NAG Library Function Document

nag_zgemqrt (f08aqc)

1 Purpose

nag_zgemqrt (f08aqc) multiplies an arbitrary complex matrix $C$ by the complex unitary matrix $Q$ from a $QR$ factorization computed by nag_zgeqrt (f08apc).

2 Specification

```c
#include <nag.h>
#include <nagf08.h>
void nag_zgemqrt (Nag_OrderType order, Nag_SideType side,
                   Nag_TransType trans, Integer m, Integer n, Integer k, Integer nb,
                   const Complex v[], Integer pdv, const Complex t[], Integer pdt,
                   Complex c[], Integer pdc, NagError *fail)
```

3 Description

nag_zgemqrt (f08aqc) is intended to be used after a call to nag_zgeqrt (f08apc), which performs a $QR$ factorization of a complex matrix $A$. The unitary matrix $Q$ is represented as a product of elementary reflectors.

This function may be used to form one of the matrix products

$$QC, Q^H C, CQ$$ or $$CQ^H$$,

overwriting the result on $C$ (which may be any complex rectangular matrix).

A common application of this function is in solving linear least squares problems, as described in the f08 Chapter Introduction and illustrated in Section 10 in nag_zgeqrt (f08apc).

4 References


5 Arguments

1: \textit{order} – Nag_OrderType \hspace{1cm} \textit{Input}

\textit{On entry}: the \textit{order} argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by \textit{order} = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

\textit{Constraint}: \textit{order} = Nag_RowMajor or Nag_ColMajor.

2: \textit{side} – Nag_SideType \hspace{1cm} \textit{Input}

\textit{On entry}: indicates how $Q$ or $Q^H$ is to be applied to $C$.

\textit{side} = Nag_LeftSide

$Q$ or $Q^H$ is applied to $C$ from the left.

\textit{side} = Nag_RightSide

$Q$ or $Q^H$ is applied to $C$ from the right.

\textit{Constraint}: \textit{side} = Nag_LeftSide or Nag_RightSide.
3: \[\text{trans} - \text{Nag\_TransType}\quad \text{Input}\]

\textit{On entry:} indicates whether \(Q\) or \(Q^H\) is to be applied to \(C\).

\text{trans} = \text{Nag\_NoTrans}
\(Q\) is applied to \(C\).

\text{trans} = \text{Nag\_ConjTrans}
\(Q^H\) is applied to \(C\).

\textit{Constraint:} \text{trans} = \text{Nag\_NoTrans} or \text{Nag\_ConjTrans}.

4: \[\text{m} - \text{Integer}\quad \text{Input}\]

\textit{On entry:} \(m\), the number of rows of the matrix \(C\).

\textit{Constraint:} \(m \geq 0\).

5: \[\text{n} - \text{Integer}\quad \text{Input}\]

\textit{On entry:} \(n\), the number of columns of the matrix \(C\).

\textit{Constraint:} \(n \geq 0\).

6: \[\text{k} - \text{Integer}\quad \text{Input}\]

\textit{On entry:} \(k\), the number of elementary reflectors whose product defines the matrix \(Q\). Usually \(k = \min(m_A, n_A)\) where \(m_A, n_A\) are the dimensions of the matrix \(A\) supplied in a previous call to \texttt{nag\_zgeqrt} (f08apc).

\textit{Constraints:}
\[
\begin{align*}
\text{if } & \text{side} = \text{Nag\_LeftSide}, \quad m \geq k \geq 0; \\
\text{if } & \text{side} = \text{Nag\_RightSide}, \quad n \geq k \geq 0.
\end{align*}
\]

7: \[\text{nb} - \text{Integer}\quad \text{Input}\]

\textit{On entry:} the block size used in the \(QR\) factorization performed in a previous call to \texttt{nag\_zgeqrt} (f08apc); this value must remain unchanged from that call.

