NAG Library Function Document

nag_zhpsv (f07pnc)

1 Purpose

nag_zhpsv (f07pnc) computes the solution to a complex system of linear equations

\[ AX = B, \]

where \( A \) is an \( n \) by \( n \) Hermitian matrix stored in packed format and \( X \) and \( B \) are \( n \) by \( r \) matrices.

2 Specification

```c
#include <nag.h>
#include <nagf07.h>

void nag_zhpsv (Nag_OrderType order, Nag_UploType uplo, Integer n,
                  Integer nrhs, Complex ap[], Integer ipiv[], Complex b[], Integer pdb,
                  NagError *fail)
```

3 Description

nag_zhpsv (f07pnc) uses the diagonal pivoting method to factor \( A \) as \( A = UDU^H \) if \( \text{uplo} = \text{Nag}_\text{Upper} \) or \( A = LDL^H \) if \( \text{uplo} = \text{Nag}_\text{Lower} \), where \( U \) (or \( L \)) is a product of permutation and unit upper (lower) triangular matrices, \( D \) is Hermitian and block diagonal with 1 by 1 and 2 by 2 diagonal blocks. The factored form of \( A \) is then used to solve the system of equations \( AX = B \).

4 References


5 Arguments

1: \textit{order} – Nag_OrderType

\textit{Input}

\textit{On entry}: the \textit{order} argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by \textit{order} = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

\textit{Constraint}: \textit{order} = Nag_RowMajor or Nag_ColMajor.

2: \textit{uplo} – Nag_UploType

\textit{Input}

\textit{On entry}: if \textit{uplo} = Nag_Upper, the upper triangle of \( A \) is stored. If \textit{uplo} = Nag_Lower, the lower triangle of \( A \) is stored.

\textit{Constraint}: \textit{uplo} = Nag_Upper or Nag_Lower.
3: \textbf{n} – Integer \hspace{1cm} \textit{Input}

\textit{On entry}: \textbf{n}, the number of linear equations, i.e., the order of the matrix \textit{A}.

\textit{Constraint}: \textbf{n} \geq 0.

4: \textbf{nrhs} – Integer \hspace{1cm} \textit{Input}

\textit{On entry}: \textbf{r}, the number of right-hand sides, i.e., the number of columns of the matrix \textit{B}.

\textit{Constraint}: \textbf{nrhs} \geq 0.

5: \textit{ap[dim]} – Complex \hspace{1cm} \textit{Input/Output}

\textit{Note}: the dimension, \textit{dim}, of the array \textit{ap} must be at least \( \max(1, \textbf{n} \times (\textbf{n} + 1)/2) \).

\textit{On entry}: the \textbf{n} by \textbf{n} Hermitian matrix \textit{A}, packed by rows or columns.

The storage of elements \textit{A}_{ij} depends on the \textit{order} and \textit{uplo} arguments as follows:

- if \textit{order} = Nag_ColMajor and \textit{uplo} = Nag_Upper,
  \textit{A}_{ij} is stored in \textit{ap}[(j - 1) \times j/2 + i - 1], for \( i \leq j \);
- if \textit{order} = Nag_ColMajor and \textit{uplo} = Nag_Lower,
  \textit{A}_{ij} is stored in \textit{ap}[(2n - j) \times (j - 1)/2 + i - 1], for \( i \geq j \);
- if \textit{order} = Nag_RowMajor and \textit{uplo} = Nag_Upper,
  \textit{A}_{ij} is stored in \textit{ap}[(2n - i) \times (i - 1)/2 + j - 1], for \( i \leq j \);
- if \textit{order} = Nag_RowMajor and \textit{uplo} = Nag_Lower,
  \textit{A}_{ij} is stored in \textit{ap}[(i - 1) \times i/2 + j - 1], for \( i \geq j \).

\textit{On exit}: the block diagonal matrix \textit{D} and the multipliers used to obtain the factor \textit{U} or \textit{L} from the factorization \( \textit{A} = \textit{UD}^\text{T} \) or \( \textit{A} = \textit{LD}^\text{T} \) as computed by \texttt{nag_zhptrf (f07prc)}, stored as a packed triangular matrix in the same storage format as \textit{A}.

6: \textbf{ipiv[n]} – Integer \hspace{1cm} \textit{Output}

\textit{On exit}: details of the interchanges and the block structure of \textit{D}. More precisely,

- if \textbf{ipiv}[i - 1] = k > 0, \textit{d}_{ki} is a 1 by 1 pivot block and the \textit{kth} row and column of \textit{A} were interchanged with the \( kth \) row and column;

- if \textit{uplo} = Nag_Upper and \textbf{ipiv}[i - 2] = \textbf{ipiv}[i - 1] = -l < 0, \( \begin{pmatrix} \textit{d}_{i-l,i-1} & \textit{d}_{i-1,i-l} \\ \textit{d}_{i-1,i-1} & \textit{d}_{i,i-1} \end{pmatrix} \) is a 2 by 2 pivot block and the \((i - 1)th\) row and column of \textit{A} were interchanged with the \( lth \) row and column;

- if \textit{uplo} = Nag_Lower and \textbf{ipiv}[i - 1] = \textbf{ipiv}[i] = -m < 0, \( \begin{pmatrix} \textit{d}_{i,i} & \textit{d}_{i+1,i} \\ \textit{d}_{i+1,i} & \textit{d}_{i+1,i+1} \end{pmatrix} \) is a 2 by 2 pivot block and the \((i + 1)th\) row and column of \textit{A} were interchanged with the \( mth \) row and column.

