NAG Library Function Document

nag_zsptrs (f07qsc)

1  Purpose

nag_zsptrs (f07qsc) solves a complex symmetric system of linear equations with multiple right-hand sides,

\[ AX = B, \]

where \( A \) has been factorized by nag_zsptrf (f07qrc), using packed storage.

2  Specification

```c
#include <nag.h>
#include <nagf07.h>

void nag_zsptrs (Nag_OrderType order, Nag_UploType uplo, Integer n, Integer nrhs, const Complex ap[], const Integer ipiv[], Complex b[], Integer pdb, NagError *fail)
```

3  Description

nag_zsptrs (f07qsc) is used to solve a complex symmetric system of linear equations \( AX = B \), the function must be preceded by a call to nag_zsptrf (f07qrc) which computes the Bunch–Kaufman factorization of \( A \), using packed storage.

If \( \text{uplo} = \text{Nag}\_\text{Upper} \), \( A = PUU^T P^T \), where \( P \) is a permutation matrix, \( U \) is an upper triangular matrix and \( D \) is a symmetric block diagonal matrix with 1 by 1 and 2 by 2 blocks; the solution \( X \) is computed by solving \( PUDY = B \) and then \( U^T P^T X = Y \).

If \( \text{uplo} = \text{Nag}\_\text{Lower} \), \( A = PLDL^T P^T \), where \( L \) is a lower triangular matrix; the solution \( X \) is computed by solving \( PLDY = B \) and then \( L^T P^T X = Y \).

4  References


5  Arguments

1:  \( \text{order} \) – Nag_OrderType  

\( \text{Input} \)

\( \text{On entry:} \) the \( \text{order} \) argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by \( \text{order} = \text{Nag}\_\text{RowMajor} \). See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

\( \text{Constraint:} \ \text{order} = \text{Nag}\_\text{RowMajor} \) or \( \text{Nag}\_\text{ColMajor} \).

2:  \( \text{uplo} \) – Nag_UploType  

\( \text{Input} \)

\( \text{On entry:} \) specifies how \( A \) has been factorized.

\( \text{uplo} = \text{Nag}\_\text{Upper} \)

\( A = PUU^T P^T \), where \( U \) is upper triangular.
\[ uplo = \text{Nag\_Lower} \]
\[ A = PLDL^T, \text{ where } L \text{ is lower triangular.} \]

**Constraint:** \( uplo = \text{Nag\_Upper} \text{ or Nag\_Lower}. \)

3: \( n \) – Integer \hspace{1cm} \text{Input}

*On entry:* \( n \), the order of the matrix \( A \).

**Constraint:** \( n \geq 0. \)

4: \( \text{nrhs} \) – Integer \hspace{1cm} \text{Input}

*On entry:* \( r \), the number of right-hand sides.

**Constraint:** \( \text{nrhs} \geq 0. \)

5: \( \text{ap}[\dim] \) – const Complex \hspace{1cm} \text{Input}

**Note:** the dimension, \( \dim \), of the array \( \text{ap} \) must be at least \( \max(1, n \times (n + 1)/2) \).

*On entry:* the factorization of \( A \) stored in packed form, as returned by \( \text{nag\_zsptrf} \) (f07qrc).

6: \( \text{ipiv}[\dim] \) – const Integer \hspace{1cm} \text{Input}

**Note:** the dimension, \( \dim \), of the array \( \text{ipiv} \) must be at least \( \max(1, n) \).

*On entry:* details of the interchanges and the block structure of \( D \), as returned by \( \text{nag\_zsptrf} \) (f07qrc).

7: \( \text{b}[\dim] \) – Complex \hspace{1cm} \text{Input/Output}

**Note:** the dimension, \( \dim \), of the array \( \text{b} \) must be at least
\[ \max(1, \text{pdb} \times \text{nrhs}) \text{ when } \text{order} = \text{Nag\_ColMajor}; \]
\[ \max(1, n \times \text{pdb}) \text{ when } \text{order} = \text{Nag\_RowMajor}. \]

The \((i, j)\)th element of the matrix \( B \) is stored in
\[ \text{b}[(j - 1) \times \text{pdb} + i - 1] \text{ when } \text{order} = \text{Nag\_ColMajor}; \]
\[ \text{b}[(i - 1) \times \text{pdb} + j - 1] \text{ when } \text{order} = \text{Nag\_RowMajor}. \]

*On entry:* the \( n \) by \( r \) right-hand side matrix \( B \).

*On exit:* the \( n \) by \( r \) solution matrix \( X \).

8: \( \text{pdb} \) – Integer \hspace{1cm} \text{Input}

*On entry:* the stride separating row or column elements (depending on the value of \( \text{order} \)) in the array \( \text{b} \).

**Constraints:**
\[ \text{if } \text{order} = \text{Nag\_ColMajor}, \text{pdb} \geq \max(1, n); \]
\[ \text{if } \text{order} = \text{Nag\_RowMajor}, \text{pdb} \geq \max(1, \text{nrhs}). \]

9: \( \text{fail} \) – NagError * \hspace{1cm} \text{Input/Output}

The NAG error argument (see Section 3.6 in the Essential Introduction).

