NAG Library Function Document

nag_zsptri (f07qwc)

1 Purpose

nag_zsptri (f07qwc) computes the inverse of a complex symmetric matrix \( A \), where \( A \) has been factorized by nag_zsptrf (f07qrc), using packed storage.

2 Specification

```c
#include <nag.h>
#include <nagf07.h>

void nag_zsptri (Nag_OrderType order, Nag_UploType uplo, Integer n,
                 Complex ap[], const Integer ipiv[], NagError *fail)
```

3 Description

nag_zsptri (f07qwc) is used to compute the inverse of a complex symmetric matrix \( A \), the function must be preceded by a call to nag_zsptrf (f07qrc), which computes the Bunch–Kaufman factorization of \( A \), using packed storage.

If \( \text{uplo} = \text{Nag} \_\text{Upper}, \) \( A = PUDU^TP^T \) and \( A^{-1} \) is computed by solving \( U^TP^TXPU = D^{-1} \).

If \( \text{uplo} = \text{Nag} \_\text{Lower}, \) \( A = PLDL^TP^T \) and \( A^{-1} \) is computed by solving \( L^TP^TXPL = D^{-1} \).

4 References


5 Arguments

1: \( \text{order} \) – Nag_OrderType

\( \text{order} \) argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by \( \text{order} = \text{Nag}_\text{RowMajor} \). See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: \( \text{order} = \text{Nag}_\text{RowMajor} \) or \( \text{Nag}_\text{ColMajor} \).

2: \( \text{uplo} \) – Nag_UploType

\( \text{uplo} \) specifies how \( A \) has been factorized.

- \( \text{uplo} = \text{Nag} \_\text{Upper} \): \( A = PUDU^TP^T \), where \( U \) is upper triangular.
- \( \text{uplo} = \text{Nag} \_\text{Lower} \): \( A = PLDL^TP^T \), where \( L \) is lower triangular.

Constraint: \( \text{uplo} = \text{Nag} \_\text{Upper} \) or \( \text{Nag} \_\text{Lower} \).

3: \( n \) – Integer

\( n \), the order of the matrix \( A \).

Constraint: \( n \geq 0 \).
4: \texttt{ap[dim]} – Complex

\textbf{Input/Output}

\textbf{Note}: the dimension, \texttt{dim}, of the array \texttt{ap} must be at least \texttt{max(1, n \times (n + 1)/2)}.

\textit{On entry}: the factorization of \(A\) stored in packed form, as returned by \texttt{nag_zsptrf (f07qrc)}.

\textit{On exit}: the factorization is overwritten by the \(n\) by \(n\) matrix \(A^{-1}\).

The storage of elements \(A_{ij}\) depends on the \texttt{order} and \texttt{uplo} arguments as follows:

\begin{enumerate}
  \item if \texttt{order} = Nag_ColMajor and \texttt{uplo} = Nag_Upper,
      \(A_{ij}\) is stored in \texttt{ap[(j - 1) \times j/2 + i - 1]}, for \(i \leq j\);
  \item if \texttt{order} = Nag_ColMajor and \texttt{uplo} = Nag_Lower,
      \(A_{ij}\) is stored in \texttt{ap[(2n - j) \times (j - 1)/2 + i - 1]}, for \(i \geq j\);
  \item if \texttt{order} = Nag_RowMajor and \texttt{uplo} = Nag_Upper,
      \(A_{ij}\) is stored in \texttt{ap[(2n - i) \times (i - 1)/2 + j - 1]}, for \(i \leq j\);
  \item if \texttt{order} = Nag_RowMajor and \texttt{uplo} = Nag_Lower,
      \(A_{ij}\) is stored in \texttt{ap[(i - 1) \times i/2 + j - 1]}, for \(i \geq j\).
\end{enumerate}

5: \texttt{ipiv[dim]} – const Integer

\textbf{Input}

\textbf{Note}: the dimension, \texttt{dim}, of the array \texttt{ipiv} must be at least \texttt{max(1, n)}.

\textit{On entry}: details of the interchanges and the block structure of \(D\), as returned by \texttt{nag_zsptrf (f07qrc)}.

6: \texttt{fail} – NagError *

\textbf{Input/Output}

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 \ Error Indicators and Warnings

\textbf{NE_ALLOC_FAIL}

Dynamic memory allocation failed.

