nag_sparse_nherm_precon_ilu_solve (f11dpc)

1 Purpose

nag_sparse_nherm_precon_ilu_solve (f11dpc) solves a system of complex linear equations involving the incomplete \( LU \) preconditioning matrix generated by nag_sparse_nherm_fac (f11dnc).

2 Specification

```c
#include <nag.h>
#include <nagf11.h>

void nag_sparse_nherm_precon_ilu_solve (Nag_TransType trans, Integer n,
const Complex a[], Integer la, const Integer irow[],
const Integer icol[], const Integer ipivp[], const Integer ipivq[],
const Integer istr[], const Integer idia[],
Nag_SparseNsym_CheckData check, const Complex y[], Complex x[],
NagError *fail)
```

3 Description

nag_sparse_nherm_precon_ilu_solve (f11dpc) solves a system of complex linear equations

\[ Mx = y, \quad \text{or} \quad M^T x = y, \]

according to the value of the argument \texttt{trans}, where the matrix \( M = PLDUQ \) corresponds to an incomplete \( LU \) decomposition of a complex sparse matrix stored in coordinate storage (CS) format (see Section 2.1.1 in the f11 Chapter Introduction), as generated by nag_sparse_nherm_fac (f11dnc).

In the above decomposition \( L \) is a lower triangular sparse matrix with unit diagonal elements, \( D \) is a diagonal matrix, \( U \) is an upper triangular sparse matrix with unit diagonal elements and, \( P \) and \( Q \) are permutation matrices. \( L, D \) and \( U \) are supplied to nag_sparse_nherm_precon_ilu_solve (f11dpc) through the matrix

\[ C = L + D^{-1} + U - 2I \]

which is an \( n \) by \( n \) sparse matrix, stored in CS format, as returned by nag_sparse_nherm_fac (f11dnc). The permutation matrices \( P \) and \( Q \) are returned from nag_sparse_nherm_fac (f11dnc) via the arrays \texttt{ipivp} and \texttt{ipivq}.

It is envisaged that a common use of nag_sparse_nherm_precon_ilu_solve (f11dpc) will be to carry out the preconditioning step required in the application of nag_sparse_nherm_basic_solver (f11bsc) to sparse complex linear systems. nag_sparse_nherm_precon_ilu_solve (f11dpc) is used for this purpose by the Black Box function nag_sparse_nherm_fac_sol (f11dqc).

nag_sparse_nherm_precon_ilu_solve (f11dpc) may also be used in combination with nag_sparse_nherm_fac (f11dnc) to solve a sparse system of complex linear equations directly (see Section 9.5 in nag_sparse_nherm_fac (f11dnc)). This use of nag_sparse_nherm_precon_ilu_solve (f11dpc) is illustrated in Section 10.

4 References

None.
5 Arguments

1: \( \text{trans} \) – Nag_TransType
\( \text{Input} \)

*On entry: specifies whether or not the matrix \( M \) is transposed.

\( \text{trans} = \text{Nag_NoTrans} \)
\( Mx = y \) is solved.

\( \text{trans} = \text{Nag_Trans} \)
\( M^T x = y \) is solved.

*Constraint: \( \text{trans} = \text{Nag_NoTrans} \) or \( \text{Nag_Trans} \).

2: \( n \) – Integer
\( \text{Input} \)

*On entry: \( n \), the order of the matrix \( M \). This \textbf{must} be the same value as was supplied in the preceding call to nag_sparse_nherm_fac (f11dnc).

*Constraint: \( n \geq 1 \).

3: \( a[\text{la}] \) – const Complex
\( \text{Input} \)

*On entry: the values returned in the array \( a \) by a previous call to nag_sparse_nherm_fac (f11dnc).

4: \( \text{la} \) – Integer
\( \text{Input} \)

*On entry: the dimension of the arrays \( a, \text{irow} \) and \( \text{icol} \). This \textbf{must} be the same value supplied in the preceding call to nag_sparse_nherm_fac (f11dnc).