\textit{Constraints:}
\[
\begin{align*}
\text{nb} & \geq 1; \\
\text{if } & k > 0, \text{nb} \leq k.
\end{align*}
\]

8: \[\text{v[dim]} - \text{const Complex}\quad \text{Input}\]

\textit{Note:} the dimension, \textit{dim}, of the array \(v\) must be at least
\[
\begin{align*}
\text{max}(1, \text{pdv} \times k) & \text{ when order} = \text{Nag\_ColMajor}; \\
\text{max}(1, \text{m} \times \text{pdv}) & \text{ when order} = \text{Nag\_RowMajor} \text{ and side} = \text{Nag\_LeftSide}; \\
\text{max}(1, \text{n} \times \text{pdv}) & \text{ when order} = \text{Nag\_RowMajor} \text{ and side} = \text{Nag\_RightSide}.
\end{align*}
\]

\textit{On entry:} details of the vectors which define the elementary reflectors, as returned by \texttt{nag\_zgeqrt} (f08apc) in the first \(k\) columns of its array argument \(a\).

9: \[\text{pdv} - \text{Integer}\quad \text{Input}\]

\textit{On entry:} the stride separating row or column elements (depending on the value of \texttt{order}) in the array \(v\).

\textit{Constraints:}
\[
\begin{align*}
\text{if } & \text{order} = \text{Nag\_ColMajor}, \\
\text{if } & \text{side} = \text{Nag\_LeftSide}, \text{pdv} \geq \text{max}(1, \text{m}); \\
\text{if } & \text{side} = \text{Nag\_RightSide}, \text{pdv} \geq \text{max}(1, \text{n}); \\
\text{if } & \text{order} = \text{Nag\_RowMajor}, \text{pdv} \geq \text{max}(1, k).
\end{align*}
\]
10: \( t[\text{dim}] \) – const Complex

**Input**

**Note:** the dimension, \( \text{dim} \), of the array \( t \) must be at least

\[
\max(1, p\text{dt} \times k) \quad \text{when order} = \text{Nag\_ColMajor};
\]

\[
\max(1, \text{nb} \times p\text{dt}) \quad \text{when order} = \text{Nag\_RowMajor}.
\]

The \((i,j)\)th element of the matrix \( T \) is stored in

\[
t[(j-1) \times p\text{dt} + i - 1] \quad \text{when order} = \text{Nag\_ColMajor};
\]

\[
t[(i-1) \times p\text{dt} + j - 1] \quad \text{when order} = \text{Nag\_RowMajor}.
\]

**On entry:** further details of the unitary matrix \( Q \) as returned by \textit{nag\_zgeqrt} (f08apc). The number of blocks is \( b = \lceil \frac{k}{\text{nb}} \rceil \), where \( k = \min(m, n) \) and each block is of order \( \text{nb} \) except for the last block, which is of order \( k - (b-1) \times \text{nb} \). For the \( b \) blocks the upper triangular block reflector factors \( T_1, T_2, \ldots, T_b \) are stored in the \( \text{nb} \) by \( n \) matrix \( T \) as \( T = [T_1 \mid T_2 \mid \ldots \mid T_b] \).

11: \( \text{pdt} \) – Integer

**Input**

**On entry:** the stride separating row or column elements (depending on the value of \textit{order}) in the array \( t \).

**Constraints:**

- if \( \text{order} = \text{Nag\_ColMajor}, \) \( \text{pdt} \geq \text{nb} \);
- if \( \text{order} = \text{Nag\_RowMajor}, \) \( \text{pdt} \geq \max(1, k) \).

