NAG Library Function Document

nag_dggrqf (f08zfc)

1 Purpose

nag_dggrqf (f08zfc) computes a generalized $RQ$ factorization of a real matrix pair $(A, B)$, where $A$ is an $m \times n$ matrix and $B$ is a $p \times n$ matrix.

2 Specification

```c
#include <nag.h>
#include <nagf08.h>

void nag_dggrqf (Nag_OrderType order, Integer m, Integer p, Integer n, 
double a[], Integer pda, double taua[], double b[], Integer pdb, 
double taub[], NagError *fail)
```

3 Description

nag_dggrqf (f08zfc) forms the generalized $RQ$ factorization of an $m \times n$ matrix $A$ and a $p \times n$ matrix $B$

$$A = RQ, \quad B = ZTQ,$$

where $Q$ is an $n \times n$ orthogonal matrix, $Z$ is a $p \times p$ orthogonal matrix and $R$ and $T$ are of the form

$$R = \begin{cases} 
m \begin{pmatrix} n-m & m \\
0 & R_{12} \end{pmatrix}; & \text{if } m \leq n, \\
m - n \begin{pmatrix} R_{11} \\
R_{21} \end{pmatrix}; & \text{if } m > n,
\end{cases}$$

with $R_{12}$ or $R_{21}$ upper triangular,

$$T = \begin{cases} 
p \begin{pmatrix} n \\
p - n \end{pmatrix}; & \text{if } p \geq n, \\
\begin{pmatrix} p & n - p \\
p & T_{12} \end{pmatrix}; & \text{if } p < n,
\end{cases}$$

with $T_{11}$ upper triangular.

In particular, if $B$ is square and nonsingular, the generalized $RQ$ factorization of $A$ and $B$ implicitly gives the $RQ$ factorization of $AB^{-1}$ as

$$AB^{-1} = (RT^{-1})Z^T.$$

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, 
Philadelphia http://www.netlib.org/lapack/lug

Anderson E, Bai Z and Dongarra J (1992) Generalized $QR$ factorization and its applications Linear 
Algebra Appl. (Volume 162–164) 243–271


5 Arguments

1:  order – Nag_OrderType

   On entry: the order argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by order = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

   Constraint: order = Nag_RowMajor or Nag_ColMajor.

2:  m – Integer

   On entry: m, the number of rows of the matrix A.

   Constraint: m ≥ 0.

3:  p – Integer

   On entry: p, the number of rows of the matrix B.

   Constraint: p ≥ 0.

4:  n – Integer

   On entry: n, the number of columns of the matrices A and B.

   Constraint: n ≥ 0.

5:  a[dim] – double

   Note: the dimension, dim, of the array a must be at least

   max(1, pda × n) when order = Nag_ColMajor;
   max(1, m × pda) when order = Nag_RowMajor.

   Where A(i, j) appears in this document, it refers to the array element

   a[(j - 1) × pda + i - 1] when order = Nag_ColMajor;
   a[(i - 1) × pda + j - 1] when order = Nag_RowMajor.

   On entry: the m by n matrix A.

   On exit: if m ≤ n, the upper triangle of the subarray A(1 : m, n - m + 1 : n) contains the m by m upper triangular matrix R_{12}.

   If m ≥ n, the elements on and above the (m - n)th subdiagonal contain the m by n upper trapezoidal matrix R; the remaining elements, with the array taua, represent the orthogonal matrix Q as a product of min(m, n) elementary reflectors (see Section 3.3.6 in the f08 Chapter Introduction).

6:  pda – Integer

   On entry: the stride separating row or column elements (depending on the value of order) in the array a.

   Constraints:

   if order = Nag_ColMajor, pda ≥ max(1, m);
   if order = Nag_RowMajor, pda ≥ max(1, n).
On exit: the scalar factors of the elementary reflectors which represent the orthogonal matrix $Q$.