5: \( \text{irow}[\text{la}] \) – const Integer
\( \text{Input} \)

6: \( \text{icol}[\text{la}] \) – const Integer
\( \text{Input} \)

7: \( \text{ipivp}[n] \) – const Integer
\( \text{Input} \)

8: \( \text{ipivq}[n] \) – const Integer
\( \text{Input} \)

9: \( \text{istr}[n + 1] \) – const Integer
\( \text{Input} \)

10: \( \text{idiag}[n] \) – const Integer
\( \text{Input} \)

*On entry: the values returned in arrays \( \text{irow, icol, ipivp, ipivq, istr} \) and \( \text{idiag} \) by a previous call to nag_sparse_nherm_fac (f11dnc).

11: \( \text{check} \) – Nag_SparseNsym_CheckData
\( \text{Input} \)

*On entry: specifies whether or not the CS representation of the matrix \( M \) should be checked.

\( \text{check} = \text{Nag_SparseNsym_Check} \)
Checks are carried on the values of \( n, \text{irow, icol, ipivp, ipivq, istr} \) and \( \text{idiag} \).

\( \text{check} = \text{Nag_SparseNsym_NoCheck} \)
None of these checks are carried out.

See also Section 9.2.

*Constraint: \( \text{check} = \text{Nag_SparseNsym_Check} \) or \( \text{Nag_SparseNsym_NoCheck} \).

12: \( y[n] \) – const Complex
\( \text{Input} \)

*On entry: the right-hand side vector \( y \).

13: \( x[n] \) – Complex
\( \text{Output} \)

*On exit: the solution vector \( x \).

14: \( \text{fail} \) – NagError *
\( \text{Input/Output} \)

The NAG error argument (see Section 3.6 in the Essential Introduction).
6  Error Indicators and Warnings

NE_ALLOC_FAIL
   Dynamic memory allocation failed.
   See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM
   On entry, argument \texttt{value} had an illegal value.

NE_INT
   On entry, \( n = \langle \text{value} \rangle \).
   Constraint: \( n \geq 1 \).

NE_INTERNAL_ERROR
   An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.
   An unexpected error has been triggered by this function. Please contact NAG.
   See Section 3.6.6 in the Essential Introduction for further information.

NE_INVALID_CS
   On entry, \( i = \langle \text{value} \rangle \), \( \text{icol}[i-1] = \langle \text{value} \rangle \), and \( n = \langle \text{value} \rangle \).
   Constraint: \( \text{icol}[i-1] \geq 1 \) and \( \text{icol}[i-1] \leq n \).
   Check that \( a \), \( \text{irow} \), \( \text{icol} \), \( \text{ipivp} \), \( \text{ipivq} \), \( \text{istr} \) and \( \text{idig} \) have not been corrupted between calls to \( \text{nag_sparse_nherm_precon_ilu_solve} \) and \( \text{nag_sparse_nherm_fac} \) (f11dnc).

   On entry, \( i = \langle \text{value} \rangle \), \( \text{irow}[i-1] = \langle \text{value} \rangle \), \( n = \langle \text{value} \rangle \).
   Constraint: \( \text{irow}[i-1] \geq 1 \) and \( \text{irow}[i-1] \leq n \).
   Check that \( a \), \( \text{irow} \), \( \text{icol} \), \( \text{ipivp} \), \( \text{ipivq} \), \( \text{istr} \) and \( \text{idig} \) have not been corrupted between calls to \( \text{nag_sparse_nherm_precon_ilu_solve} \) and \( \text{nag_sparse_nherm_fac} \) (f11dnc).

NE_INVALID_CS_PRECOND
   On entry, \( i = \langle \text{value} \rangle \), \( \text{ipivp}[i-1] = \langle \text{value} \rangle \), \( n = \langle \text{value} \rangle \).
   Constraint: \( \text{ipivp}[i-1] \geq 1 \) and \( \text{ipivp}[i-1] \leq n \).
   Check that \( a \), \( \text{irow} \), \( \text{icol} \), \( \text{ipivp} \), \( \text{ipivq} \), \( \text{istr} \) and \( \text{idig} \) have not been corrupted between calls to \( \text{nag_sparse_nherm_precon_ilu_solve} \) and \( \text{nag_sparse_nherm_fac} \) (f11dnc).

