NAG Library Function Document
nag_dgerqf (f08chc)

1 Purpose
nag_dgerqf (f08chc) computes an RQ factorization of a real \( m \) by \( n \) matrix \( A \).

2 Specification

```c
#include <nag.h>
#include <nagf08.h>

void nag_dgerqf (Nag_OrderType order, Integer m, Integer n, double a[],
                 Integer pda, double tau[], NagError *fail)
```

3 Description

nag_dgerqf (f08chc) forms the \( RQ \) factorization of an arbitrary rectangular real \( m \) by \( n \) matrix. If \( m \leq n \), the factorization is given by

\[
A = (0 \quad R)Q,
\]

where \( R \) is an \( m \) by \( m \) lower triangular matrix and \( Q \) is an \( n \) by \( n \) orthogonal matrix. If \( m > n \) the factorization is given by

\[
A = RQ,
\]

where \( R \) is an \( m \) by \( n \) upper trapezoidal matrix and \( Q \) is again an \( n \) by \( n \) orthogonal matrix. In the case where \( m < n \) the factorization can be expressed as

\[
A = (0 \quad R)\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = RQ_2,
\]

where \( Q_1 \) consists of the first \( (n - m) \) rows of \( Q \) and \( Q_2 \) the remaining \( m \) rows.

The matrix \( Q \) is not formed explicitly, but is represented as a product of \( \min(m, n) \) elementary reflectors (see the f08 Chapter Introduction for details). Functions are provided to work with \( Q \) in this representation (see Section 9).

4 References


5 Arguments

1: \textbf{order} – Nag_OrderType \hspace{1cm} \textit{Input}

\textit{On entry}: the \textbf{order} argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by \textbf{order} = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

\textit{Constraint}: \textbf{order} = Nag_RowMajor or Nag_ColMajor.
m – Integer

**Input**

*On entry:* $m$, the number of rows of the matrix $A$.

*Constraint:* $m \geq 0$.

n – Integer

**Input**

*On entry:* $n$, the number of columns of the matrix $A$.

*Constraint:* $n \geq 0$.

$a[\dim]$ – double

**Input/Output**

*Note:* the dimension, $\dim$, of the array $a$ must be at least

$$
\max(1, pda \times n) \text{ when } order = \text{Nag\_ColMajor}; \\
\max(1, m \times pda) \text{ when } order = \text{Nag\_RowMajor}.
$$

Where $A(i, j)$ appears in this document, it refers to the array element

$$
a[(j - 1) \times pda + i - 1] \text{ when } order = \text{Nag\_ColMajor}; \\
a[(i - 1) \times pda + j - 1] \text{ when } order = \text{Nag\_RowMajor}.
$$

*On entry:* the $m$ by $n$ matrix $A$.

*On exit:* if $m \leq n$, the upper triangle of the subarray $A(1 : m, n - m + 1 : n)$ contains the $m$ by $m$ upper triangular matrix $R$.

If $m \geq n$, the elements on and above the $(m - n)$th subdiagonal contain the $m$ by $n$ upper trapezoidal matrix $R$; the remaining elements, with the array $tau$, represent the orthogonal matrix $Q$ as a product of $\min(m, n)$ elementary reflectors (see Section 3.3.6 in the f08 Chapter Introduction).

pda – Integer

**Input**

*On entry:* the stride separating row or column elements (depending on the value of $order$) in the array $a$.

*Constraints:*

$$
\text{if } order = \text{Nag\_ColMajor}, \quad pda \geq \max(1, m); \\
\text{if } order = \text{Nag\_RowMajor}, \quad pda \geq \max(1, n).
$$

tau[\dim] – double

**Output**

*Note:* the dimension, $\dim$, of the array $tau$ must be at least $\max(1, \min(m, n))$.

*On exit:* the scalar factors of the elementary reflectors.

fail – NagError *

**Input/Output**

The NAG error argument (see Section 3.6 in the Essential Introduction).

### 6 Error Indicators and Warnings

**NE_ALLOC_FAIL**

Dynamic memory allocation failed.

See Section 3.2.1.2 in the Essential Introduction for further information.

**NE_BAD_PARAM**

On entry, argument (value) had an illegal value.
NE_INT
On entry, $m = \langle \text{value} \rangle$.
Constraint: $m \geq 0$.