See Section 3.2.1.2 in the Essential Introduction for further information.

\textbf{NE_BAD_PARAM}

On entry, argument \langle value\rangle had an illegal value.

\textbf{NE_INT}

On entry, \texttt{n} = \langle value\rangle.

Constraint: \texttt{n} \geq 0.

\textbf{NE_INTERNAL_ERROR}

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 3.6.6 in the Essential Introduction for further information.

\textbf{NE_NO_LICENCE}

Your licence key may have expired or may not have been installed correctly.

See Section 3.6.5 in the Essential Introduction for further information.

\textbf{NE_SINGULAR}

Element \langle value\rangle of the diagonal is exactly zero. \(D\) is singular and the inverse of \(A\) cannot be computed.
7 Accuracy

The computed inverse $X$ satisfies a bound of the form

$$
|D^{1/2}P^TXPU - I| \leq c(n)\epsilon\left(|D||U^T|P^TX|P|U| + |D||D^{-1}|\right);
$$

if uplo = Nag_Upper,

$$
|DL^TP^TXPL - I| \leq c(n)\epsilon\left(|D||L^T|P^TX|P|L| + |D||D^{-1}|\right),
$$

if uplo = Nag_Lower,

$c(n)$ is a modest linear function of $n$, and $\epsilon$ is the machine precision.

8 Parallelism and Performance

nag_zsptri (f07qwc) is not threaded by NAG in any implementation.

nag_zsptri (f07qwc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users’ Note for your implementation for any additional implementation-specific information.

9 Further Comments

The total number of real floating-point operations is approximately $\frac{4}{3}n^3$.

The real analogue of this function is nag_dsptri (f07pjc).

10 Example

This example computes the inverse of the matrix $A$, where

$$
A = \begin{pmatrix}
-0.39 - 0.71i & 5.14 - 0.64i & -7.86 - 2.96i & 3.80 + 0.92i \\
5.14 - 0.64i & 8.86 + 1.81i & -3.52 + 0.58i & 5.32 - 1.59i \\
-7.86 - 2.96i & -3.52 + 0.58i & -2.83 - 0.03i & -1.54 - 2.86i \\
3.80 + 0.92i & 5.32 - 1.59i & -1.54 - 2.86i & -0.56 + 0.12i
\end{pmatrix}.
$$

Here $A$ is symmetric, stored in packed form, and must first be factorized by nag_zsptrf (f07qrc).

10.1 Program Text

/* nag_zsptri (f07qwc) Example Program. */
/* Copyright 2014 Numerical Algorithms Group. */
/* Mark 7, 2001. */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
    Integer ap_len, i, j, n;
    Integer exit_status = 0;
    NagError fail;
    Nag_UploType uplo;
    Nag_OrderType order;
    /* Arrays */
    Integer *ipiv = 0;
    char nag_enum_arg[40];
    Complex *ap = 0;

    // Other code...
}
#include NAG_LOAD_FP
#define NAG_LOAD_FP_F
/* The following line is needed to force the Microsoft linker
to load floating point support */
float force_loading_of_ms_float_support = 0;
#endif /* NAG_LOAD_FP */

#define A_UPPER(I, J) ap[J*(J-1)/2 + I - 1]
#define A_LOWER(I, J) ap[(2*n-J)*(J-1)/2 + I - 1]
order = Nag_ColMajor;
#else
#define A_LOWER(I, J) ap[I*(I-1)/2 + J - 1]
#define A_UPPER(I, J) ap[(2*n-I)*(I-1)/2 + J - 1]
order = Nag_RowMajor;
#endif

INIT_FAIL(fail);
printf("nag_zsptri (f07qwc) Example Program Results\n\n");

/* Skip heading in data file */
#ifdef _WIN32
scanf_s("%*[\n ] ");
#else
scanf("%*[\n ] ");
#endif

ap_len = n*( n+ 1)/2;
/* Allocate memory */
if (!(ipiv = NAG_ALLOC(n, Integer)) ||
!(ap = NAG_ALLOC(ap_len, Complex))){
printf("Allocation failure\n");
exit_status = -1;
goto END;
}
/* Read A from data file */
#ifdef _WIN32
scanf_s("%39s%*[\n] ", Nag_enum_arg, _countof(nag_enum_arg));
#else
scanf("%39s%*[\n] ", Nag_enum_arg);
#endif

