1 Purpose

nag_sparse_herm_precon_ichol_solve (f11jpc) solves a system of complex linear equations involving the incomplete Cholesky preconditioning matrix generated by nag_sparse_herm_chol_fac (f11jnc).

2 Specification

```c
#include <nag.h>
#include <nagf11.h>

void nag_sparse_herm_precon_ichol_solve (Integer n, const Complex a[],
   Integer la, const Integer irow[], const Integer icol[],
   const Integer ipiv[], const Integer istr[],
   Nag_SparseSym_CheckData check, const Complex y[], Complex x[],
   NagError *fail)
```

3 Description

nag_sparse_herm_precon_ichol_solve (f11jpc) solves a system of linear equations

\[ Mx = y \]

involving the preconditioning matrix \( M = PLD^\top P^\top \), corresponding to an incomplete Cholesky decomposition of a complex sparse Hermitian matrix stored in symmetric coordinate storage (SCS) format (see Section 2.1.2 in the f11 Chapter Introduction), as generated by nag_sparse_herm_chol_fac (f11jnc).

In the above decomposition \( L \) is a complex lower triangular sparse matrix with unit diagonal, \( D \) is a real diagonal matrix and \( P \) is a permutation matrix. \( L \) and \( D \) are supplied to nag_sparse_herm_precon_ichol_solve (f11jpc) through the matrix

\[ C = L + D^{-1} - I \]

which is a lower triangular \( n \) by \( n \) complex sparse matrix, stored in SCS format, as returned by nag_sparse_herm_chol_fac (f11jnc). The permutation matrix \( P \) is returned from nag_sparse_herm_chol_fac (f11jnc) via the array \( \text{ipiv} \).

nag_sparse_herm_precon_ichol_solve (f11jpc) may also be used in combination with nag_sparse_herm_chol_fac (f11jnc) to solve a sparse complex Hermitian positive definite system of linear equations directly (see nag_sparse_herm_chol_fac (f11jnc)). This is illustrated in Section 10.

4 References

None.

5 Arguments

1:  
   \( n \) – Integer

\( \text{Input} \)

\( \text{On entry:} \ n, \ \text{the order of the matrix} \ M. \ \text{This must be the same value as was supplied in the preceding call to nag_sparse_herm_chol_fac (f11jnc).} \)

\( \text{Constraint:} \ n \geq 1. \)
2: \(a\) – const Complex

On entry: the values returned in the array \(a\) by a previous call to `nag_sparse_herm_chol_fac` (f11jnc).

3: \(l_a\) – Integer

On entry: the dimension of the arrays \(a\), \(irow\) and \(icol\). This must be the same value supplied in the preceding call to `nag_sparse_herm_chol_fac` (f11jnc).

4: \(irow[l_a]\) – const Integer

5: \(icol[l_a]\) – const Integer

6: \(ipiv[n]\) – const Integer

7: \(istr[n + 1]\) – const Integer

On entry: the values returned in arrays \(irow\), \(icol\), \(ipiv\) and \(istr\) by a previous call to `nag_sparse_herm_chol_fac` (f11jnc).

8: \(check\) – `Nag_SparseSym_CheckData`

On entry: specifies whether or not the input data should be checked.

\(check = \text{Nag}\_\text{SparseSym}\_\text{Check}\)

Checks are carried out on the values of \(n\), \(irow\), \(icol\), \(ipiv\) and \(istr\).

\(check = \text{Nag}\_\text{SparseSym}\_\text{NoCheck}\)

None of these checks are carried out.

Constraint: \(check = \text{Nag}\_\text{SparseSym}\_\text{Check}\) or \(\text{Nag}\_\text{SparseSym}\_\text{NoCheck}\).

9: \(y[n]\) – const Complex

On entry: the right-hand side vector \(y\).

10: \(x[n]\) – Complex

On exit: the solution vector \(x\).

11: \(fail\) – `NagError`

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

**NE_ALLOC_FAIL**

Dynamic memory allocation failed.

See Section 3.2.1.2 in the Essential Introduction for further information.

**NE_BAD_PARAM**

On entry, argument \(\langle value\rangle\) had an illegal value.

**NE_INT**

On entry, \(n = \langle value\rangle\).

Constraint: \(n \geq 1\).