On entry, $n = \langle \text{value} \rangle$.
Constraint: $n \geq 0$.

On entry, $pda = \langle \text{value} \rangle$.
Constraint: $pda > 0$.

NE_INT_2
On entry, $pda = \langle \text{value} \rangle$ and $m = \langle \text{value} \rangle$.
Constraint: $pda \geq \max(1, m)$.

On entry, $pda = \langle \text{value} \rangle$ and $n = \langle \text{value} \rangle$.
Constraint: $pda \geq \max(1, n)$.

NE_INTERNAL_ERROR
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG. See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE
Your licence key may have expired or may not have been installed correctly. See Section 3.6.5 in the Essential Introduction for further information.

7 Accuracy
The computed factorization is the exact factorization of a nearby matrix $A + E$, where
\[ \|E\|_2 = O(\epsilon\|A\|_2) \]
and $\epsilon$ is the machine precision.

8 Parallelism and Performance
nag_dgerqf (f08chc) is not threaded by NAG in any implementation.

nag_dgerqf (f08chc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users’ Note for your implementation for any additional implementation-specific information.

9 Further Comments
The total number of floating-point operations is approximately $\frac{2}{3}m^2(3n - m)$ if $m \leq n$, or $\frac{2}{3}n^2(3m - n)$ if $m > n$.

To form the orthogonal matrix $Q$ nag_dgerqf (f08chc) may be followed by a call to nag_dorgrq (f08cjc):
\[
\text{nag_dorgrq}(\text{order}, n, n, \min(m, n), a, pda, tau, &\text{fail})
\]
where $\min(m, n)$, but note that the first dimension of the array $a$ must be at least $n$, which may be larger than was required by nag_dgerqf (f08chc). When $m \leq n$, it is often only the first $m$ rows of $Q$ that are required and they may be formed by the call:
\[
\text{nag_dorgrq}(\text{order}, m, n, m, a, pda, tau, c, pdc, &\text{fail})
\]
To apply $Q$ to an arbitrary real rectangular matrix $C$, nag_dgerqf (f08chc) may be followed by a call to nag_dormrq (f08ckc). For example:

\[
\text{nag_dormrq(Nag_LeftSide, Nag_Trans, n, p, minmn, a, pda, tau, c, pdc, &fail)}
\]

forms $C = Q^T C$, where $C$ is $n$ by $p$.

The complex analogue of this function is nag_zgerqf (f08cvc).

10 Example

This example finds the minimum norm solution to the underdetermined equations

\[ Ax = b \]

where

\[
A = \begin{pmatrix}
-5.42 & 3.28 & -3.68 & 0.27 & 2.06 & 0.46 \\
-1.65 & -3.40 & -3.20 & -1.03 & -4.06 & -0.01 \\
-0.37 & 2.35 & 1.90 & 4.31 & -1.76 & 1.13 \\
-3.15 & -0.11 & 1.99 & -2.70 & 0.26 & 4.50 \\
\end{pmatrix}
\]

and

\[
b = \begin{pmatrix}
-2.87 \\
1.63 \\
-3.52 \\
0.45 \\
\end{pmatrix}
\]

The solution is obtained by first obtaining an $RQ$ factorization of the matrix $A$.

10.1 Program Text

/* nag_dgerqf (f08chc) Example Program. */
/* Copyright 2014 Numerical Algorithms Group. */
/* Mark 23, 2011. */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagf08.h>
#include <nagf16.h>

int main(void)
{
    /* Scalars */
    Integer i, j, m, n, nrhs, pda, pdb, pdx;
    Integer exit_status = 0;
    /* Arrays */
    double *a = 0, *b = 0, *tau = 0, *x = 0;
    /* Nag Types */
    Nag_OrderType order;
    NagError fail;

    #ifdef NAG_COLUMN_MAJOR
    #define A(I, J) a[(J - 1) * pda + I - 1]
    #define B(I, J) b[(J - 1) * pdb + I - 1]
    order = Nag_ColMajor;
    #else
    #define A(I, J) a[(I - 1) * pda + J - 1]
    #define B(I, J) b[(I - 1) * pdb + J - 1]
    order = Nag_RowMajor;
    #endif

    INIT_FAIL(fail);

    printf("nag_dgerqf (f08chc) Example Program Results\n\n");

