NAG Library Function Document
nag_deviates_f_dist (g01fdc)

1 Purpose
nag_deviates_f_dist (g01fdc) returns the deviate associated with the given lower tail probability of the \( F \) or variance-ratio distribution with real degrees of freedom.

2 Specification
#include <nag.h>
#include <nagg01.h>
double nag_deviates_f_dist (double p, double df1, double df2, NagError *fail)

3 Description
The deviate, \( f_p \), associated with the lower tail probability, \( p \), of the \( F \)-distribution with degrees of freedom \( \nu_1 \) and \( \nu_2 \) is defined as the solution to

\[
P(F \leq f_p : \nu_1, \nu_2) = \frac{\nu_1^{\frac{\nu_1}{2}} \nu_2^{\frac{\nu_2}{2}} \Gamma\left(\frac{\nu_1+\nu_2}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}\right)} \int_0^{f_p} F^{\frac{\nu_1}{2}+\nu_2} (\nu_2 + \nu_1 f)^{-\frac{\nu_1+\nu_2}{2}} dF,
\]

where \( \nu_1, \nu_2 > 0; 0 \leq f_p < \infty \).

The value of \( f_p \) is computed by means of a transformation to a beta distribution, \( P_\beta(B \leq \beta : a, b) \):

\[
P(F \leq f : \nu_1, \nu_2) = P_\beta\left(B \leq \frac{\nu_1 f}{\nu_1 f + \nu_2} : \nu_1/2, \nu_2/2\right)
\]

and using a call to nag_deviates_beta (g01fec).

For very large values of both \( \nu_1 \) and \( \nu_2 \), greater than \( 10^5 \), a normal approximation is used. If only one of \( \nu_1 \) or \( \nu_2 \) is greater than \( 10^5 \) then a \( \chi^2 \) approximation is used; see Abramowitz and Stegun (1972).

4 References

5 Arguments
1: \hspace{1cm} p – double \hspace{1cm} Input
   \hspace{1cm} On entry: \( p \), the lower tail probability from the required \( F \)-distribution.
   \hspace{1cm} Constraint: \( 0.0 \leq p < 1.0 \).

2: \hspace{1cm} df1 – double \hspace{1cm} Input
   \hspace{1cm} On entry: the degrees of freedom of the numerator variance, \( \nu_1 \).
   \hspace{1cm} Constraint: \( \text{df1} > 0.0 \).
3: df2 – double
   
   *Input*
   
   *On entry:* the degrees of freedom of the denominator variance, \( \nu_2 \).
   
   *Constraint:* \( df2 > 0.0 \).

4: fail – NagError *
   
   *Input/Output*
   
   The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

On any of the error conditions listed below except *fail* \( = \) NE_SOL_NOT_CONV
nag_deviates_f_dist (g01fdc) returns 0.0.

**NE_ALLOC_FAIL**

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

**NE_INTERNAL_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

**NE_NO_LICENCE**

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

**NE_PROBAB_CLOSE_TO_TAIL**

The probability is too close to 0.0 or 1.0. The value of \( f_p \) cannot be computed. This will only occur when the large sample approximations are used.

**NE_REAL_ARG_GE**

On entry, \( p = \langle value \rangle \).
Constraint: \( p < 1.0 \).

**NE_REAL_ARG_LE**

On entry, \( df1 = \langle value \rangle \) and \( df2 = \langle value \rangle \).
Constraint: \( df1 > 0.0 \) and \( df2 > 0.0 \).

**NE_REAL_ARG_LT**

On entry, \( p = \langle value \rangle \).
Constraint: \( p \geq 0.0 \).

**NE_SOL_NOT_CONV**

The solution has failed to converge. However, the result should be a reasonable approximation. Alternatively, nag_deviates_beta (g01fec) can be used with a suitable setting of the argument *tol*.

7 Accuracy

The result should be accurate to five significant digits.
8 Parallelism and Performance

Not applicable.

9 Further Comments

For higher accuracy nag_deviates_beta (g01fec) can be used along with the transformations given in Section 3.

10 Example

This example reads the lower tail probabilities for several $F$-distributions, and calculates and prints the corresponding deviates until the end of data is reached.

10.1 Program Text

```c
/* nag_deviates_f_dist (g01fdc) Example Program. */
* Copyright 2014 Numerical Algorithms Group.
* Mark 1, 1990.
*/
#include <nag.h>
#include <stdio.h>
#include <nagg01.h>
int main(void)
{
    Integer exit_status = 0;
    double df1, df2, f, p;
    NagError fail;
    INIT_FAIL(fail);
    /* Skip heading in data file */
    #ifdef _WIN32
    scanf_s("%*[\n]");
    #else
    scanf("%*[\n]");
    #endif
    printf("nag_deviates_f_dist (g01fdc) Example Program Results
    p df1 df2 f
    ");
    #ifdef _WIN32
    while (scanf_s("%lf %lf %lf", &p, &df1, &df2) != EOF)
    #else
    while (scanf("%lf %lf %lf", &p, &df1, &df2) != EOF)
    #endif
    {
        /* nag_deviates_f_dist (g01fdc).
           Deviates for the F-distribution */
        f = nag_deviates_f_dist(p, df1, df2, &fail);
        if (fail.code != NE_NOERROR)
        {
            printf("Error from nag_deviates_f_dist (g01fdc).\n%s\n", fail.message);
            exit_status = 1;
            goto END;
        }
        printf("%8.3f%8.3f%8.3f%8.3f\n", p, df1, df2, f);
    }
    END:
    return exit_status;
}
```

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10.2 Program Data

nag_deviates_f_dist (g01fdc) Example Program Data
0.9837 10.0 25.5
0.9000 1.0 1.0
0.5342 20.25 1.0

10.3 Program Results

nag_deviates_f_dist (g01fdc) Example Program Results
    p    df1    df2    f
0.984  10.000  25.500  2.837
0.900  1.000  1.000  39.863
0.534  20.250  1.000  2.500