   On entry, \( \text{istr} \) appears to be invalid.
   Check that \( a \), \( \text{irow} \), \( \text{icol} \), \( \text{ipivp} \), \( \text{ipivq} \), \( \text{istr} \) and \( \text{idig} \) have not been corrupted between calls to \( \text{nag_sparse_nherm_precon_ilu_solve} \) and \( \text{nag_sparse_nherm_fac} \) (f11dnc).

   On entry, \( \text{istr}[i-1] \) is inconsistent with \( \text{irow} \): \( i = \langle \text{value} \rangle \).
   Check that \( a \), \( \text{irow} \), \( \text{icol} \), \( \text{ipivp} \), \( \text{ipivq} \), \( \text{istr} \) and \( \text{idig} \) have not been corrupted between calls to \( \text{nag_sparse_nherm_precon_ilu_solve} \) and \( \text{nag_sparse_nherm_fac} \) (f11dnc).

NE_INVALID_ROWCOL_PIVOT
   On entry, \( i = \langle \text{value} \rangle \), \( \text{ipivp}[i-1] = \langle \text{value} \rangle \), \( n = \langle \text{value} \rangle \).
   Constraint: \( \text{ipivq}[i-1] \geq 1 \) and \( \text{ipivq}[i-1] \leq n \).
   Check that \( a \), \( \text{irow} \), \( \text{icol} \), \( \text{ipivp} \), \( \text{ipivq} \), \( \text{istr} \) and \( \text{idig} \) have not been corrupted between calls to \( \text{nag_sparse_nherm_precon_ilu_solve} \) and \( \text{nag_sparse_nherm_fac} \) (f11dnc).

   On entry, \( \text{ipivp}[i-1] \) is a repeated value: \( i = \langle \text{value} \rangle \).
   Check that \( a \), \( \text{irow} \), \( \text{icol} \), \( \text{ipivp} \), \( \text{ipivq} \), \( \text{istr} \) and \( \text{idig} \) have not been corrupted between calls to \( \text{nag_sparse_nherm_precon_ilu_solve} \) and \( \text{nag_sparse_nherm_fac} \) (f11dnc).
On entry, \( \text{ipivq}[i - 1] \) is a repeated value: \( i = \langle \text{value} \rangle \).
Check that \( a, \text{irow}, \text{icol}, \text{ipivp}, \text{ipivq}, \text{istr} \) and \( \text{idag} \) have not been corrupted between calls to nag_sparse_nherm_precon_ilu_solve (f11dpc) and nag_sparse_nherm_fac (f11dnc).

**NE_NO_LICENCE**

Your licence key may have expired or may not have been installed correctly. See Section 3.6.5 in the Essential Introduction for further information.

**NE_NOT_STRICTLY_INCREASING**

On entry, \( a[i - 1] \) is out of order: \( i = \langle \text{value} \rangle \).
Check that \( a, \text{irow}, \text{icol}, \text{ipivp}, \text{ipivq}, \text{istr} \) and \( \text{idag} \) have not been corrupted between calls to nag_sparse_nherm_precon_ilu_solve (f11dpc) and nag_sparse_nherm_fac (f11dnc).

On entry, the location \( \text{irow}[i - 1], \text{icol}[i - 1] \) is a duplicate: \( i = \langle \text{value} \rangle \).
Check that \( a, \text{irow}, \text{icol}, \text{ipivp}, \text{ipivq}, \text{istr} \) and \( \text{idag} \) have not been corrupted between calls to nag_sparse_nherm_precon_ilu_solve (f11dpc) and nag_sparse_nherm_fac (f11dnc).

7 Accuracy

If \( \text{trans} = \text{Nag_NoTrans} \) the computed solution \( x \) is the exact solution of a perturbed system of equations \( (M + \delta M)x = y \), where

\[
|\delta M| \leq c(n)\epsilon P|L||D||U|Q,
\]

\( c(n) \) is a modest linear function of \( n \), and \( \epsilon \) is the *machine precision*. An equivalent result holds when \( \text{trans} = \text{Nag_Trans} \).

