom uniform bivariates \( u_1, u_2 \in [0,1]^2 \) whose joint distribution is the Plackett copula \( C_\theta \) with parameter \( \theta \), given by
\[
C_\theta = \frac{[1 + (\theta - 1)(u_1 + u_2)] - \sqrt{[1 + (\theta - 1)(u_1 + u_2)]^2 - 4u_1u_2\theta(\theta - 1)}}{2(\theta - 1)}, \quad \theta \in (0, \infty) \setminus \{1\}
\]
with the special cases:
- \( C_0 = \max(u_1 + u_2 - 1, 0) \), the Fréchet–Hoeffding lower bound;
- \( C_1 = u_1u_2 \), the product copula;
- \( C_\infty = \min(u_1, u_2) \), the Fréchet–Hoeffding upper bound.

The generation method uses conditional sampling.

One of the initialization functions nag_rand_init_repeatable (g05kfc) (for a repeatable sequence if computed sequentially) or nag_rand_init_nonrepeatable (g05kgc) (for a non-repeatable sequence) must be called prior to the first call to nag_rand_bivariate_copula_plackett (g05rgc).

4 References

5 Arguments
1: order – Nag_OrderType
   Input
   On entry: the order argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by order = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.
   Constraint: order = Nag_RowMajor or Nag_ColMajor.

2: state[dim] – Integer
   Communication Array
   Note: the dimension, dim, of this array is dictated by the requirements of associated functions that must have been previously called. This array MUST be the same array passed as argument state in the previous call to nag_rand_init_repeatable (g05kfc) or nag_rand_init_nonrepeatable (g05kgc).
On entry: contains information on the selected base generator and its current state.
On exit: contains updated information on the state of the generator.

3: \( \theta \) – double
   \textit{Input}
   On entry: \( \theta \), the copula parameter.
   Constraint: \( \theta \geq 0.0 \).

4: \( n \) – Integer
   \textit{Input}
   On entry: \( n \), the number of bivariates to generate.
   Constraint: \( n \geq 0 \).

5: \( X[pdx \times sdx] \) – double
   \textit{Output}
   Note: where \( X(i, j) \) appears in this document, it refers to the array element
   \[ X[(j - 1) \times \text{pdx} + i - 1] \]
   when \( \text{order} = \text{Nag\_ColMajor} \);
   \[ X[(i - 1) \times \text{pdx} + j - 1] \]
   when \( \text{order} = \text{Nag\_RowMajor} \).
   On exit: the \( n \) bivariate uniforms with joint distribution described by \( C_0 \), with \( X(i, j) \) holding the
   \( i \)th value for the \( j \)th dimension if \( \text{order} = \text{Nag\_ColMajor} \) and the \( j \)th value for the \( i \)th dimension
   if \( \text{order} = \text{Nag\_RowMajor} \).

6: \( pdx \) – Integer
   \textit{Input}
   On entry: the stride separating row or column elements (depending on the value of \( \text{order} \)) in the
   array \( x \).
   Constraints:
   - if \( \text{order} = \text{Nag\_ColMajor} \), \( pdx \geq n \);
   - if \( \text{order} = \text{Nag\_RowMajor} \), \( pdx \geq 2 \).

7: \( sdx \) – Integer
   \textit{Input}
   On entry: the secondary dimension of \( X \).
   Constraints:
   - if \( \text{order} = \text{Nag\_ColMajor} \), \( sdx \geq 2 \);
   - if \( \text{order} = \text{Nag\_RowMajor} \), \( sdx \geq n \).

8: \( \text{fail} \) – NagError*
   \textit{Input/Output}
   The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL
Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM
On entry, argument \( \langle \text{value} \rangle \) had an illegal value.

NE_INT
On entry, \( n = \langle \text{value} \rangle \).
Constraint: \( n \geq 0 \).
NE_INT_2
On entry, pdx must be at least \langle value\rangle: \texttt{pdx = \langle value\rangle}.
On entry, sdx must be at least \langle value\rangle: \texttt{sdx = \langle value\rangle}.

NE_INTERNAL_ERROR
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.
An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_INVALID_STATE
On entry, corrupt state argument.

NE_NO_LICENCE
Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

NE_REAL
On entry, invalid theta: \texttt{theta = \langle value\rangle}.
Constraint: \texttt{theta} \geq 0.0.

