1 Purpose

nag_prob_chi_sq (g01ecc) returns the lower or upper tail probability for the $\chi^2$-distribution with real degrees of freedom.

2 Specification

```c
#include <nag.h>
#include <nagg01.h>
double nag_prob_chi_sq (Nag_TailProbability tail, double x, double df,
                      NagError *fail)
```

3 Description

The lower tail probability for the $\chi^2$-distribution with $\nu$ degrees of freedom, $P(X \leq x : \nu)$ is defined by:

$$P(X \leq x : \nu) = \frac{1}{2^\nu/2 \Gamma(\nu/2)} \int_0^x X^{\nu/2-1} e^{-X/2} dX, \quad x \geq 0, \nu > 0.$$ 

To calculate $P(X \leq x : \nu)$ a transformation of a gamma distribution is employed, i.e., a $\chi^2$-distribution with $\nu$ degrees of freedom is equal to a gamma distribution with scale parameter 2 and shape parameter $\nu/2$.

4 References


5 Arguments

1:  `tail` – Nag_TailProbability
    
    **Input**
    
    On entry: indicates whether the upper or lower tail probability is required.
    
    `tail = Nag_LowerTail`
    
    The lower tail probability is returned, i.e., $P(X \leq x : \nu)$.
    
    `tail = Nag_UpperTail`
    
    The upper tail probability is returned, i.e., $P(X \geq x : \nu)$.
    
    **Constraint:** `tail = Nag_LowerTail` or `Nag_UpperTail`.

2:  `x` – double
    
    **Input**
    
    On entry: $x$, the value of the $\chi^2$ variate with $\nu$ degrees of freedom.
    
    **Constraint:** $x \geq 0.0$.

3:  `df` – double
    
    **Input**
    
    On entry: $\nu$, the degrees of freedom of the $\chi^2$-distribution.
    
    **Constraint:** $df > 0.0$. 
fail — NagError *

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALG_NOT_CONV
The series used to calculate the gamma probabilities has failed to converge. The result returned should represent an approximation to the solution.

NE_ALLOC_FAIL
Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM
On entry, argument <value> had an illegal value.

NE_INTERNAL_ERROR
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG. See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE
Your licence key may have expired or may not have been installed correctly. See Section 3.6.5 in the Essential Introduction for further information.

NE_REAL_ARG_LE
On entry, df = <value>.
Constraint: df > 0.0.

NE_REAL_ARG_LT
On entry, x = <value>.
Constraint: x ≥ 0.0.

7 Accuracy
A relative accuracy of five significant figures is obtained in most cases.

8 Parallelism and Performance
Not applicable.

9 Further Comments
For higher accuracy the transformation described in Section 3 may be used with a direct call to nag_incomplete_gamma (s14bac).

10 Example
Values from various χ²-distributions are read, the lower tail probabilities calculated, and all these values printed out, until the end of data is reached.
10.1 Program Text

/* nag_prob_chi_sq (g01ecc) Example Program.
 * Copyright 2014 Numerical Algorithms Group.
 * Mark 1, 1990.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
    Integer exit_status = 0;
    double df, prob, x;
    NagError fail;

    INIT_FAIL(fail);

    /* Skip heading in data file */
    #ifdef _WIN32
    scanf_s("%*[\n");
    #else
    scanf("%*[\n");
    #endif

    printf("nag_prob_chi_sq (g01ecc) Example Program Results\n");
    printf(" x df prob\n\n");

    #ifdef _WIN32
    while (scanf_s("%lf %lf", &x, &df) != EOF)
    #else
    while (scanf("%lf %lf", &x, &df) != EOF)
    #endif
    {
        /* nag_prob_chi_sq (g01ecc).
         * Probabilities for chi^2 distribution
         */
        prob = nag_prob_chi_sq(Nag_LowerTail, x, df, &fail);
        if (fail.code != NE_NOERROR)
        {
            printf("Error from nag_prob_chi_sq (g01ecc).\n\n", fail.message);
            exit_status = 1;
            goto END;
        }
        printf("%6.3f%8.3f%8.4f\n", x, df, prob);
    }

    END:
    return exit_status;
}

10.2 Program Data

nag_prob_chi_sq (g01ecc) Example Program Data
  8.26  20.0
  6.2   7.5
  55.76 45.0
10.3 Program Results

nag_prob_chi_sq (g01ecc) Example Program Results

\[
\begin{array}{ccc}
  x & df & prob \\
  8.260 & 20.000 & 0.0100 \\
  6.200 & 7.500 & 0.4279 \\
  55.760 & 45.000 & 0.8694 \\
\end{array}
\]