/* nag_enum_name_to_value (x04nac).
* Converts NAG enum member name to value */
uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);
if (uplo == Nag_Upper){
for (i = 1; i <= n; ++i){
for (j = i; j <= n; ++j){
#ifdef _WIN32
scanf_s(" ( %lf , %lf )", &A_UPPER(i, j).re,
&A_UPPER(i, j).im);
#else
scanf(" ( %lf , %lf )", &A_UPPER(i, j).re,
&A_UPPER(i, j).im);
#endif
}
}
#ifdef _WIN32
scanf_s("%*[\n ] ");
#else
scanf("%*[\n ] ");
#endif

/* The following line is needed to force the Microsoft linker
to load floating point support */
float force_loading_of_ms_float_support = 0;
#endif /* NAG_LOAD_FP */
#ifdef _WIN32
    scanf_s(" ( %lf , %lf )", &A_LOWER(i, j).re,
         &A_LOWER(i, j).im);
#else
    scanf(" ( %lf , %lf )", &A_LOWER(i, j).re,
         &A_LOWER(i, j).im);
#endif

#else
    scanf("%*[\n"]");
#endif

/* Factorize A */
/* nag_zsptrf (f07qrc). */
* Bunch-Kaufman factorization of complex symmetric matrix, * packed storage */
nag_zsptrf(order, uplo, n, ap, ipiv, &fail);
if (fail.code != NE_NOERROR)
    { printf("Error from nag_zsptrf (f07qrc).\n\s\n", fail.message);
      exit_status = 1;
      goto END;
    }

/* Compute inverse of A */
/* nag_zsptri (f07qwc). */
* Inverse of complex symmetric matrix, matrix already * factorized by nag_zsptrf (f07qrc), packed storage */
nag_zsptri(order, uplo, n, ap, ipiv, &fail);
if (fail.code != NE_NOERROR)
    { printf("Error from nag_zsptri (f07qwc).\n\s\n", fail.message);
      exit_status = 1;
      goto END;
    }

/* Print inverse */
/* nag_pack_complx_mat_print_comp (x04ddc). */
* Print complex packed triangular matrix (comprehensive) */
fflush(stdout);
nag_pack_complx_mat_print_comp(order, uplo, Nag_NonUnitDiag, n, ap,
                             Nag_BracketForm, "%7.4f", "Inverse",
                             Nag_IntegerLabels, 0, Nag_IntegerLabels, 0,
                             80, 0, 0, &fail);
if (fail.code != NE_NOERROR)
    { printf("Error from nag_pack_complx_mat_print_comp (x04ddc).\n\s\n", fail.message);
      exit_status = 1;
      goto END;
    }

END:
NAG_FREE(ipiv);
NAG_FREE(ap);
return exit_status;
}
10.2 Program Data

nag_zsptri (f07qwc) Example Program Data

<table>
<thead>
<tr>
<th>Value of n</th>
<th>Value of uplo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nag_Lower</td>
<td></td>
</tr>
<tr>
<td>(-0.39, -0.71)</td>
<td></td>
</tr>
<tr>
<td>( 5.14, -0.64)</td>
<td>( 8.86, 1.81)</td>
</tr>
<tr>
<td>(-7.86, -2.96)</td>
<td>(-3.52, 0.58)</td>
</tr>
<tr>
<td>( 3.80, 0.92)</td>
<td>( 5.32, -1.59)</td>
</tr>
</tbody>
</table>

( -7.86, -2.96) (-2.83, -0.03) ( -3.52, 0.58) ( -0.39, -0.71) : End of matrix A

10.3 Program Results

nag_zsptri (f07qwc) Example Program Results

<table>
<thead>
<tr>
<th>Inverse</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(-0.1562, -0.1014)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( 0.0400, 0.1527)</td>
<td>( 0.0946, -0.1475)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( 0.0550, 0.0845)</td>
<td>(-0.0326, -0.1370)</td>
<td>(-0.1320, -0.0102)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>( 0.2162, -0.0742)</td>
<td>(-0.0995, -0.0461)</td>
<td>(-0.1793, 0.1183)</td>
<td>(-0.2269, 0.2383)</td>
</tr>
</tbody>
</table>