8. $b[dim]$ – double

*Input/Output*

Note: the dimension, $dim$, of the array $b$ must be at least

$max(1, p \times n)$ when $order = \text{Nag\_ColMajor};$

$max(1, p \times pdb)$ when $order = \text{Nag\_RowMajor}.$

The $(i, j)$th element of the matrix $B$ is stored in

$b[(j - 1) \times pdb + i - 1]$ when $order = \text{Nag\_ColMajor};$

$b[(i - 1) \times pdb + j - 1]$ when $order = \text{Nag\_RowMajor}.$

On exit: the elements on and above the diagonal of the array contain the $\min(p, n)$ by $n$ upper trapezoidal matrix $T$ ($T$ is upper triangular if $p \geq n$); the elements below the diagonal, with the array $taub$, represent the orthogonal matrix $Z$ as a product of elementary reflectors (see Section 3.3.6 in the f08 Chapter Introduction).

9. $pdb$ – Integer

*Input*

On entry: the stride separating row or column elements (depending on the value of $order$) in the array $b$.

Constraints:

if $order = \text{Nag\_ColMajor}, pdb \geq max(1, p);$ 

if $order = \text{Nag\_RowMajor}, pdb \geq max(1, n).$

10. $taub[min(p, n)]$ – double

*Output*

On exit: the scalar factors of the elementary reflectors which represent the orthogonal matrix $Z$.

11. $fail$ – NagError *

*Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

**NE_ALLOC_FAIL**

Dynamic memory allocation failed.

See Section 3.2.1.2 in the Essential Introduction for further information.

**NE_BAD_PARAM**

On entry, argument $\langle value \rangle$ had an illegal value.

**NE_INT**

On entry, $m = \langle value \rangle$.

Constraint: $m \geq 0$.

On entry, $n = \langle value \rangle$.

Constraint: $n \geq 0$.

On entry, $p = \langle value \rangle$.

Constraint: $p \geq 0$.

On entry, $pda = \langle value \rangle$.

Constraint: $pda > 0$. 

Mark 25
On entry, \( \text{pdb} = \langle \text{value} \rangle \).
Constraint: \( \text{pdb} > 0 \).

**NE_INT_2**

On entry, \( \text{pda} = \langle \text{value} \rangle \) and \( \text{m} = \langle \text{value} \rangle \).
Constraint: \( \text{pda} \geq \max(1, \text{m}) \).

On entry, \( \text{pda} = \langle \text{value} \rangle \) and \( \text{n} = \langle \text{value} \rangle \).
Constraint: \( \text{pda} \geq \max(1, \text{n}) \).

On entry, \( \text{pdb} = \langle \text{value} \rangle \) and \( \text{n} = \langle \text{value} \rangle \).
Constraint: \( \text{pdb} \geq \max(1, \text{n}) \).

**NE_INTERNAL_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG. See Section 3.6.6 in the Essential Introduction for further information.

**NE_NO_LICENCE**

Your licence key may have expired or may not have been installed correctly. See Section 3.6.5 in the Essential Introduction for further information.

7 Accuracy

The computed generalized \( RQ \) factorization is the exact factorization for nearby matrices \( (A+E) \) and \( (B+F) \), where

\[
\|E\|_2 = O \varepsilon A \|_2 \quad \text{and} \quad \|F\|_2 = O \varepsilon B \|_2,
\]

and \( \varepsilon \) is the machine precision.

8 Parallelism and Performance

\( \text{nag_dggrqf (f08zfc)} \) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

\( \text{nag_dggrqf (f08zfc)} \) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users’ Note for your implementation for any additional implementation-specific information.

9 Further Comments

The orthogonal matrices \( Q \) and \( Z \) may be formed explicitly by calls to \( \text{nag_dorgrq (f08cjc)} \) and \( \text{nag_dorgqr (f08afc)} \) respectively. \( \text{nag_dormrq (f08ckc)} \) may be used to multiply \( Q \) by another matrix and \( \text{nag_dormqr (f08agc)} \) may be used to multiply \( Z \) by another matrix.

