NAG Library Function Document

nag_prob_non_central_students_t (g01gbc)

1 Purpose

nag_prob_non_central_students_t (g01gbc) returns the lower tail probability for the noncentral Student’s t-distribution.

2 Specification

```c
#include <nag.h>
#include <nagg01.h>

double nag_prob_non_central_students_t (double t, double df, double delta,
                                    double tol, Integer max_iter, NagError *fail)
```

3 Description

The lower tail probability of the noncentral Student’s t-distribution with \( \nu \) degrees of freedom and noncentrality parameter \( \delta \), \( P(T \leq t : \nu; \delta) \), is defined by

\[
P(T \leq t : \nu; \delta) = C_\nu \int_0^\infty \left( \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha - \delta} e^{-x^2/2} \, dx \right) \nu^{\nu-1} e^{-\nu u^2/2} \, du, \quad \nu > 0.0
\]

with

\[
C_\nu = \frac{1}{\Gamma(\frac{\nu}{2}) 2^{(\nu-2)/2}}
\]

The probability is computed in one of two ways.

(i) When \( t = 0.0 \), the relationship to the normal is used:

\[
P(T \leq t : \nu; \delta) = \frac{1}{\sqrt{2\pi}} \int_{-\delta}^{\infty} e^{-u^2/2} \, du.
\]

(ii) Otherwise the series expansion described in Equation 9 of Amos (1964) is used. This involves the sums of confluent hypergeometric functions, the terms of which are computed using recurrence relationships.

4 References


5 Arguments

1:  \( t \) – double

   *Input*

   *On entry:* \( t \), the deviate from the Student’s t-distribution with \( \nu \) degrees of freedom.

2:  \( df \) – double

   *Input*

   *On entry:* \( \nu \), the degrees of freedom of the Student’s t-distribution.

   *Constraint:* \( df \geq 1.0 \).

3:  \( delta \) – double

   *Input*

   *On entry:* \( \delta \), the noncentrality argument of the Students t-distribution.
4:  tol – double

    Input
    On entry: the absolute accuracy required by you in the results. If nag_prob_non_central_students_t
    (g01gbc) is entered with tol greater than or equal to 1.0 or less than $10 \times \text{machine precision}$ (see
    nag_machine_precision (X02AJC)), then the value of $10 \times \text{machine precision}$ is used instead.

5:  max_iter – Integer

    Input
    On entry: the maximum number of terms that are used in each of the summations.
    Suggested value: 100. See Section 9 for further comments.
    Constraint: $\text{max_iter} \geq 1$.

6:  fail – NagError*

    Input/Output
    The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL
    Dynamic memory allocation failed.
    See Section 3.2.1.2 in the Essential Introduction for further information.

NE_INT_ARG_LT
    On entry, $\text{max_iter} = \langle\text{value}\rangle$.
    Constraint: $\text{max_iter} \geq 1$.

NE_INTERNAL_ERROR
    An internal error has occurred in this function. Check the function call and any array sizes. If the
    call is correct then please contact NAG for assistance.
    An unexpected error has been triggered by this function. Please contact NAG.
    See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE
    Your licence key may have expired or may not have been installed correctly.
    See Section 3.6.5 in the Essential Introduction for further information.

NE_PROB_LIMIT
    The probability is too close to 0 or 1.

NE_PROBABILITY
    The probability is too small to calculate accurately.

NE_REAL_ARG_LT
    On entry, $\text{df} = \langle\text{value}\rangle$.
    Constraint: $\text{df} \geq 1.0$.

NE.Series
    One of the series has failed to converge with $\text{max_iter} = \langle\text{value}\rangle$ and $\text{tol} = \langle\text{value}\rangle$. Reconsider
    the requested tolerance and/or the maximum number of iterations.
7 Accuracy

The series described in Amos (1964) are summed until an estimated upper bound on the contribution of future terms to the probability is less than $\text{tol}$. There may also be some loss of accuracy due to calculation of gamma functions.

8 Parallelism and Performance

Not applicable.

9 Further Comments

The rate of convergence of the series depends, in part, on the quantity $t^2/(t^2 + \nu)$. The smaller this quantity the faster the convergence. Thus for large $t$ and small $\nu$ the convergence may be slow. If $\nu$ is an integer then one of the series to be summed is of finite length.

If two tail probabilities are required then the relationship of the $t$-distribution to the $F$-distribution can be used:

$$F = T^2, \lambda = \delta^2, \nu_1 = 1 \quad \text{and} \quad \nu_2 = \nu,$$

and a call made to nag_prob_non_central_f_dist (g01gdc).

Note that nag_prob_non_central_students_t (g01gbc) only allows degrees of freedom greater than or equal to 1 although values between 0 and 1 are theoretically possible.

10 Example

This example reads values from, and degrees of freedom for, and noncentrality arguments of the noncentral Student’s $t$-distributions, calculates the lower tail probabilities and prints all these values until the end of data is reached.

10.1 Program Text

/* nag_prob_non_central_students_t (g01gbc) Example Program. */
/* Copyright 2014 Numerical Algorithms Group. */
/* Mark 6a revised, 2001. */

#include <stdio.h>
#include <nag.h>
#include <nagg01.h>

int main(void)
{
    Integer exit_status = 0, max_iter;
    NagError fail;
    double delta, df, prob, t, tol;
    INIT_FAIL(fail);

    printf("nag_prob_non_central_students_t (g01gbc) Example Program Results\n\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif

    printf(" t df delta prob\n\n");
}
tol = 5e-6;
max_iter = 50;
#ifdef _WIN32
while ((scanf_s("%lf %lf %lf %*[\n]", &t, &df, &delta)) != EOF)
#else
while ((scanf("%lf %lf %lf %*[\n]", &t, &df, &delta)) != EOF)
#endif
{ /* nag_prob_non_central_students_t (g01gbc).
* Computes probabilities for the non-central Student’s
* t-distribution
*/
prob = nag_prob_non_central_students_t(t, df, delta, tol, max_iter, &fail);
if (fail.code == NE_NOERROR)
    printf(" %8.3f%8.3f%8.3f%8.4f
", t, df, delta, prob);
else
    printf("Error from nag_prob_non_central_students_t (g01gbc).
%s
", fail.message);
    exit_status = 1;
goto END;
}
END:
return exit_status;
}

10.2 Program Data
nag_prob_non_central_students_t (g01gbc) Example Program Data
-1.528 20.000 2.000 :t df delta
-0.188 7.500 1.000 :t df delta
1.138 45.000 0.000 :t df delta

10.3 Program Results
nag_prob_non_central_students_t (g01gbc) Example Program Results

<table>
<thead>
<tr>
<th>t</th>
<th>df</th>
<th>delta</th>
<th>prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.528</td>
<td>20.000</td>
<td>2.000</td>
<td>0.0003</td>
</tr>
<tr>
<td>-0.188</td>
<td>7.500</td>
<td>1.000</td>
<td>0.1189</td>
</tr>
<tr>
<td>1.138</td>
<td>45.000</td>
<td>0.000</td>
<td>0.8694</td>
</tr>
</tbody>
</table>