1 Purpose

nag_prob_studentized_range (g01emc) returns the probability associated with the lower tail of the distribution of the Studentized range statistic.

2 Specification

#include <nag.h>
#include <nag01.h>
double nag_prob_studentized_range (double q, double v, Integer ir,
NagError *fail)

3 Description

The externally Studentized range, \( q \), for a sample, \( x_1, x_2, \ldots, x_r \), is defined as:

\[
q = \frac{\max(x_i) - \min(x_i)}{\hat{\sigma}_e},
\]

where \( \hat{\sigma}_e \) is an independent estimate of the standard error of the \( x_i \)'s. The most common use of this statistic is in the testing of means from a balanced design. In this case for a set of group means, \( T_1, T_2, \ldots, T_r \), the Studentized range statistic is defined to be the difference between the largest and smallest means, \( T_{\text{largest}} \) and \( T_{\text{smallest}} \), divided by the square root of the mean-square experimental error, \( MS_{\text{error}} \), over the number of observations in each group, \( n \), i.e.,

\[
q = \frac{T_{\text{largest}} - T_{\text{smallest}}}{\sqrt{MS_{\text{error}}/n}}.
\]

The Studentized range statistic can be used as part of a multiple comparisons procedure such as the Newman–Keuls procedure or Duncan’s multiple range test (see Montgomery (1984) and Winer (1970)).

For a Studentized range statistic the probability integral, \( P(q; v, r) \), for \( v \) degrees of freedom and \( r \) groups can be written as:

\[
P(q; v, r) = C \int_0^\infty x^{v/2-1}e^{-wx^2/2} \left\{ r \int_{-\infty}^{\phi(y)} \phi(y) - \Phi(y - qx) \right\}^{-1} dy \, dx,
\]

where

\[
C = \frac{v^{v/2}}{\Gamma(v/2)2^{v/2-1}}, \quad \phi(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \quad \text{and} \quad \Phi(y) = \int_{-\infty}^{y} \phi(t) \, dt.
\]

The above two-dimensional integral is evaluated using numerical quadrature with the upper and lower limits computed to give stated accuracy (see Section 7).

If the degrees of freedom \( v \) are greater than 2000 the probability integral can be approximated by its asymptotic form:

\[
P(q; r) = r \int_{-\infty}^{\infty} \phi(y) \left[ \Phi(y) - \Phi(y - q) \right]^{-1} dy.
\]

This integral is evaluated using nag_1d_quad_inf_1 (d01smc).
4 References

5 Arguments
1:  q – double  
   On entry: q, the Studentized range statistic.  
   Constraint: q > 0.0.
2:  v – double  
   On entry: v, the number of degrees of freedom for the experimental error.  
   Constraint: v ≥ 1.0.
3:  ir – Integer  
   On entry: r, the number of groups.  
   Constraint: ir ≥ 2.
4:  fail – NagError *  
   The NAG error argument (see Section 3.6 in the Essential Introduction).  
   If on exit fail.code = NE_INT or NE_REAL, then nag_prob_studentized_range (g01emc) returns to 0.0.

6 Error Indicators and Warnings

NE_ACCURACY
   Warning – There is some doubt as to whether full accuracy has been achieved.

NE_ALLOC_FAIL
   Dynamic memory allocation failed.  
   See Section 3.2.1.2 in the Essential Introduction for further information.

NE_INT
   On entry, ir = (value).  
   Constraint: ir ≥ 2.

NE_INTERNAL_ERROR
   An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.  
   An unexpected error has been triggered by this function. Please contact NAG.  
   See Section 3.6.6 in the Essential Introduction for further information.
NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

NE_REAL

On entry, \( q = \langle \text{value} \rangle \).
Constraint: \( q > 0.0 \).

On entry, \( v = \langle \text{value} \rangle \).
Constraint: \( v \geq 1.0 \).

7 Accuracy

The returned value will have absolute accuracy to at least four decimal places (usually five), unless fail.code = NE_ACCURACY. When fail.code = NE_ACCURACY it is usual that the returned value will be a good estimate of the true value.

8 Parallelism and Performance

nag_prob_studentized_range (g01emc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users’ Note for your implementation for any additional implementation-specific information.

9 Further Comments

None.

10 Example

The lower tail probabilities for the distribution of the Studentized range statistic are computed and printed for a range of values of \( q, v \) and \( r \).

10.1 Program Text

/* nag_prob_studentized_range (g01emc) Example Program. */
* * Copyright 2014 Numerical Algorithms Group.
* * Mark 7, 2001.
* * Mark 7b revised, 2004.
* */
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nag_g01.h>

int main(void)
{
    /* Scalars */
    Integer exit_status = 0;
    double q, v, valp;
    Integer i, ir;
    NagError fail;

    INIT_FAIL(fail);

    printf("nag_prob_studentized_range (g01emc) Example Program Results\n");

    /* Scalars */
/* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    printf("%7.4f%2s%4.1f%1s%3"NAG_IFMT"%1s%10.4f
", q, "", v, "", ir, "", valp);
}

END:
    return exit_status;
}

10.2 Program Data

nag_prob_studentized_range (g01emc) Example Program Data

4.6543 10.0 5
2.8099 60.0 12
4.2636 5.0 4

10.3 Program Results

nag_prob_studentized_range (g01emc) Example Program Results

<table>
<thead>
<tr>
<th>q</th>
<th>v</th>
<th>ir</th>
<th>Quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.6543</td>
<td>10.0</td>
<td>5</td>
<td>0.9500</td>
</tr>
<tr>
<td>2.8099</td>
<td>60.0</td>
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<td>0.3000</td>
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<tr>
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<td>5.0</td>
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</tbody>
</table>