    /* Skip heading in data file */
    #ifdef _WIN32
    scanf_s("%*[\n]");
    #else
    scanf("%*[\n]");
    #endif
```c
scanf("%*[\n"]);
#endif
#if defined _WIN32
scanf_s("%"NAG_IFMT"%"NAG_IFMT"%"NAG_IFMT"%*[\n]", &m, &n, &nrhs);
#else
scanf("%"NAG_IFMT"%"NAG_IFMT"%"NAG_IFMT"%*[\n]", &m, &n, &nrhs);
#endif
#endif
#define NAG_COLUMN_MAJOR
pda = m;
pdb = m;
pdx = n;
#else
pda = n;
pdb = nrhs;
pdx = 1;
#endif
/* Allocate memory */
if (!((a = NAG_ALLOC(m*n, double)) ||
     (b = NAG_ALLOC(m*nrhs, double)) ||
     (tau = NAG_ALLOC(MAX(1, MIN(m, n)), double)) ||
     (x = NAG_ALLOC(n, double))))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}
/* Read the matrix A and the vectors b from data file */
for (i = 1; i <= m; ++i)
    for (j = 1; j <= n; ++j)
#if defined _WIN32
    scanf_s("%lf", &A(i, j));
#else
    scanf("%lf", &A(i, j));
#endif
#endif
#if defined _WIN32
scanf_s("%*[\n]");
#else
scanf("%*[\n]");
#endif
/* nag_dgerqf (f08chc).
* Compute the RQ factorization of A.
*/
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_dgerqf (f08chc).\n", fail.message);
    exit_status = 1;
    goto END;
}
/* nag_dge_copy (f16qfc).
* Copy the m element vector b into x2, where x2 is the vector
* containing the elements x(n-m+1), ..., x(n) of x.
*/
```
nag_dge_copy(order, Nag_NoTrans, m, 1, &B(1, 1), pdb, &x[n - m], pdx, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_dge_copy (f16qfc).\n\n", fail.message);
    exit_status = 1;
    goto END;
}

/* nag_dtrtrs (f07tec).
* Solve R*y2 = b, storing the result in x2.
*/
      nag_dtrtrs(order, Nag_Upper, Nag_NoTrans, Nag_NonUnitDiag, m, 1,
               &A(1, n - m + 1), pda, &x[n - m], pdx, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_dtrtrs (f07tec).\n\n", fail.message);
    exit_status = 1;
    goto END;
}

/* nag_dload (f16fbc).
* Set y1 to zero (stored in rows 1 to (n-m) of x).
*/
      nag_dload(n - m, 0.0, x, 1, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_dload (f16fbc).\n\n", fail.message);
    exit_status = 1;
    goto END;
}

/* nag_dormrq (f08ckc).
* Compute the minimum-norm solution x = (Q**T)*y.
*/
      nag_dormrq(order, Nag_LeftSide, Nag_Trans, n, 1, m, a, pda, tau, x, pdx,
               &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_dormrq (f08ckc).\n\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Print minimum-norm solution */
printf("Minimum-norm solution\n");
for (i = 0; i < n; ++i)
    printf("%9.4f%s", x[i], (i+1)%8 == 0?"\n":" ");

END:
NAG_FREE(a);
NAG_FREE(b);
NAG_FREE(tau);
NAG_FREE(x);

return exit_status;
}

#undef A
#undef B

10.2 Program Data

nag_dgerqf (f08chc) Example Program Data

<table>
<thead>
<tr>
<th>4</th>
<th>6</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5.42</td>
<td>3.28</td>
<td>-3.68</td>
</tr>
<tr>
<td>-1.65</td>
<td>-3.40</td>
<td>-3.20</td>
</tr>
<tr>
<td>-0.37</td>
<td>2.35</td>
<td>1.90</td>
</tr>
<tr>
<td>-3.15</td>
<td>-0.11</td>
<td>1.99</td>
</tr>
</tbody>
</table>

Values of m, n and nrhs

End of matrix A
10.3 Program Results

nag_dgerqf (f08chc) Example Program Results

Minimum-norm solution

\[
\begin{array}{cccccc}
0.2371 & -0.4575 & -0.0085 & -0.5192 & 0.0239 & -0.0543 \\
\end{array}
\]