7: \textbf{b[dim]} – Complex \hspace{1cm} \textit{Input/Output}

\textit{Note}: the dimension, \textit{dim}, of the array \textit{b} must be at least

- \( \max(1, \textbf{pdb} \times \textbf{nrhs}) \) when \textit{order} = Nag_ColMajor;
- \( \max(1, \textbf{n} \times \textbf{pdb}) \) when \textit{order} = Nag_RowMajor.

The \((i, j)th\) element of the matrix \textit{B} is stored in

- \textbf{b}[(j - 1) \times \textbf{pdb} + i - 1] when \textit{order} = Nag_ColMajor;
- \textbf{b}[(i - 1) \times \textbf{pdb} + j - 1] when \textit{order} = Nag_RowMajor.

\textit{On entry}: the \textbf{n} by \textbf{r} right-hand side matrix \textit{B}.

\textit{On exit}: if \textbf{fail.code} = NE_NOERROR, the \textbf{n} by \textbf{r} solution matrix \textit{X}. 

\textit{Mark 25}
8:  
   **pdb** – Integer  
   
   *Input*
   
   *On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **b**.
   
   *Constraints:*
   
   if **order** = Nag_ColMajor, **pdb** ≥ max(1, **n**);
   
   if **order** = Nag_RowMajor, **pdb** ≥ max(1, **nrhs**).

9:  
   **fail** – NagError*
   
   *Input/Output*
   
   The NAG error argument (see Section 3.6 in the Essential Introduction).

### 6 Error Indicators and Warnings

**NE_ALLOC_FAIL**

Dynamic memory allocation failed.  
See Section 3.2.1.2 in the Essential Introduction for further information.

**NE_BAD_PARAM**

On entry, argument <value> had an illegal value.

**NE_INT**

On entry, **n** = <value>.  
Constraint: **n** ≥ 0.

On entry, **nrhs** = <value>.  
Constraint: **nrhs** ≥ 0.

On entry, **pdb** = <value>.  
Constraint: **pdb** > 0.

**NE_INT_2**

On entry, **pdb** = <value> and **n** = <value>.  
Constraint: **pdb** ≥ max(1, **n**).

On entry, **pdb** = <value> and **nrhs** = <value>.  
Constraint: **pdb** ≥ max(1, **nrhs**).

**NE_INTERNAL_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.  
See Section 3.6.6 in the Essential Introduction for further information.

**NE_NO_LICENCE**

Your licence key may have expired or may not have been installed correctly.  
See Section 3.6.5 in the Essential Introduction for further information.

**NE_SINGULAR**

Element <value> of the diagonal is exactly zero. The factorization has been completed, but the block diagonal matrix **D** is exactly singular, so the solution could not be computed.
7 Accuracy

The computed solution for a single right-hand side, \( \hat{x} \), satisfies an equation of the form

\[(A + E)\hat{x} = b,\]

where

\[\|E\|_1 = O(\epsilon)\|A\|_1\]

and \( \epsilon \) is the **machine precision**. An approximate error bound for the computed solution is given by

\[\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\|E\|_1,\]

where \( \kappa(A) = \|A^{-1}\|_1\|A\|_1 \), the condition number of \( A \) with respect to the solution of the linear equations. See Section 4.4 of Anderson et al. (1999) and Chapter 11 of Higham (2002) for further details.

\n
8 Parallelism and Performance

nag_zhpsvx (f07ppc) is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, nag_herm_packed_lin_solve (f04cjc) solves \( AX = B \) and returns a forward error bound and condition estimate. nag_herm_packed_lin_solve (f04cjc) calls nag_zhpsv (f07pnc) to solve the equations.

9 Further Comments

The total number of floating-point operations is approximately \( \frac{1}{3}n^3 + 8n^2r \), where \( r \) is the number of right-hand sides.

The real analogue of this function is nag_dspsv (f07pac). The complex symmetric analogue of this function is nag_zspsv (f07qnc).