The complex analogue of this function is \( \text{nag_zggrqf (f08ztc)} \).
10 Example

This example solves the least squares problem

$$\minimize_x \|c - Ax\|_2 \quad \text{subject to} \quad Bx = d$$

where

$$A = \begin{pmatrix}
-0.57 & -1.28 & -0.39 & 0.25 \\
-1.93 & 1.08 & -0.31 & -2.14 \\
2.30 & 0.24 & 0.40 & -0.35 \\
-1.93 & 0.64 & -0.66 & 0.08 \\
0.15 & 0.30 & 0.15 & -2.13 \\
-0.02 & 1.03 & -1.43 & 0.50
\end{pmatrix}, \quad B = \begin{pmatrix}
1 & 0 & -1 & 0 \\
0 & 1 & 0 & -1
\end{pmatrix},$$

$$c = \begin{pmatrix}
-1.50 \\
-2.14 \\
1.23 \\
-0.54 \\
-1.68 \\
0.82
\end{pmatrix} \quad \text{and} \quad d = \begin{pmatrix}
0 \\
0
\end{pmatrix}.$$

The constraints $Bx = d$ correspond to $x_1 = x_3$ and $x_2 = x_4$.

The solution is obtained by first computing a generalized $RQ$ factorization of the matrix pair $(B, A)$. The example illustrates the general solution process.

10.1 Program Text

.IsAny("nag_dggrqf (f08zfc) Example Program.
* * Copyright 2014 Numerical Algorithms Group.
* * Mark 23, 2011.
* /
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagf08.h>
#include <nagf16.h>

int main(void)
{
    /* Scalars */
    double alpha, beta, rnorm;
    Integer i, j, m, n, p, pda, pdb, pdc, pdd, pdx;
    Integer y1rows, y2rows, y3rows;
    Integer exit_status = 0;
    /* Arrays */
    double *a = 0, *b = 0, *c = 0, *d = 0, *taua = 0, *taub = 0, *x = 0;
    /* Nag Types */
    NagError fail;
    Nag_OrderType order;
    
    #ifdef NAG_COLUMN_MAJOR
    #define A(I, J) a[(J-1)*pda +I-1 ]
    #define B(I, J) b[(J-1)*pdb +I-1 ]
    order = Nag_ColMajor;
    #else
    #define A(I, J) a[(I-1)*pda +J-1 ]
    #define B(I, J) b[(I-1)*pdb +J-1 ]
    order = Nag_RowMajor;
    #endif
    
    * Nag Types */
    NagError fail;
    Nag_OrderType order;
    
    * Scalars */
    double alpha, beta, rnorm;
    Integer i, j, m, n, p, pda, pdb, pdc, pdd, pdx;
    Integer y1rows, y2rows, y3rows;
    Integer exit_status = 0;
    
    * Arrays */
    double *a = 0, *b = 0, *c = 0, *d = 0, *taua = 0, *taub = 0, *x = 0;
    
    return 0;
}
#endif

INIT_FAIL(fail);

printf("nag_dggrqf (f08zfc) Example Program Results\n\n");

/* Skip heading in data file */
#else _WIN32
    scanf_s("%*[\n"]);
#else
    scanf("%*[\n"]);
#endif
#endif _WIN32
    scanf_s("%"NAG_IFMT "%"NAG_IFMT "%"NAG_IFMT"%*[\n"]", &m, &n, &p);
#else
    scanf("%"NAG_IFMT "%"NAG_IFMT "%"NAG_IFMT"%*[\n"]", &m, &n, &p);
#endif

if( n<0 | | m<0 | | p<0 )
{
    printf("Invalid n, m or p\n");
    exit_status = 1;
    goto END;
}