8 Parallelism and Performance

Not applicable.

9 Further Comments

9.1 Timing

The time taken for a call to nag_sparse_nherm_precon_ilu_solve (f11dpc) is proportional to the value of \( \text{nnzc} \) returned from nag_sparse_nherm_fac (f11dnc).

9.2 Use of check

It is expected that a common use of nag_sparse_nherm_precon_ilu_solve (f11dpc) will be to carry out the preconditioning step required in the application of nag_sparse_nherm_basic_solver (f11bsc) to sparse complex linear systems. In this situation nag_sparse_nherm_precon_ilu_solve (f11dpc) is likely to be called many times with the same matrix \( M \). In the interests of both reliability and efficiency, you are recommended to set \( \text{check} = \text{Nag_SparseNsym_Check} \) for the first of such calls, and to set \( \text{check} = \text{Nag_SparseNsym_NoCheck} \) for all subsequent calls.

10 Example

This example reads in a complex sparse non-Hermitian matrix \( A \) and a vector \( y \). It then calls nag_sparse_nherm_fac (f11dnc), with \( \text{ilfill} = -1 \) and \( \text{dtol} = 0.0 \), to compute the complete \( LU \) decomposition

\[
A = PLDUQ.
\]

Finally it calls nag_sparse_nherm_precon_ilu_solve (f11dpc) to solve the system

\[
PLDUQx = y.
\]
10.1 Program Text

/* nag_sparse_nherm_precon_ilu_solve (f11dpc) Example Program. 
* Copyright 2014 Numerical Algorithms Group. 
* Mark 23, 2011. 
*/
#include <nag.h>
#include <nag_stdlib.h>
#include <naga02.h>
#include <nagf11.h>
int main(void)
{
    /* Scalars */
    Integer exit_status = 0;
    double dtol;
    Integer i, la, lfill, n, nnz, nnzc, npivm;
    /* Arrays */
    Complex *a = 0, *x = 0, *y = 0;
    Integer *icol = 0, *idiag = 0, *ipivp = 0, *ipivq = 0,
    *irow = 0, *istr = 0;
    /* NAG types */
    Nag_SparseNsym_Piv pstrat;
    Nag_SparseNsym_Fact milu;
    Nag_SparseNsym_CheckData check;
    Nag_TransType trans;
    NagError fail;

    INIT_FAIL(fail);

    printf("nag_sparse_nherm_precon_ilu_solve (f11dpc) Example Program Results\n");
    printf("\n\n");
    /* Skip heading in data file*/
    #ifdef _WIN32
    scanf_s("%*[^
");
    #else
    scanf("%*[^
");
    #endif
    #ifdef _WIN32
    scanf_s("%"NAG_IFMT"%*[\n]", &n);
    #else
    scanf("%"NAG_IFMT"%*[\n]", &n);
    #endif
    #ifdef _WIN32
    scanf_s("%"NAG_IFMT"%*[\n]", &nnz);
    #else
    scanf("%"NAG_IFMT"%*[\n]", &nnz);
    #endif
    la = 3 * nnz;
    if (a = NAG_ALLOC((la), Complex)) ||
    (x = NAG_ALLOC((n), Complex)) ||
    (y = NAG_ALLOC((n), Complex)) ||
    (icol = NAG_ALLOC((la), Integer)) ||
    (idiag = NAG_ALLOC((n), Integer)) ||
    (ipivp = NAG_ALLOC((n), Integer)) ||
    (ipivq = NAG_ALLOC((n), Integer)) ||
    (irow = NAG_ALLOC((la), Integer)) ||
    (istr = NAG_ALLOC((n + 1), Integer))
    )
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}
    /* Read the non-zero elements of the matrix a*/
    for (i = 0; i < nnz; i++)
    #ifdef _WIN32
    scanf_s(" ( %lf , %lf ) %"NAG_IFMT"%"NAG_IFMT"%*[\n]",
        &a[i].re, &a[i].im, &irow[i], &icol[i]);
    #else
    scanf(" ( %lf , %lf ) %"NAG_IFMT"%"NAG_IFMT"%*[\n]",
        &a[i].re, &a[i].im, &irow[i], &icol[i]);
    #endif
    */