7 Accuracy
Not applicable.

8 Parallelism and Performance
nag_rand_bivariate_copula_plackett (g05rgc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.
Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users’ Note for your implementation for any additional implementation-specific information.

9 Further Comments
In practice, the need for numerical stability restricts the range of \theta such that:
if \theta < \epsilon_s, the function returns pseudorandom uniform variates with \texttt{C0} joint distribution;
if |\theta - 1| < \epsilon, the function returns pseudorandom uniform variates with \texttt{C1} joint distribution;
if \theta > \epsilon_s^{-1/2}, the function returns pseudorandom uniform variates with \texttt{C\infty} joint distribution;
where \epsilon_s is the safe-range parameter, the value of which is returned by nag_real_safe_small_number (X02AMC); and \epsilon is the \textit{machine precision} returned by nag_machine_precision (X02AJC).

10 Example
This example generates thirteen variates for copula \texttt{C2.0}.
10.1 Program Text

/* nag_rand_bivariate_copula_plackett (g05rgc) Example Program. */
/* Copyright 2014 Numerical Algorithms Group. */
/* Mark 9, 2009. */
/* Pre-processor includes */
#include <stdio.h>
#include <math.h>
#include <nag.h>
#include <nagg05.h>
#define X(I, J) x[order == Nag_ColMajor?((J-1)*pdx + I-1):((I-1)*pdx + J-1)]

int main(void)
{
  /* Integer scalar and array declarations */
  Integer exit_status = 0;
  Integer i, lstate, pdx, sdx;
  Integer *state = 0;
  /* Double scalar and array declarations */
  double *x = 0;
  /* NAG structures */
  NagError fail;

  /* Use row major order */
  Nag_OrderType order = Nag_RowMajor;

  /* Set the number of variates */
  Integer n = 13;

  /* Choose the base generator */
  Nag_BaseRNG genid = Nag_Basic;
  Integer subid = 0;

  /* Set the seed */
  Integer seed[] = { 1762543 };
  Integer lseed = 1;

  /* Set the theta parameter value */
  double theta = 2.0e0;

  /* Initialise the error structure */
  INIT_FAIL(fail);
  printf("nag_rand_bivariate_copula_plackett (g05rgc) "
         "Example Program Results\n\n");

  /* Get the length of the state array */
  lstate = -1;
  nag_rand_init_repeatable(genid, subid, seed, lseed, state, &lstate, &fail);
  if (fail.code != NE_NOERROR)
  {
    printf("Error from nag_rand_init_repeatable (g05kfc).\n\n",
            fail.message);
    exit_status = 1;
    goto END;
  }

  /* Set matrix size and principal dimension according to storage order */
  pdx = (order == Nag_ColMajor)?n:2;
  sdx = (order == Nag_ColMajor)?2:n;

  /* Allocate arrays */

END:
exit_status = exit_status;
return exit_status;
}
if (!(x = NAG_ALLOC((pdx*sdx), double)) ||
   !(state = NAG_ALLOC(lstate, Integer)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Initialise the generator to a repeatable sequence */
   nag_rand_init_repeatable(genid, subid, seed, lseed, state, &lstate, &fail);
   if (fail.code != NE_NOERROR)
   {
      printf("Error from nag_rand_init_repeatable (g05kfc).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
   }

/* Generate variates */
   nag_rand_bivariate_copula_plackett(order, state, theta, n, x, pdx, sdx,
      &fail);
   if (fail.code != NE_NOERROR)
   {
      printf("Error from nag_rand_bivariate_copula_plackett (g05rgc).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
   }

/* Display the results */
   printf("Uniform variates with copula joint distrbution\n");
   for (i = 1; i <= n; i++)
   {
      printf(" %9.6f %9.6f\n", X(i, 1), X(i, 2));
   }

END:
   NAG_FREE(x);
   NAG_FREE(state);

   return exit_status;

10.2 Program Data

None.

10.3 Program Results

nag_rand_bivariate_copula_plackett (g05rgc) Example Program Results

Uniform variates with copula joint distribution

0.636373  0.069548
0.106480  0.458594
0.746035  0.358630
0.798270  0.326669
0.104592  0.988775
0.492451  0.891998
0.384260  0.490260
0.787070  0.424763
0.498187  0.578279
0.671703  0.441908
0.050537  0.580173
0.258030  0.662903
0.623806  0.329071