#ifdef NAG_COLUMN_MAJOR
    pda = m;
    pdb = p;
    pdc = m;
    pdd = p;
    pdx = n;
#else
    pda = n;
    pdb = n;
    pdc = 1;
    pdd = 1;
    pdx = 1;
#endif

/* Allocate memory */
if (!(a = NAG_ALLOC(m*n, double)) ||
    !(b = NAG_ALLOC(p*n, double)) ||
    !(c = NAG_ALLOC(m, double)) ||
    !(d = NAG_ALLOC(p, double)) ||
    !(taua = NAG_ALLOC(MIN(m, n), double)) ||
    !(taub = NAG_ALLOC(MIN(n, p), double)) ||
    !(x = NAG_ALLOC(n, double)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read A, B, c and d from data file for the problem
 * min{||c-Ax||_2, x in R^n and Bx = d}.
 */
    for (i = 1; i <= m; ++i)
#ifdef _WIN32
        for (j = 1; j <= n; ++j) scanf_s("%lf", &A(i, j));
#else
        for (j = 1; j <= n; ++j) scanf("%lf", &A(i, j));
#endif
#ifdef _WIN32
    scanf_s("%*[\n"]);
#else
    scanf("%*[\n"]);
#endif
    for (i = 1; i <= p; ++i)
#ifdef _WIN32
        for (j = 1; j <= n; ++j) scanf_s("%lf", &B(i, j));
#else
        for (j = 1; j <= n; ++j) scanf("%lf", &B(i, j));
#endif
#ifdef _WIN32
    scanf_s("%*[\n"]);
#else
    scanf("%*[\n"]);
#endif
    for (i = 1; i <= p; ++i)
#ifdef _WIN32
        for (j = 1; j <= n; ++j) scanf_s("%lf", &B(i, j));
#else
        for (j = 1; j <= n; ++j) scanf("%lf", &B(i, j));
#endif
#ifdef _WIN32
            scanf_s("%*[\n"]);
#else
            scanf("%*[\n"]);
#endif
#endif
/* First compute the generalized RQ factorization of (B,A) as
   * B = (0 R12)*Q,   A = Z*(T11 T12 T13)*Q = T*Q.
   * ( 0 T22 T23)
   * where R12, T11 and T22 are upper triangular,
   * using nag_dggrqf (f08zfc).
*/
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_dggrqf (f08zfc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Now, trans(z)*(c-Ax) = trans(z)*c - T*Q*x, and
   * let f = (f1) = trans(z) * (c1) => minimize ||f - T*Q*x||
   *(f2) (c2)
   * Compute f using nag_dormqr (f08agc), storing result in c
*/
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_dormqr (f08agc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Putting Q*x = (y1), B * x = d becomes (0 R12) (y1) = d;
   * (w )
   * => R12 * w = d.
   * Solve for w using nag_dtrtrs (f07tec), storing result in d;
   * R12 is (p by p) triangular submatrix starting at B(1,n-p+1).
*/
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_dtrtrs (f07tec).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* The problem now reduces to finding the minimum norm of
* g = (g1) = (f1) - T11*y1 - (T12 T13)*w
  * (g2) = (f2) - (T22 T23)*w.
* Form c1 = f1 - (T12 T13)*w using nag_dgemv (f16pac).
*/
alpha = -1.0;
beta = 1.0;
y1rows = n - p;
nag_dgemv(order, Nag_NoTrans, y1rows, p, alpha, &A(1, n-p+1), pda, d, 1,
beta, c, 1, &fail);
if (fail.code != NE_NOERROR)
  { printf("Error from nag_dgemv (f16pac).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }
/* => now (g1) = c - T11*y1 and ||g1|| = 0 when T11 * y1 = c1.
* So now (g1) = c - T11*y1 and ||g1|| = 0 when T11 * y1 = c1.
* Solve this for y1 using nag_dtrtrs (f07tec) storing result in c1.
*/
nag_dtrtrs(order, Nag_Upper, Nag_NoTrans, Nag_NonUnitDiag, y1rows, 1, a, pda,
c, pdc, &fail);
if (fail.code != NE_NOERROR)
  { printf("Error from nag_dtrtrs (f07tec).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }
/* So now Q*x = (y1) is stored in (c1), which is now copied to x.
  * (w )
  *( d )
*/
for (i = 0; i < y1rows; ++i) x[i] = c[i];
for (i = y1rows; i < n; ++i) x[i] = d[i-y1rows];
/* Compute x by applying transpose of Q using nag_dormrq (f08ckc). */
nag_dormrq(order, Nag_LeftSide, Nag_Trans, n, 1, p, b, pdb, taub, x, pdx,
&fail);
if (fail.code != NE_NOERROR)
  { printf("Error from nag_dormrq (f08ckc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }
/* It remains to minimize ||g2||, g2 = f2 - (T22 T23)*w.
* Putting w = (y2), gives g2 = f2 - T22*y2 - T23*y3
* (y3)
* [y2 stored in d1, first y2rows of d; y3 stored in d2, next n-m rows of d.]
* First form T22*y2 using nag_dtrmv (f16pfc) where y2 is held in d.
* / 
y2rows = MIN(m, n) - y1rows;
algebra = 1.0;
nag_dtrmv(order, Nag_Upper, Nag_NoTrans, Nag_NonUnitDiag, y2rows, alpha,
&A(n-p+1, n-p+1), pda, d, 1, &fail);
if (fail.code != NE_NOERROR)
  { printf("Error from nag_dtrmv (f16pfc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }
/* Then, f2 - T22*y2 (c2 = c2 - d) */
for (i = 0; i < y2rows; ++i) c[y1rows + i] -= d[i];
y2rows = m - y1rows;
if (m < n)
  { y3rows = n - m;
    /* Then g2 = f2 - T22*y2 - T23*y3 (c2 = c2 - T23*d2) */
    alpha = -1.0;
beta = 1.0;
nag_dgemv(order, Nag_NoTrans, y2rows, y3rows, alpha, &A(n-p+1, m+1), pda,

