NAG Library Function Document

nag_mv_ordinal_multidimscale (g03fcc)

1 Purpose
nag_mv_ordinal_multidimscale (g03fcc) performs non-metric (ordinal) multidimensional scaling.

2 Specification
#include <nag.h>
#include <nagg03.h>

void nag_mv_ordinal_multidimscale (Nag_ScaleCriterion type, Integer n,
       Integer ndim, const double d[], double x[], Integer tdx, double *stress,
       double dfit[], Nag_E04_Opt *options, NagError *fail)

3 Description
For a set of \( n \) objects, a distance or dissimilarity matrix \( D \) can be calculated such that \( d_{ij} \) is a measure of how ‘far apart’ objects \( i \) and \( j \) are. If \( p \) variables \( x_k \) have been recorded for each observation this measure may be based on Euclidean distance, \( d_{ij} = \sum_{k=1}^{p} (x_{ki} - x_{kj})^2 \), or some other calculation such as the number of variables for which \( x_{kj} \neq x_{ki} \). Alternatively, the distances may be the result of a subjective assessment. For a given distance matrix, multidimensional scaling produces a configuration of \( n \) points in a chosen number of dimensions, \( m \), such that the distance between the points in some way best matches the distance matrix. For some distance measures, such as Euclidean distance, the size of distance is meaningful, for other measures of distance all that can be said is that one distance is greater or smaller than another. For the former, metric scaling can be used, see nag_mv_prin_coord_analysis (g03fac), for the latter, a non-metric scaling is more appropriate.

For non-metric multidimensional scaling, the criterion used to measure the closeness of the fitted distance matrix to the observed distance matrix is known as stress. Stress is given by,

\[
\sqrt{\frac{\sum_{i=1}^{n} \sum_{j=1}^{n} (d_{ij} - \bar{d}_{ij})^2}{\sum_{i=1}^{n} \sum_{j=1}^{n} \bar{d}_{ij}^2}}
\]

where \( \bar{d}_{ij}^2 \) is the Euclidean squared distance between points \( i \) and \( j \) and \( \bar{d}_{ij} \) is the fitted distance obtained when \( \bar{d}_{ij} \) is monotonically regressed on \( d_{ij} \), that is, \( \bar{d}_{ij} \) is monotonic relative to \( d_{ij} \) and is obtained from \( \bar{d}_{ij} \) with the smallest number of changes. So stress is a measure of by how much the set of points preserve the order of the distances in the original distance matrix. Non-metric multidimensional scaling seeks to find the set of points that minimize the stress.

An alternate measure is squared stress, SSTRESS,

\[
\sqrt{\frac{\sum_{i=1}^{n} \sum_{j=1}^{n-1} (d_{ij}^2 - \bar{d}_{ij}^2)^2}{\sum_{i=1}^{n} \sum_{j=1}^{n} \bar{d}_{ij}^2}}
\]

in which the distances in stress are replaced by squared distances.
In order to perform a non-metric scaling, an initial configuration of points is required. This can be obtained from principal coordinate analysis, see nag_mv_prin_coord_analysis (g03fac). Given an initial configuration, nag_mv_ordinal_multidimscale (g03fcc) uses the optimization function nag_opt_conj_grad (e04dgc) to find the configuration of points that minimizes stress or SSTRESS. The function nag_opt_conj_grad (e04dgc) uses a conjugate gradient algorithm. nag_mv_ordinal_multidimscale (g03fcc) will find an optimum that may only be a local optimum, to be more sure of finding a global optimum several different initial configurations should be used; these can be obtained by randomly perturbing the original initial configuration using functions from the g05 Chapter Introduction.

4 References

5 Arguments
1:  type – Nag_ScaleCriterion
    On entry: indicates whether stress or SSTRESS is to be used as the criterion.
    type = Nag_Stress
    stress is used.
    type = Nag_SSStress
    SSTRESS is used.
    Constraint: type = Nag_Stress or Nag_SSStress.

2:  n – Integer
    On entry: the number of objects in the distance matrix , n.
    Constraint: n > ndim.

3:  ndim – Integer
    On entry: the number of dimensions used to represent the data, m.
    Constraint: ndim ≥ 1.

4:  d[n × (n − 1)/2] – const double
    On entry: the lower triangle of the distance matrix D stored packed by rows. That is
    d[(i − 1) × (i − 2)/2 + j − 1] must contain d_{ij}, for i = 2, 3, . . . , n and j = 1, 2, . . . , i − 1. If d_{ij} is
    missing then set d_{ij} < 0; For further comments on missing values see Section 9.

5:  x[n × tdx] – double
    Note: the (i, j)th element of the matrix X is stored in x[(i − 1) × tdx + j − 1].
    On entry: the ith row must contain an initial estimate of the coordinates for the ith point,
    i = 1, 2, . . . , n. One method of computing these is to use nag_mv_prin_coord_analysis (g03fac).
    On exit: the ith row contains m coordinates for the ith point, i = 1, 2, . . . , n.

