NAG Library Function Document

nag_deviates_students_t (g01fbc)

1 Purpose

nag_deviates_students_t (g01fbc) returns the deviate associated with the given tail probability of Student’s *t*-distribution with real degrees of freedom.

2 Specification

```c
#include <nag.h>
#include <nagg01.h>
double nag_deviates_students_t (Nag_TailProbability tail, double p,
                              double df, NagError *fail)
```

3 Description

The deviate, \( t_p \) associated with the lower tail probability, \( p \), of the Student’s *t*-distribution with \( \nu \) degrees of freedom is defined as the solution to

\[
\Pr(T < t_p : \nu) = p = \frac{\nu/(\nu+1)}{\sqrt{\nu\pi}\Gamma(\nu/2)} \int_{-\infty}^{t_p} \left( 1 + \frac{T^2}{\nu} \right)^{-\nu/2} dT, \quad \nu \geq 1; -\infty < t_p < \infty.
\]

For \( \nu = 1 \) or \( 2 \) the integral equation is easily solved for \( t_p \).

For other values of \( \nu < 3 \) a transformation to the beta distribution is used and the result obtained from nag_deviates_beta (g01fec).

For \( \nu \geq 3 \) an inverse asymptotic expansion of Cornish–Fisher type is used. The algorithm is described by Hill (1970).

4 References


Hill G W (1970) Student’s *t*-distribution *Comm. ACM* 13(10) 617–619

5 Arguments

1: \( \text{tail} \) – Nag_TailProbability

*Input*

\( \text{tail} = \) Nag_UpperTail

The upper tail probability, i.e., \( \Pr(T \geq t_p : \nu) \).

\( \text{tail} = \) Nag_LowerTail

The lower tail probability, i.e., \( \Pr(T \leq t_p : \nu) \).

\( \text{tail} = \) Nag_TwoTailSignif

The two tail (significance level) probability, i.e., \( \Pr(T \geq |t_p| : \nu) + \Pr(T \leq -|t_p| : \nu) \).

\( \text{tail} = \) Nag_TwoTailConfid

The two tail (confidence interval) probability, i.e., \( \Pr(T \leq |t_p| : \nu) - \Pr(T \leq -|t_p| : \nu) \).

*Constraint:* \( \text{tail} = \) Nag_UpperTail, Nag_LowerTail, Nag_TwoTailSignif or Nag_TwoTailConfid.
2: \( p \) – double

\textit{Input}

\textit{On entry:} \( p \), the probability from the required Student’s \( t \)-distribution as defined by \texttt{tail}.
\textit{Constraint:} \( 0.0 < p < 1.0 \).

3: \( df \) – double

\textit{Input}

\textit{On entry:} \( \nu \), the degrees of freedom of the Student’s \( t \)-distribution.
\textit{Constraint:} \( df \geq 1.0 \).

4: \( \texttt{fail} \) – \texttt{NagError} *

\textit{Input/Output}

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 \ Error Indicators and Warnings

On any of the error conditions listed below except \texttt{fail.code} = \texttt{NE_SOL_NOT_CONV}
\texttt{nag_deviates_students_t} (\texttt{g01fbc}) returns 0.0.

\textbf{NE_ALLOC_FAIL}

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

\textbf{NE_BAD_PARAM}

On entry, argument \( \langle \text{value} \rangle \) had an illegal value.

\textbf{NE_INTERNAL_ERROR}

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

\textbf{NE_NO_LICENCE}

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

\textbf{NE_REAL_ARG_GE}

On entry, \( p = \langle \text{value} \rangle \).
\textit{Constraint:} \( p < 1.0 \).

\textbf{NE_REAL_ARG_LE}

On entry, \( p = \langle \text{value} \rangle \).
\textit{Constraint:} \( p > 0.0 \).

\textbf{NE_REAL_ARG_LT}

On entry, \( df = \langle \text{value} \rangle \).
\textit{Constraint:} \( df \geq 1.0 \).

\textbf{NE_SOL_NOT_CONV}

The solution has failed to converge. However, the result should be a reasonable approximation.
7 Accuracy
The results should be accurate to five significant digits, for most argument values. The error behaviour for various argument values is discussed in Hill (1970).

8 Parallelism and Performance
Not applicable.

9 Further Comments
The value \( t_p \) may be calculated by using the transformation described in Section 3 and using nag_deviates_beta (g01fec). This function allows you to set the required accuracy.

10 Example
This example reads the probability, the tail that probability represents and the degrees of freedom for a number of Student's \( t \)-distributions and computes the corresponding deviates.

10.1 Program Text
/* nag_deviates_students_t (g01fbc) Example Program.
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 4, 1996.
 * Mark 5 revised, 1998.
 * Mark 7 revised, 2001.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
    Integer exit_status=0;
    double df, p, t;
    int i;
    static Nag_TailProbability tail[] = { Nag_LowerTail, Nag_UpperTail,
                                          Nag_TwoTailSignif, Nag_TwoTailConfid };
    static const char *tailmess[] = { "Nag_LowerTail", "Nag_UpperTail",
                                      "Nag_TwoTailSignif",
                                      "Nag_TwoTailConfid" };

    NagError fail;
    INIT_FAIL(fail);

    printf("nag_deviates_students_t (g01fbc) Example Program Results\n\n");
    /* Skip heading in data file */
    #ifdef _WIN32
    scanf_s("%*[\n"]);
    #else
    scanf("%*[\n"]);
    #endif
    #ifdef _WIN32
    printf("%lf %lf %d\n".pivot, df, p, i);
    #else
    printf("%lf %lf %d\n", df, p, i);
    #endif
    #ifdef _WIN32
    while (scanf_s("%lf %lf %d", &p, &df, &i) != EOF)
    #else
    while (scanf("%lf %lf %d", &p, &df, &i) != EOF)
    #endif
    {
        /* nag_deviates_students_t (g01fbc).
* Deviates for Student’s t-distribution
  
  \[
  t = \text{nag\_deviates\_students\_t}(\text{tail[i]}, \ p, \ \text{df}, \ &\text{fail});
  \]
  
  if (\text{fail\_code} != \text{NE\_NOERROR})
  {
    printf("Error from nag\_deviates\_students\_t (g01fbc).\n\s\n", \text{fail\_message});
    \text{exit\_status} = 1;
    \text{goto END;}
  }
  
  printf("%8.3f%8.3f %-19s %8.3f\n", \ p, \ \text{df}, \ \text{tailmess[i]}, \ t);

END:
  \text{return exit\_status;}
}

10.2 Program Data

\text{nag\_deviates\_students\_t (g01fbc) Example Program Data}
0.0100 20.00 2
0.01 7.50 0
0.99 45.00 3

10.3 Program Results

\text{nag\_deviates\_students\_t (g01fbc) Example Program Results}

\begin{tabular}{cccc}
 p & df & tail & t \\
 0.010 & 20.000 & Nag\_TwoTailSignif & 2.845 \\
 0.010 & 7.500 & Nag\_LowerTail & -2.943 \\
 0.990 & 45.000 & Nag\_TwoTailConfid & 2.690 \\
\end{tabular}