Mark 25

f11dpc5

f11 – Large Scale Linear Systems
```c
scanf(" %lf , %lf ") %"NAG_IPMT"%"NAG_IPMT"%*[\`n]",
   &a[i].re, &a[i].im, &irow[i], &icol[i]);
#endif
/* Read the vector y */
#ifdef _WIN32
for (i = 0; i < n; i++) scanf_s(" %lf , %lf ", &y[i].re, &y[i].im);
#else
for (i = 0; i < n; i++) scanf(" %lf , %lf ", &y[i].re, &y[i].im);
#endif
/* Calculate LU factorization*/
lfill = -1;
dtol = 0.0;
pstrat = Nag_SparseNsym_CompletePiv;
milu = Nag_SparseNsym_UnModFact;
/* nag_sparse_nherm_fac (f11dnc).
   * Complex sparse non-Hermitian linear systems, incomplete LU factorization
   */
nag_sparse_nherm_fac(n, nnz, a, la, irow, icol, lfill, dtol, pstrat, milu,
   ipivp, ipivq, istr, idiag, &nnzc, &npivm, &fail);
if (fail.code != NE_NOERROR) {
   printf("Error from nag_sparse_nherm_fac (f11dnc).\n%s\n", fail.message);
   exit_status = 1;
   goto END;
}
/* Check value of npivm */
if (npivm > 0) {
   printf("Factorization is not complete\n");
} else {
   /* Solve P L D U x = y */
   check = Nag_SparseNsym_Check;
   trans = Nag_NoTrans;
   /* nag_sparse_nherm_precon_ilu_solve (f11dpc).
      * Solution of complex linear system involving incomplete LU
      * preconditioning matrix
      */
nag_sparse_nherm_precon_ilu_solve(trans, n, a, la, irow, icol, ipivp, ipivq,
   istr, idiag, check, y, x, &fail);
   if (fail.code != NE_NOERROR) {
      printf("Error from nag_sparse_nherm_precon_ilu_solve.\n%s\n", fail.message);
      exit_status = 2;
      goto END;
   }
   /* Output results*/
   printf(" Solution of linear system \n");
   for (i = 0; i < n; i++) printf(" ( %13.4e, %13.4e) \n", x[i].re, x[i].im);
}
END:
NAG_FREE(a);
NAG_FREE(x);
NAG_FREE(y);
NAG_FREE(icol);
NAG_FREE(idiag);
NAG_FREE(ipivp);
NAG_FREE(ipivq);
NAG_FREE(irow);
NAG_FREE(istr);
return exit_status;
```
10.2 Program Data

nag_sparse_nherm_precon_ilu_solve (f11dpc) Example Program Data

4 : n
11 : nnz
( 1., 2. ) 1 2
( 1., 3. ) 1 3
(-1.,-3. ) 2 1
( 2., 0. ) 2 3
( 0., 4. ) 2 4
( 3., 4. ) 3 1
(-2., 0. ) 3 4
( 1.,-1. ) 4 1
(-2.,-1. ) 4 2
( 1., 0. ) 4 3
( 1., 3. ) 4 4 : a[i], irow[i], icol[i], i=0,...,nnz-1
( 5.0, 14.0)
( 21.0, 5.0)
(-21.0, 18.0)
( 14.0, 4.0) : y[i], i=0,...,n-1

10.3 Program Results

nag_sparse_nherm_precon_ilu_solve (f11dpc) Example Program Results

Solution of linear system
( 1.0000e+00, 4.0000e+00)
( 2.0000e+00, 3.0000e+00)
( 3.0000e+00, -2.0000e+00)
( 4.0000e+00, -1.0000e+00)