6:  tdx – Integer
    On entry: the stride separating matrix column elements in the array x.
    Constraint: tdx ≥ ndim.
7: stress – double *  
On exit: the value of stress or SSTRESS at the final iteration.

8: dfit[2 × n × (n – 1)] – double  
On exit: auxiliary outputs. If type = Nag_Stress, the first n(n – 1)/2 elements contain the distances, \( \hat{d}_{ij} \), for the points returned in \( x \), the second set of n(n – 1)/2 contains the distances \( \hat{d}_{ij} \) ordered by the input distances, \( d_{ij} \), the third set of n(n – 1)/2 elements contains the monotonic distances, \( \tilde{d}_{ij} \), ordered by the input distances, \( d_{ij} \) and the final set of n(n – 1)/2 elements contains fitted monotonic distances, \( \tilde{d}_{ij} \), for the points in \( x \). The \( \hat{d}_{ij} \) corresponding to distances which are input as missing are set to zero. If type = Nag_SSStress, the results are as above except that the squared distances are returned.

Each distance matrix is stored in lower triangular packed form in the same way as the input matrix \( D \).

9: options – Nag_E04_Opt *  
Input/Output
On entry/exit: a pointer to a structure of type Nag_E04_Opt whose members are optional arguments for nag_opt_conj_grad (e04dgc). These structure members offer the means of adjusting some of the argument values of the algorithm and on output will supply further details of the results. You are referred to the nag_opt_conj_grad (e04dgc) document for further details.

The default values used by nag_mv_ordinal_multidimscale (g03fcc) when the options argument is set to the NAG defined null pointer, E04_DEFAULT, are as follows:

\[
\begin{align*}
\text{optim_tol} & = 0.00001; \\
\text{print_level} & = \text{Nag_NoPrint}; \\
\text{list} & = \text{Nag_FALSE}; \\
\text{verify_grad} & = \text{Nag_FALSE}; \\
\text{max_iter} & = \text{max}(50, n \times ndim).
\end{align*}
\]

If a different value is required for any of these four structure members or if other options available in nag_opt_conj_grad (e04dgc) are to be used, then the structure options should be declared and initialized by a call to nag_opt_conj_grad (e04dgc) and supplied as an argument to nag_mv_ordinal_multidimscale (g03fcc). In this case, the structure members listed above except for list will have the default values as specified above; list = Nag_TRUE in this case.

10: fail – NagError *  
Input/Output
The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

**NE_2_INT_ARG_LE**

On entry, \( n = \langle \text{value} \rangle \) while \( ndim = \langle \text{value} \rangle \). These arguments must satisfy \( n > ndim \).

**NE_2_INT_ARG_LT**

On entry, \( tdx = \langle \text{value} \rangle \) while \( ndim = \langle \text{value} \rangle \). These arguments must satisfy \( tdx \geq ndim \).

**NE_ALLOC_FAIL**

Dynamic memory allocation failed.

**NE_BAD_PARAM**

On entry, argument type had an illegal value.
On entry, \( \text{ndim} = \langle \text{value} \rangle \).

Constraint: \( \text{ndim} \geq 1 \).

Additional error messages are output if the optimization fails to converge or if the options are set incorrectly. Details of these can be found in the \text{g04dgc} document.

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

All elements of array \( \text{d} \leq 0.0 \).

Constraint: At least one element of \( \text{d} \) must be positive.

After a successful optimization, the relative accuracy of \( \text{stress} \) should be approximately \( \epsilon \), as specified by \text{optim} tol.

Not applicable.

Missing values in the input distance matrix can be specified by a negative value and providing there are not more than about two thirds of the values missing, the algorithm may still work. However, the function \text{g03fac} does not allow for missing values so an alternative method of obtaining an initial set of coordinates is required. It may be possible to estimate the missing values with some form of average and then use \text{g03fac} to give an initial set of coordinates.

The data, given by Krzanowski (1990), are dissimilarities between water vole populations in Europe. Initial estimates are provided by the first two principal coordinates computed.

/* \text{g03fcc} Example Program. */
* Copyright 2014 Numerical Algorithms Group.
* Mark 6 revised, 2000.
* Mark 7 revised, 2001.
* /