### 6 Error Indicators and Warnings

**NE\_ALLOC\_FAIL**

Dynamic memory allocation failed.

See Section 3.2.1.2 in the Essential Introduction for further information.
NE_BAD_PARAM
On entry, argument <value> had an illegal value.

NE_INT
On entry, n = <value>.  
Constraint: n ≥ 0.

On entry, nrhs = <value>.  
Constraint: nrhs ≥ 0.

On entry, pdb = <value>.  
Constraint: pdb > 0.

NE_INT_2
On entry, pdb = <value> and n = <value>.  
Constraint: pdb ≥ max(1, n).

On entry, pdb = <value> and nrhs = <value>.  
Constraint: pdb ≥ max(1, nrhs).

NE_INTERNAL_ERROR
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG. 
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE
Your licence key may have expired or may not have been installed correctly.  
See Section 3.6.5 in the Essential Introduction for further information.

7 Accuracy
For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations (A + E)x = b, where

if uplo = Nag_Upper, |E| ≤ c(n)εP|D||D^T|P^T;

if uplo = Nag_Lower, |E| ≤ c(n)εP|L||L^T|P^T,

where c(n) is a modest linear function of n, and ε is the machine precision.  
If \( \hat{x} \) is the true solution, then the computed solution x satisfies a forward error bound of the form

\[
\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(n) \text{cond}(A, x) \epsilon
\]

where \( \text{cond}(A, x) \) is \( \|A^{-1}\|_\infty \|A\|_\infty / \|x\|_\infty \).  
Note that \( \text{cond}(A, x) \) can be much smaller than \( \text{cond}(A) \).

Forward and backward error bounds can be computed by calling nag_zsprfs (f07qvc), and an estimate for \( \kappa_\infty(A) \) ( = \( \kappa_1(A) \)) can be obtained by calling nag_zspcon (f07quc).

8 Parallelism and Performance
nag_zsptrs (f07qsc) is not threaded by NAG in any implementation.

nag_zsptrs (f07qsc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.
Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users’ Note for your implementation for any additional implementation-specific information.

9 Further Comments

The total number of real floating-point operations is approximately $8n^2r$.

This function may be followed by a call to nag_zsprfs (f07qvc) to refine the solution and return an error estimate.

The real analogue of this function is nag_dsptrs (f07pec).

10 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} -0.39 - 0.71i & 5.14 - 0.64i & -7.86 - 2.96i & 3.80 + 0.92i \\ 5.14 - 0.64i & 8.86 + 1.81i & -3.52 + 0.58i & 5.32 - 1.59i \\ -7.86 - 2.96i & -3.52 + 0.58i & -2.83 - 0.03i & -1.54 - 2.86i \\ 3.80 + 0.92i & 5.32 - 1.59i & -1.54 - 2.86i & -0.56 + 0.12i \end{pmatrix}$$

and

$$B = \begin{pmatrix} -55.64 + 41.22i & -19.09 - 35.97i \\ -48.18 + 66.00i & -12.08 - 27.02i \\ -0.49 - 1.47i & 6.95 + 20.49i \\ -6.43 + 19.24i & -4.59 - 35.53i \end{pmatrix}.$$

Here $A$ is symmetric, stored in packed form, and must first be factorized by nag_zsptrf (f07qrc).

10.1 Program Text

/* nag_zsptrs (f07qsc) Example Program. *
* * Copyright 2014 Numerical Algorithms Group. *
* * Mark 7, 2001. *
* Mark 7b revised, 2004. *
*/

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
    Integer i, j, n, nrhs, pdb;
    Integer exit_status = 0;
    NagError fail;
    Nag_UploType uplo;
    Nag_OrderType order;
    /* Arrays */
    Integer *ipiv = 0;
    char nag_enum_arg[40];
    Complex *ap = 0, *b = 0;

#define NAG_LOAD_FP
    /* The following line is needed to force the Microsoft linker to load floating point support */
    float force_loading_of_ms_float_support = 0;
#undef NAG_LOAD_FP

