NAG Library Function Document

nag_dpbstf (f08ufc)

1 Purpose
nag_dpbstf (f08ufc) computes a split Cholesky factorization of a real symmetric positive definite band matrix.

2 Specification

```c
#include <nag.h>
#include <nagf08.h>
void nag_dpbstf (Nag_OrderType order, Nag_UploType uplo, Integer n,
    Integer kb, double bb[], Integer pdbb, NagError *fail)
```

3 Description
nag_dpbstf (f08ufc) computes a split Cholesky factorization of a real symmetric positive definite band matrix $B$. It is designed to be used in conjunction with nag_dsbgst (f08uec).

The factorization has the form $B = S^T S$, where $S$ is a band matrix of the same bandwidth as $B$ and the following structure: $S$ is upper triangular in the first $(n+k)/2$ rows, and transposed — hence, lower triangular — in the remaining rows. For example, if $n = 9$ and $k = 2$, then

$$
S = \begin{pmatrix}
S_{11} & S_{12} & S_{13} \\
S_{22} & S_{23} & S_{24} \\
S_{33} & S_{34} & S_{35} \\
S_{44} & S_{45} & & \\
S_{55} & & & \\
S_{64} & S_{65} & S_{66} & \\
S_{75} & S_{76} & S_{77} & \\
S_{86} & S_{87} & S_{88} & \\
S_{97} & S_{98} & S_{99} & \\
\end{pmatrix}.
$$

4 References

None.

5 Arguments

1: `order` – Nag_OrderType
   
   *Input*
   
   On entry: the order argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by `order = Nag_RowMajor`. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.
   
   Constraint: `order = Nag_RowMajor` or `Nag_ColMajor`.

2: `uplo` – Nag_UploType
   
   *Input*
   
   On entry: indicates whether the upper or lower triangular part of $B$ is stored.
   
   `uplo = Nag_Upper`
   
   The upper triangular part of $B$ is stored.
uplo = Nag_Lower
The lower triangular part of $B$ is stored.

Constraint: $\text{uplo} = \text{Nag_Upper}$ or $\text{Nag_Lower}$.

3: $n$ – Integer

Input

On entry: $n$, the order of the matrix $B$.

Constraint: $n \geq 0$.

4: $\text{kb}$ – Integer

Input

On entry: if $\text{uplo} = \text{Nag_Upper}$, the number of superdiagonals, $k_b$, of the matrix $B$.

If $\text{uplo} = \text{Nag_Lower}$, the number of subdiagonals, $k_b$, of the matrix $B$.

Constraint: $\text{kb} \geq 0$.

5: $\text{bb}[\text{dim}]$ – double

Input/Output

Note: the dimension, $\text{dim}$, of the array $\text{bb}$ must be at least $\max(1, \text{pdbb} \times n)$.

On entry: the $n$ by $n$ symmetric positive definite band matrix $B$.

This is stored as a notional two-dimensional array with row elements or column elements stored contiguously. The storage of elements of $B_{ij}$, depends on the order and uplo arguments as follows:

if order = Nag_ColMajor and uplo = Nag_Upper,

$B_{ij}$ is stored in $\text{bb}[k_b + i - j + (j - 1) \times \text{pdbb}]$, for $j = 1, \ldots, n$ and $i = \max(1, j - k_b), \ldots, j$;

if order = Nag_ColMajor and uplo = Nag_Lower,

$B_{ij}$ is stored in $\text{bb}[i - j + (j - 1) \times \text{pdbb}]$, for $j = 1, \ldots, n$ and $i = j, \ldots, \min(n, j + k_b)$;

if order = Nag_RowMajor and uplo = Nag_Upper,

$B_{ij}$ is stored in $\text{bb}[j - i + (i - 1) \times \text{pdbb}]$, for $i = 1, \ldots, n$ and $j = i, \ldots, \min(n, i + k_b)$;

if order = Nag_RowMajor and uplo = Nag_Lower,

$B_{ij}$ is stored in $\text{bb}[k_b + j - i + (i - 1) \times \text{pdbb}]$, for $i = 1, \ldots, n$ and $j = \max(1, i - k_b), \ldots, i$.

On exit: $B$ is overwritten by the elements of its split Cholesky factor $S$.

6: $\text{pdbb}$ – Integer

Input

On entry: the stride separating row or column elements (depending on the value of order) of the matrix $B$ in the array $\text{bb}$.

Constraint: $\text{pdbb} \geq \text{kb} + 1$.

7: $\text{fail}$ – NagError*

Input/Output

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM

On entry, argument $\langle\text{value}\rangle$ had an illegal value.
On entry, $kb = \langle value\rangle$.
Constraint: $kb \geq 0$.

On entry, $n = \langle value\rangle$.
Constraint: $n \geq 0$.

On entry, $pdbb = \langle value\rangle$.
Constraint: $pdbb > 0$.

On entry, $pdbb = \langle value\rangle$ and $kb = \langle value\rangle$.
Constraint: $pdbb \geq kb + 1$.

An internal error has occurred in this function. Check the function call and any array sizes. If the
call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

The factorization could not be completed, because the updated element $b(\langle value\rangle, \langle value\rangle)$ would
be the square root of a negative number. Hence $B$ is not positive definite. This may indicate an
error in forming the matrix $B$.

The computed factor $S$ is the exact factor of a perturbed matrix $(B + E)$, where
\[ |E| \leq c(k + 1) \epsilon |S^T||S|, \]
$c(k + 1)$ is a modest linear function of $k + 1$, and $\epsilon$ is the machine precision. It follows that
\[ |e_{ij}| \leq c(k + 1) \epsilon \sqrt{(b_{ii}b_{jj})}. \]

nag_dpbstf (f08ufc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the
vendor library used by this implementation. Consult the documentation for the vendor library for further
information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the
OpenMP environment used within this function. Please also consult the Users’ Note for your
implementation for any additional implementation-specific information.

The total number of floating-point operations is approximately $n(k + 1)^2$, assuming $n \gg k$.
A call to nag_dpbstf (f08ufc) may be followed by a call to nag_dsbgst (f08uec) to solve the generalized
eigenproblem $Az = \lambda Bz$, where $A$ and $B$ are banded and $B$ is positive definite.
The complex analogue of this function is nag_zpbstf (f08utc).
10 Example

See Section 10 in nag_dsbgst (f08uec).