**NE_INTERNAL_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG. See Section 3.6.6 in the Essential Introduction for further information.
NE_INVALID_ROWCOL_PIVOT
Check that a, irow, icol, ipiv and istr have not been corrupted between calls to
nag_sparse_herm_chol_fac (f11jnc) and nag_sparse_herm_precon_ichol_solve (f11jpc).

NE_INVALID_SCS
Check that a, irow, icol, ipiv and istr have not been corrupted between calls to
nag_sparse_herm_chol_fac (f11jnc) and nag_sparse_herm_precon_ichol_solve (f11jpc).

NE_INVALID_SCS_PRECOND
Check that a, irow, icol, ipiv and istr have not been corrupted between calls to
nag_sparse_herm_chol_fac (f11jnc) and nag_sparse_herm_precon_ichol_solve (f11jpc).

NE_NO_LICENCE
Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

NE_NOT STRICTLY_INCREASING
Check that a, irow, icol, ipiv and istr have not been corrupted between calls to
nag_sparse_herm_chol_fac (f11jnc) and nag_sparse_herm_precon_ichol_solve (f11jpc).

7 Accuracy
The computed solution $x$ is the exact solution of a perturbed system of equations $(M + \delta M)x = y,$
where
$$|\delta M| \leq c(n)\epsilon P|L||D||L^H|P^T,$$
$c(n)$ is a modest linear function of $n$, and $\epsilon$ is the machine precision.

8 Parallelism and Performance
Not applicable.

9 Further Comments

9.1 Timing
The time taken for a call to nag_sparse_herm_precon_ichol_solve (f11jpc) is proportional to the value of
nnz returned from nag_sparse_herm_chol_fac (f11jnc).

10 Example
This example reads in a complex sparse Hermitian positive definite matrix $A$ and a vector $y$. It then calls
nag_sparse_herm_chol_fac (f11jnc), with lfill = -1 and dtol = 0.0, to compute the complete Cholesky
decomposition of $A$:
$$A = PLDL^H P^T.$$ 
Finally it calls nag_sparse_herm_precon_ichol_solve (f11jpc) to solve the system
$$PLDL^H P^T x = y.$$
10.1 Program Text

/* nag_sparse_herm_precon_ichol_solve (f11jpc) Example Program. */
* Copyright 2014 Numerical Algorithms Group.
* Mark 23, 2011.
*/
#include <nag.h>
#include <nag_stdlib.h>
#include <naga02.h>
#include <nagf11.h>

int main(void)
{
    /* Scalars */
    Integer exit_status = 0;
    double dscale, dtol;
    Integer i, la, lfill, n, nnz, nnzc, npivm;
    /* Arrays */
    Complex *a = 0, *x = 0, *y = 0;
    Integer *icol = 0, *ipiv = 0, *irow = 0, *istr = 0;
    /* NAG types */
    Nag_SparseSym_Fact mic;
    Nag_SparseNsym_Piv pstrat;
    Nag_SparseSym_CheckData check;
    NagError fail;

    INIT_FAIL(fail);

    printf("nag_sparse_herm_precon_ichol_solve (f11jpc) Example Program Results\n");
    printf("\n\n");
    /* Skip heading in data file*/
    #ifdef _WIN32
    scanf_s("%*[^
\] ");
    #else
    scanf("%*[^
\] ");
    #endif
    #ifdef _WIN32
    scanf_s("%"NAG_IFMT"%*[\n]", &n);
    #else
    scanf("%"NAG_IFMT"%*[\n]", &n);
    #endif
    #ifdef _WIN32
    scanf_s("%"NAG_IFMT"%*[\n]", &nnz);
    #else
    scanf("%"NAG_IFMT"%*[\n]", &nnz);
    #endif
    #ifdef _WIN32
    scanf_s("%"NAG_IFMT"%*[\n]", &nnz);
    #else
    scanf("%"NAG_IFMT"%*[\n]", &nnz);
    #endif
    /* Allocate memory */
    la = 3 * nnz;
    if (!
        !(a = NAG_ALLOC(la, Complex)) ||
        !(x = NAG_ALLOC(n, Complex)) ||
        !(y = NAG_ALLOC(n, Complex)) ||
        !(icol = NAG_ALLOC(la, Integer)) ||
        !(ipiv = NAG_ALLOC(n, Integer)) ||
        !(irow = NAG_ALLOC(la, Integer)) ||
        !(istr = NAG_ALLOC(n + 1, Integer))
    )
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
    /* Read the matrix a */
    for (i = 0; i <= nnz - 1; i++)
    #ifdef _WIN32
    scanf_s(" ( %lf , %lf ) %"NAG_IFMT"%"NAG_IFMT"%*[\n] ",
        &a[i].re, &a[i].im, &irow[i], &icol[i]);
    #endif
    