10 Example

This example solves the equations

\[Ax = b,\]

where \( A \) is the Hermitian matrix

\[
A = \begin{pmatrix}
-1.84 & 0.11 - 0.11i & -1.78 - 1.8i & 3.91 - 1.50i \\
0.11 + 0.11i & -4.63 & -1.84 + 0.03i & 2.21 + 0.21i \\
-1.78 + 1.8i & -1.84 - 0.03i & -8.87 & 1.58 - 0.90i \\
3.91 + 1.50i & 2.21 - 0.21i & 1.58 + 0.90i & -1.36
\end{pmatrix}
\]

and

\[b = \begin{pmatrix}
2.98 - 10.18i \\
-9.58 + 3.88i \\
-0.77 - 16.05i \\
7.79 + 5.48i
\end{pmatrix}.
\]

Details of the factorization of \( A \) are also output.
10.1 Program Text

/* nag_zhpsv (f07pnc) Example Program. */
* Copyright 2014 Numerical Algorithms Group.
* Mark 23, 2011. */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>

int main(void)
{
    /* Scalars */
    Integer exit_status = 0, i, j, n, nrhs, pdb;

    /* Arrays */
    Complex *ap = 0, *b = 0;
    Integer *ipiv = 0;
    char nag_enum_arg[40];

    /* Nag Types */
    NagError fail;
    Nag_UploType uplo;
    Nag_OrderType order;

    #ifdef NAG_COLUMN_MAJOR
    #define A_UPPER(I, J) ap[J*(J-1)/2 + I - 1]
    #define A_LOWER(I, J) ap[(2*n-J)*(J-1)/2 + I - 1]
    #define B(I, J) b[(J-1)*pdb + I - 1]
    order = Nag_ColMajor;
    #else
    #define A_LOWER(I, J) ap[I*(I-1)/2 + J - 1]
    #define A_UPPER(I, J) ap[(2*n-I)*(I-1)/2 + J - 1]
    #define B(I, J) b[(I-1)*pdb + J - 1]
    order = Nag_RowMajor;
    #endif

    INIT_FAIL(fail);
    printf("nag_zhpsv (f07pnc) Example Program Results\n\n");

    /* Skip heading in data file */
    #ifdef _WIN32
    scanf_s("%*\n");
    #else
    scanf("%*\n");
    #endif

    /* nag_enum_name_to_value (x04nac).
    * Converts NAG enum member name to value
    * Mark 25 f07pnc.5 */
uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);

/* Allocate memory */
if (!ap = NAG_ALLOC(n*(n+1)/2, Complex) ||
    !(b = NAG_ALLOC(n*nrhs, Complex)) ||
    !(ipiv = NAG_ALLOC(n, Integer)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}
#endif NAG_COLUMN_MAJOR
pdb = n;
#else
pdb = nrhs;
#endif

/* Read the triangular part of the matrix A from data file */
if (uplo == Nag_Upper)
    for (i = 1; i <= n; ++i)
        for (j = i; j <= n; ++j)
#endif _WIN32
    scanf_s(" ( %lf , %lf )", &A_UPPER(i, j).re, &A_UPPER(i, j).im);
#else
    scanf(" ( %lf , %lf )", &A_UPPER(i, j).re, &A_UPPER(i, j).im);
#endif if (uplo == Nag_Lower)
    for (i = 1; i <= n; ++i)
        for (j = 1; j <= i; ++j)
#endif _WIN32
    scanf_s(" ( %lf , %lf )", &A_LOWER(i, j).re, &A_LOWER(i, j).im);
#else
    scanf(" ( %lf , %lf )", &A_LOWER(i, j).re, &A_LOWER(i, j).im);
#endif
#endif _WIN32
scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif

/* Read b from data file */
for (i = 1; i <= n; ++i)
    for (j = 1; j <= nrhs; ++j)
#endif _WIN32
    scanf_s(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
#else
    scanf(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
#endif
#endif _WIN32
scanf_s("%*[\n]"");
#else
    scanf("%*[\n]"");
#endif

/* Solve the equations Ax = b for x using nag_zhpsv (f07pnc). */
if (fail.code != NE_NOERROR)
{ printf("Error from nag_zhpsv (f07pnc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Print solution */
printf(" Solution\n");
for (i = 1; i <= n; ++i)
{ for (j = 1; j <= nrhs; ++j)
    printf(" (%7.4f, %7.4f)%s", B(i, j).re, B(i, j).im, j%4 == 0?"\n":"");
printf("\n");

END:
    NAG_FREE(ap);
    NAG_FREE(b);
    NAG_FREE(ipiv);

    return exit_status;
}
#endif A_UPPER
#endif A_LOWER
#endif B

10.2 Program Data

nag_zhpsv (f07pnc) Example Program Data

        4   1
    Nag_Lower  : n, nrhs
            uplo
(  -1.84,  0.00)
(  0.11,  0.11)  ( -4.63,  0.00)
( -1.78,  1.18)  ( -1.84, -0.03)  ( -8.87,  0.00)
(  3.91,  1.50)  (  2.21, -0.21)  (  1.58,  0.90)  ( -1.36 ,  0.00) : matrix A
(  2.98,-10.18)  ( -9.58,  3.88)  ( -0.77,-16.05)  (  7.79,  5.48) : vector b

10.3 Program Results

nag_zhpsv (f07pnc) Example Program Results

    Solution
    (  2.0000,  1.0000)
    (  3.0000, -2.0000)
    ( -1.0000,  2.0000)
    (  1.0000, -1.0000)