&d[y2rows], 1, beta, &c[y1rows], 1, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_dgemv (f16pac).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Compute ||g|| = ||g2|| = norm(f2 - T22*y2 - T23*y3)
* using nag_dge_norm (f16rac).
*/
ag_dge_norm(Nag_ColMajor, Nag_FrobeniusNorm, y2rows, 1, &c[y1rows], y2rows,
    &rnorm, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_dge_norm (f16rac).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Print least squares solution x */
printf("Constrained least squares solution
");
for (i = 0; i < n; ++i) printf(" %10.4f%s", x[i], i%7 == 6?"\n":"");

/* Print the square root of the residual sum of squares */
printf("\nSquare root of the residual sum of squares\n");
printf("%11.2e\n", rnorm);

END:
NAG_FREE(a);
NAG_FREE(b);
NAG_FREE(c);
NAG_FREE(d);
NAG_FREE(taua);
NAG_FREE(taub);
NAG_FREE(x);
return exit_status;

10.2 Program Data

nag_dggrqf (f08zfc) Example Program Data

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 6 | 4 | 2 :

-0.57 -1.28 -0.39 0.25
-1.93 1.08 -0.31 -2.14
2.30 0.24 0.40 -0.35
-1.93 0.64 -0.66 0.08
0.15 0.30 0.15 -2.13
-0.02 1.03 -1.43 0.50 : matrix A[m*n]

1.00 0.00 -1.00 0.00
0.00 1.00 0.00 -1.00 : matrix B[p*n]

-1.50
-2.14
1.23
-0.54
-1.68
0.82 : vector c[m]
0.00
0.00 : vector d[p]
10.3 Program Results

nag_dggrqf (f08zfc) Example Program Results

Constrained least squares solution

0.4890  0.9975  0.4890  0.9975

Square root of the residual sum of squares

2.51e-02