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg03.h>

#define NMAX 14
#define NNMAX NMAX*(NMAX-1)/2
#define X(I, J) x[(I-1)*tdx + (J-1)]
```c
#define XTMP(I) xtmp[(I) -1]
#define YTMP(I) ytmp[(I) -1]

int main(void)
{
    Integer exit_status = 0, i, j, n, ndim, nn, tdx = NMAX;
    double *d = 0, *dfit = 0, *e = 0, stress, *x = 0;
    char nag_enum_arg[40];
    Nag_ScaleCriterion type;
    NagError fail;

    INIT_FAIL(fail);

    printf(
        "nag_mv_ordinal_multidimscale (g03fcc) Example Program Results\n\n";

    /* Skip heading in data file */
    ifndef _WIN32
        scanf("%*[\n"]);
    else
        scanf("%*[\n"]);
    endif
    ifndef _WIN32
        scanf("%"NAG_IFMT"", &n);
    else
        scanf("%"NAG_IFMT"", &n);
    endif
    ifndef _WIN32
        scanf("%39s", nag_enum_arg, _countof(nag_enum_arg));
    else
        scanf("%39s", nag_enum_arg);
    endif
    ifndef _WIN32
        scanf("%39s", nag_enum_arg, _countof(nag_enum_arg));
    else
        scanf("%39s", nag_enum_arg);
    endif
    /* nag_enum_name_to_value (x04nac).
     * Converts NAG enum member name to value 
     */
    type = (Nag_ScaleCriterion) nag_enum_name_to_value(nag_enum_arg);
    if (ndim >= 1 && n > ndim)
    {
        if (!(d = NAG_ALLOC(n*(n-1)/2, double)) ||
            ! (dfit = NAG_ALLOC(4*(n*(n-1)/2), double)) ||
            ! (e = NAG_ALLOC(n, double)) ||
            ! (x = NAG_ALLOC(n*n, double)))
        {
            printf("Allocation failure\n");
            exit_status = -1;
            goto END;
        }
        tdx = n;
    }
    else
    {
        printf("Invalid ndim or n.\n");
        exit_status = 1;
        return exit_status;
    }

    nn = n * (n - 1) / 2;
    for (i = 1; i <= nn; ++i)
    #ifndef _WIN32
        scanf("%lf", &d[i-1]);
    #else
        scanf("%lf", &d[i-1]);
    #endif
    /* nag_mv_prin_coord_analysis (g03fac).
     * Principal co-ordinate analysis 
     */
    ```
nag_mv_prin_coord_analysis(Nag_LargeEigVals, n, d, ndim, x, tdx, e, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_mv_prin_coord_analysis (g03fac).\n\n", fail.message);
    exit_status = 1;
    goto END;
}

/* nag_mv_ordinal_multidimscale (g03fcc).
 Multidimensional scaling
*/
nag_mv_ordinal_multidimscale(type, n, ndim, d, x, tdx, &stress, dfit, E04_DEFAULT, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_mv_ordinal_multidimscale (g03fcc).\n\n", fail.message);
    exit_status = 1;
    goto END;
}

printf("\n STRESS = 1.2557e-01\n\n");  
printf("Co-ordinates\n\n");
for (i = 1; i <= n; ++i)
{
    for (j = 1; j <= ndim; ++j)
        printf("%10.4f", X(i, j));
    printf("\n");
}

10.2 Program Data

nag_mv_ordinal_multidimscale (g03fcc) Example Program Data

14 2 Nag_Stress

0.099
0.033 0.022
0.183 0.114 0.042
0.148 0.224 0.059 0.068
0.198 0.039 0.053 0.085 0.051
0.462 0.266 0.322 0.435 0.268 0.025
0.628 0.442 0.444 0.406 0.240 0.129 0.014
0.113 0.070 0.046 0.047 0.034 0.002 0.106 0.129
0.173 0.119 0.162 0.331 0.177 0.039 0.089 0.237 0.071
0.434 0.419 0.339 0.505 0.469 0.390 0.315 0.349 0.151 0.430
0.762 0.633 0.781 0.700 0.758 0.625 0.469 0.618 0.440 0.538 0.607
0.530 0.389 0.482 0.579 0.597 0.498 0.374 0.562 0.247 0.383 0.387 0.084
0.586 0.435 0.550 0.530 0.552 0.509 0.369 0.471 0.234 0.346 0.456 0.090 0.038

10.3 Program Results

nag_mv_ordinal_multidimscale (g03fcc) Example Program Results

    STRESS = 1.2557e-01

Co-ordinates
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2060</td>
<td>0.2439</td>
</tr>
<tr>
<td>0.1063</td>
<td>0.1418</td>
</tr>
<tr>
<td>0.2224</td>
<td>0.0817</td>
</tr>
<tr>
<td>0.3032</td>
<td>0.0355</td>
</tr>
<tr>
<td>0.2645</td>
<td>-0.0698</td>
</tr>
<tr>
<td>0.1554</td>
<td>-0.0435</td>
</tr>
<tr>
<td>-0.0070</td>
<td>-0.1612</td>
</tr>
<tr>
<td>0.0749</td>
<td>-0.3275</td>
</tr>
<tr>
<td>0.0488</td>
<td>0.0289</td>
</tr>
<tr>
<td>0.0124</td>
<td>-0.0267</td>
</tr>
<tr>
<td>-0.1649</td>
<td>-0.2500</td>
</tr>
<tr>
<td>-0.5073</td>
<td>0.1267</td>
</tr>
<tr>
<td>-0.3093</td>
<td>0.1590</td>
</tr>
<tr>
<td>-0.3498</td>
<td>0.0700</td>
</tr>
</tbody>
</table>