12: \( c[\text{dim}] \) – Complex

**Input/Output**

**Note:** the dimension, \( \text{dim} \), of the array \( c \) must be at least

\[
\max(1, \text{pdc} \times n) \quad \text{when order} = \text{Nag\_ColMajor};
\]

\[
\max(1, m \times \text{pdc}) \quad \text{when order} = \text{Nag\_RowMajor}.
\]

The \((i,j)\)th element of the matrix \( C \) is stored in

\[
c[(j-1) \times \text{pdc} + i - 1] \quad \text{when order} = \text{Nag\_ColMajor};
\]

\[
c[(i-1) \times \text{pdc} + j - 1] \quad \text{when order} = \text{Nag\_RowMajor}.
\]

**On entry:** the \( m \) by \( n \) matrix \( C \).

**On exit:** \( c \) is overwritten by \( QC \) or \( Q^H C \) or \( CQ \) or \( CQ^H \) as specified by \textit{side} and \textit{trans}.

13: \( \text{pdc} \) – Integer

**Input**

**On entry:** the stride separating row or column elements (depending on the value of \textit{order}) in the array \( c \).

**Constraints:**

- if \( \text{order} = \text{Nag\_ColMajor}, \) \( \text{pdc} \geq \max(1, m) \);
- if \( \text{order} = \text{Nag\_RowMajor}, \) \( \text{pdc} \geq \max(1, n) \).

14: \( \text{fail} \) – NagError*

**Input/Output**

The NAG error argument (see Section 3.6 in the Essential Introduction).

### 6 Error Indicators and Warnings

**NE\_ALLOC\_FAIL**

Dynamic memory allocation failed.

See Section 3.2.1.2 in the Essential Introduction for further information.

**NE\_BAD\_PARAM**

On entry, argument \( \langle \text{value} \rangle \) had an illegal value.
NE_ENUM_INT_3
On entry, side = \langle value \rangle, m = \langle value \rangle, n = \langle value \rangle and k = \langle value \rangle.
Constraint: if side = Nag_LeftSide, m \geq k \geq 0;
if side = Nag_RightSide, n \geq k \geq 0.
On entry, side = \langle value \rangle, m = \langle value \rangle, n = \langle value \rangle and pdv = \langle value \rangle.
Constraint: if side = Nag_LeftSide, pdv \geq \max(1, m);
if side = Nag_RightSide, pdv \geq \max(1, n).

NE_INT
On entry, m = \langle value \rangle.
Constraint: m \geq 0.
On entry, n = \langle value \rangle.
Constraint: n \geq 0.

NE_INT_2
On entry, nb = \langle value \rangle and k = \langle value \rangle.
Constraint: nb \geq 1 and
if k > 0, nb \leq k.
On entry, pdc = \langle value \rangle and m = \langle value \rangle.
Constraint: pdc \geq \max(1, m).
On entry, pdc = \langle value \rangle and n = \langle value \rangle.
Constraint: pdc \geq \max(1, n).
On entry, pdt = \langle value \rangle and k = \langle value \rangle.
Constraint: pdt \geq \max(1, k).
On entry, pdt = \langle value \rangle and nb = \langle value \rangle.
Constraint: pdt \geq nb.
On entry, pdv = \langle value \rangle and k = \langle value \rangle.
Constraint: pdv \geq \max(1, k).

NE_INTERNAL_ERROR
An internal error has occurred in this function. Check the function call and any array sizes. If the
call is correct then please contact NAG for assistance.
An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE
Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

7 Accuracy
The computed result differs from the exact result by a matrix E such that
\[ \|E\|_2 = O(\epsilon)\|C\|_2, \]
where \( \epsilon \) is the machine precision.

8 Parallelism and Performance
nag_zgemqrt (f08aqc) is not threaded by NAG in any implementation.
nag_zgemqrt (f08aqc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users’ Note for your implementation for any additional implementation-specific information.

9 Further Comments

The total number of real floating-point operations is approximately $8nk(2m - k)$ if \texttt{side} = \texttt{Nag\_LeftSide} and $8nk(2n - k)$ if \texttt{side} = \texttt{Nag\_RightSide}.

The real analogue of this function is nag_dgemqrt (f08acc).

10 Example

See Section 10 in nag_zgeqrt (f08apc).