    END:
    exit_status = 0;
    return (exit_status);
}
```c
/* Read the vector y*/
for (i = 0; i <= n - 1; i++)
#ifdef _WIN32
    scanf_s(" ( %lf , %lf ) ", &y[i].re, &y[i].im);
#else
    scanf(" ( %lf , %lf ) ", &y[i].re, &y[i].im);
#endif

lfill = -1;
dtol = 0.0;
dscale = 0.0;
mic = Nag_SparseSym_UnModFact;
pstrat = Nag_SparseSym_MarkPiv;
/* Calculate Cholesky factorization using nag_sparse_herm_chol_fac (f11jnc). */
nag_sparse_herm_chol_fac(n, nnz, a, la, irow, icol, lfill, dtol, mic, dscale, pstrat, ipiv, istr, &nnzc, &npivm, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_sparse_herm_chol_fac (f11jnc). \n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Check the output value of npivm */
if (npivm != 0)
    printf("Factorization is not complete \n");
else {
    /* Solve complex linear system involving incomplete Cholesky factorization */
    /* H^T P L D L^H P x = y */
    /* using nag_sparse_herm_precon_ichol_solve (f11jpc). */
    check = Nag_SparseSym_Check;
nag_sparse_herm_precon_ichol_solve(n, a, la, irow, icol, ipiv, istr, check, y, x, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_sparse_herm_precon_ichol_solve (f11jpc). \n%s\n", fail.message);
        exit_status = 2;
        goto END;
    }
    /* Output results*/
    printf("Solution of linear system \n");
    for (i = 0; i <= n - 1; i++)
        printf(" (%13.4e, %13.4e) \n", x[i].re, x[i].im);
}
END:
NAG_FREE(a);
NAG_FREE(x);
NAG_FREE(y);
NAG_FREE(icol);
NAG_FREE(ipiv);
NAG_FREE(irow);
NAG_FREE(istr);
return exit_status;
```
10.2 Program Data

nag_sparse_herm_precon_ichol_solve (f11jpc) Example Program Data

9 : n
23 : nnz
  ( 6., 0.)  1  1
( -1., 1.)  2  1
  ( 6., 0.)  2  2
  ( 0., 1.)  3  3
  ( 5., 0.)  4  4
  ( 2.,-2.)  5  1
  ( 4., 0.)  5  5
  ( 1., 1.)  6  3
  ( 2., 0.)  6  4
  ( 6., 0.)  6  6
( -4., 3.)  7  2
  ( 0., 1.)  7  5
( -1., 0.)  7  6
  ( 6., 0.)  7  7
( -1., 1.)  8  4
  ( 0., 1.)  8  6
  ( 9., 0.)  8  8
  ( 1., 3.)  9  1
  ( 1., 2.)  9  5
( -1., 0.)  9  6
  ( 1., 4.)  9  8
  ( 9., 0.)  9  9
: a[i], irow[i], icol[i], i=0,...,nnz-1
  ( 8.,54.) (-10.,-92.)
( 25.,27.) (26.,-28.)
( 54.,12.) (26.,-22.)
( 47.,65.) (71.,-57.)
( 60.,70.)
: y[i], i=0,...,n-1

10.3 Program Results

nag_sparse_herm_precon_ichol_solve (f11jpc) Example Program Results

Solution of linear system
( 1.0000e+00,  9.0000e+00)
( 2.0000e+00, -8.0000e+00)
( 3.0000e+00,  7.0000e+00)
( 4.0000e+00, -6.0000e+00)
( 5.0000e+00,  5.0000e+00)
( 6.0000e+00, -4.0000e+00)
( 7.0000e+00,  3.0000e+00)
( 8.0000e+00, -2.0000e+00)
( 9.0000e+00,  1.0000e+00)