NAG Library Function Document

nag_deviates_chi_sq (g01fcc)

1 Purpose
nag_deviates_chi_sq (g01fcc) returns the deviate associated with the given lower tail probability of the \( \chi^2 \)-distribution with real degrees of freedom.

2 Specification

```c
#include <nag.h>
#include <nagg01.h>
double nag_deviates_chi_sq (double p, double df, NagError *fail)
```

3 Description

The deviate, \( x_p \), associated with the lower tail probability \( p \) of the \( \chi^2 \)-distribution with \( \nu \) degrees of freedom is defined as the solution to

\[
P(X \leq x_p : \nu) = p = \frac{1}{2^{\nu/2} \Gamma(\nu/2)} \int_0^{x_p} e^{-X/2} X^{\nu/2-1} dX, \quad 0 \leq x_p < \infty; \nu > 0.
\]

The required \( x_p \) is found by using the relationship between a \( \chi^2 \)-distribution and a gamma distribution, i.e., a \( \chi^2 \)-distribution with \( \nu \) degrees of freedom is equal to a gamma distribution with scale parameter 2 and shape parameter \( \nu/2 \).

For very large values of \( \nu \), greater than 10^5, Wilson and Hilferty’s normal approximation to the \( \chi^2 \) is used; see Kendall and Stuart (1969).

4 References


5 Arguments

1: \( p \) – double  
   \textit{Input}
   
   \textit{On entry:} \( p \), the lower tail probability from the required \( \chi^2 \)-distribution.
   
   \textit{Constraint:} 0.0 \leq p < 1.0.

2: \( df \) – double  
   \textit{Input}
   
   \textit{On entry:} \( \nu \), the degrees of freedom of the \( \chi^2 \)-distribution.
   
   \textit{Constraint:} df > 0.0.

3: \( fail \) – NagError *  
   \textit{Input/Output}
   
   The NAG error argument (see Section 3.6 in the Essential Introduction).
6 Error Indicators and Warnings

On any of the error conditions listed below except fail.code = NE_ALG_NOT_CONV nag_deviates_chisq (g01fcc) returns 0.0.

**NE_ALG_NOT_CONV**

The algorithm has failed to converge in \(\text{value} \) iterations. The result should be a reasonable approximation.

**NE_ALLOC_FAIL**

Dynamic memory allocation failed.

See Section 3.2.1.2 in the Essential Introduction for further information.

**NE_GAM_NOT_CONV**

The series used to calculate the gamma function has failed to converge. This is an unlikely error exit.

**NE_INTERNAL_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 3.6.6 in the Essential Introduction for further information.

**NE_NO_LICENCE**

Your licence key may have expired or may not have been installed correctly.

See Section 3.6.5 in the Essential Introduction for further information.

**NE_PROBAB_CLOSE_TO_TAIL**

The probability is too close to 0.0 or 1.0.

**NE_REAL_ARG_GE**

On entry, \( p = \text{value} \).

Constraint: \( p < 1.0 \).

**NE_REAL_ARG_LE**

On entry, \( df = \text{value} \).

Constraint: \( df > 0.0 \).

**NE_REAL_ARG_LT**

On entry, \( p = \text{value} \).

Constraint: \( p \geq 0.0 \).

7 Accuracy

The results should be accurate to five significant digits for most argument values. Some accuracy is lost for \( p \) close to 0.0.

8 Parallelism and Performance

Not applicable.
9 Further Comments

For higher accuracy the relationship described in Section 3 may be used and a direct call to nag_deviates_gamma_dist (g01ffc) made.

10 Example

This example reads lower tail probabilities for several $\chi^2$-distributions, and calculates and prints the corresponding deviates until the end of data is reached.

10.1 Program Text

/* nag_deviates_chi_sq (g01fcc) Example Program. 
* 
* Copyright 2014 Numerical Algorithms Group. 
* 
* Mark 1, 1990. 
* */
#include <nag.h>
#include <stdio.h>
#include <nagg01.h>

int main(void)
{
    Integer exit_status = 0;
double df, p, x;
NagError fail;
    INIT_FAIL(fail);
    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
    printf("nag_deviates_chi_sq (g01fcc) Example Program Results\n");
    printf(" p df x\n");
#ifdef _WIN32
    while (scanf_s("%lf %lf", &p, &df) != EOF)
#else
    while (scanf("%lf %lf", &p, &df) != EOF)
#endif
    {
        /* nag_deviates_chi_sq (g01fcc). 
         * Deviates for the chi^2 distribution 
         */
        x = nag_deviates_chi_sq(p, df, &fail);
        if (fail.code != NE_NOERROR)
        {
            printf("Error from nag_deviates_chi_sq (g01fcc).\n\n", fail.message);
            exit_status = 1;
            goto END;
        }
        printf("%8.3f%8.3f%8.3f\n", p, df, x);
    }

END:
    return exit_status;
}
10.2 Program Data

nag_deviates_chi_sq (g01fcc) Example Program Data
0.0100 20.0
0.4279 7.50
0.8694 45.0

10.3 Program Results

nag_deviates_chi_sq (g01fcc) Example Program Results

<table>
<thead>
<tr>
<th>p</th>
<th>df</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010</td>
<td>20.00</td>
<td>8.260</td>
</tr>
<tr>
<td>0.428</td>
<td>7.500</td>
<td>6.200</td>
</tr>
<tr>
<td>0.869</td>
<td>45.000</td>
<td>55.759</td>
</tr>
</tbody>
</table>