#define NAG_COLUMN_MAJOR

}}
```c
#define A_UPPER(I, J) ap[J*(J-1)/2 + I - 1]
#define A_LOWER(I, J) ap[(2*n-J)*(J-1)/2 + I - 1]
#define B(I, J) b[(J-1)*pdb + I - 1]

order = Nag_ColMajor;
#else
#define A_LOWER(I, J) ap[I*(I-1)/2 + J - 1]
#define A_UPPER(I, J) ap[(2*n-I)*(I-1)/2 + J - 1]
#define B(I, J) b[(I-1)*pdb + J - 1]

order = Nag_RowMajor;
#endif

INIT_FAIL(fail);

printf("nag_zsptrs (f07qsc) Example Program Results\n\n");

/* Skip heading in data file */
#ifdef _WIN32
scanf_s("%*[\n"]);
#else
scanf("%*[\n"]);
#endif
#endif
#define _WIN32
scanf_s("%"NAG_IFMT"%"NAG_IFMT"%*[\n]", &n, &nrhs);
#else
scanf("%"NAG_IFMT"%"NAG_IFMT"%*[\n]", &n, &nrhs);
#endif
#ifdef NAG_COLUMN_MAJOR
pdb = n;
#else
pdb = nrhs;
#endif

/* Allocate memory */
if (!ipiv = NAG_ALLOC(n, Integer) ||
    !ap = NAG_ALLOC(n * (n + 1)/2, Complex) ||
    !b = NAG_ALLOC(n * nrhs, Complex))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read A and B from data file */
#ifdef _WIN32
scanf_s(" %39s%*[\n"] , nag_enum_arg, _countof(nag_enum_arg));
#else
scanf(" %39s%*[\n"] , nag_enum_arg);
#endif
/* nag_enum_name_to_value (x04nac).
 * Converts NAG enum member name to value */
uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);

if (uplo == Nag_Upper)
{
    for (i = 1; i <= n; ++i)
        for (j = i; j <= n; ++j)
        {
            #ifdef _WIN32
                scanf_s(" ( %lf , %lf )", &A_UPPER(i, j).re, &A_UPPER(i, j).im);  
            #else
                scanf(" ( %lf , %lf )", &A_UPPER(i, j).re, &A_UPPER(i, j).im);  
            #endif
        }
}
else
{
    for (i = 1; i <= n; ++i)
        {
            #ifdef _WIN32
                scanf_s("%*[\n"]);
            #else
                scanf("%*[\n"]);
            #endif
        }
}
```

Mark 25 f07qsc.5
for (j = 1; j <= i; ++j)
  #ifdef _WIN32
    scanf_s(" ( %lf , %lf )", &A_LOWER(i, j).re, &A_LOWER(i, j).im);
  #else
    scanf(" ( %lf , %lf )", &A_LOWER(i, j).re, &A_LOWER(i, j).im);
  #endif
  #ifdef _WIN32
    scanf_s("%*[\n] ");
  #else
    scanf("%*[\n] ");
  #endif
  #ifdef _WIN32
    scanf_s(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
  #else
    scanf(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
  #endif
  #ifdef _WIN32
    scanf_s("%*[\n] ");
  #else
    scanf("%*[\n] ");
  #endif
/* Factorize A */
/* nag_zsptrf (f07qrc). *
 * Bunch-Kaufman factorization of complex symmetric matrix, *
 * packed storage *
 */
nag_zsptrf(order, uplo, n, ap, ipiv, &fail);
if (fail.code != NE_NOERROR)
{
  printf("Error from nag_zsptrf (f07qrc).\n%s\n", fail.message);
  exit_status = 1;
  goto END;
}
/* Compute solution */
/* nag_zsprs (f07qsc). *
 * Solution of complex symmetric system of linear equations, *
 * multiple right-hand sides, matrix already factorized by *
 * nag_zsptrf (f07qrc), packed storage *
 */
nag_zsprs(order, uplo, n, nrhs, ap, ipiv, b, pdb, &fail);
if (fail.code != NE_NOERROR)
{
  printf("Error from nag_zsprs (f07qsc).\n%s\n", fail.message);
  exit_status = 1;
  goto END;
}
/* Print solution */
/* nag_gen_complx_mat_print_comp (x04dbc). *
 * Print complex general matrix (comprehensive) *
 */
fflush(stdout);
nag_gen_complx_mat_print_comp(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, 
nrhs, b, pdb, Nag_BracketForm, "%7.4f",  
"Solution(s)", Nag_IntegerLabels, 
0, Nag_IntegerLabels, 0, 80, 0, 0, &fail);
if (fail.code != NE_NOERROR)
{
  printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n%s\n", 
fail.message);
  exit_status = 1;
  goto END;
}
END:
NAG_FREE(ipiv);
NAG_FREE(ap);
NAG_FREE(b);
return exit_status;
}

10.2 Program Data

nag_zsptrs (f07qsc) Example Program Data

4 2 :Values of n and nrhs
Nag_Lower :Value of uplo
(-0.39,-0.71)
( 5.14,-0.64) ( 8.86, 1.81)
(-7.86,-2.96) (-3.52, 0.58) (-2.83,-0.03)
( 3.80, 0.92) ( 5.32,-1.59) (-1.54,-2.86) (-0.56, 0.12) :End of matrix A
(-55.64, 41.22) (-19.09,-35.97)
(-48.18, 66.00) (-12.08,-27.02)
( -0.49, -1.47) ( 6.95, 20.49)
( -6.43, 19.24) ( -4.59,-35.53) :End of matrix B

10.3 Program Results

nag_zsptrs (f07qsc) Example Program Results

Solution(s) 1 2
1 ( 1.0000,-1.0000) (-2.0000,-1.0000)
2 (-2.0000, 5.0000) ( 1.0000,-3.0000)
3 ( 3.0000,-2.0000) ( 3.0000, 2.0000)
4 (-4.0000, 3.0000) (-1.0000, 1.0000)