NAG Library Function Document

nag_zsyrfs (f07nvc)

1 Purpose

nag_zsyrfs (f07nvc) returns error bounds for the solution of a complex symmetric system of linear equations with multiple right-hand sides, \( AX = B \). It improves the solution by iterative refinement, in order to reduce the backward error as much as possible.

2 Specification

```c
#include <nag.h>
#include <nagf07.h>
void nag_zsyrfs (Nag_OrderType order, Nag_UploType uplo, Integer n,
    Integer nrhs, const Complex a[], Integer pda, const Complex af[],
    Integer pdaf, const Integer ipiv[], const Complex b[], Integer pdb,
    Complex x[], Integer pdx, double ferr[], double berr[], NagError *fail)
```

3 Description

nag_zsyrfs (f07nvc) returns the backward errors and estimated bounds on the forward errors for the solution of a complex symmetric system of linear equations with multiple right-hand sides \( AX = B \). The function handles each right-hand side vector (stored as a column of the matrix \( B \)) independently, so we describe the function of nag_zsyrfs (f07nvc) in terms of a single right-hand side \( b \) and solution \( x \).

Given a computed solution \( \hat{x} \), the function computes the component-wise backward error \( \beta \). This is the size of the smallest relative perturbation in each element of \( A \) and \( b \) such that \( \hat{x} \) is the exact solution of a perturbed system

\[
(A + \delta A)\hat{x} = b + \delta b
\]

\[
|\delta a_{ij}| \leq \beta |a_{ij}| \quad \text{and} \quad |\delta b_i| \leq \beta |b_i|.
\]

Then the function estimates a bound for the component-wise forward error in the computed solution, defined by:

\[
\max_i |x_i - \hat{x}_i| / \max_i |x_i|
\]

where \( \hat{x} \) is the true solution.

For details of the method, see the f07 Chapter Introduction.

4 References


5 Arguments

1: \( \text{order} \) – Nag_OrderType

\text{Input}

On entry: the \text{order} argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by \text{order} = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: \text{order} = Nag_RowMajor or Nag_ColMajor.
2: \texttt{uplo} – Nag_UploType \hspace{1cm} \textit{Input}
   
   \textit{On entry:} specifies whether the upper or lower triangular part of \(A\) is stored and how \(A\) is to be factorized.

   \texttt{uplo} = Nag_Upper
   
   The upper triangular part of \(A\) is stored and \(A\) is factorized as \(PUDU^TP^T\), where \(U\) is upper triangular.

   \texttt{uplo} = Nag_Lower
   
   The lower triangular part of \(A\) is stored and \(A\) is factorized as \(PLDL^TP^T\), where \(L\) is lower triangular.

   \textit{Constraint:} \texttt{uplo} = Nag_Upper or Nag_Lower.

3: \(n\) – Integer \hspace{1cm} \textit{Input}

   \textit{On entry:} \(n\), the order of the matrix \(A\).

   \textit{Constraint:} \(n \geq 0\).

4: \(nrhs\) – Integer \hspace{1cm} \textit{Input}

   \textit{On entry:} \(r\), the number of right-hand sides.

   \textit{Constraint:} \(nrhs \geq 0\).

5: \(a[\text{dim}]\) – const Complex \hspace{1cm} \textit{Input}

   \textit{Note:} the dimension, \(\text{dim}\), of the array \(a\) must be at least \(\max(1, \text{pda} \times n)\).

   \textit{On entry:} the \(n\) by \(n\) original symmetric matrix \(A\) as supplied to nag_zsytrf (f07nrc).

6: \(\text{pda}\) – Integer \hspace{1cm} \textit{Input}

   \textit{On entry:} the stride separating row or column elements (depending on the value of \texttt{order}) of the matrix in the array \(a\).

   \textit{Constraint:} \(\text{pda} \geq \max(1, n)\).

7: \(af[\text{dim}]\) – const Complex \hspace{1cm} \textit{Input}

   \textit{Note:} the dimension, \(\text{dim}\), of the array \(af\) must be at least \(\max(1, \text{pdaf} \times n)\).

   \textit{On entry:} details of the factorization of \(A\), as returned by nag_zsytrf (f07nrc).

8: \(\text{pdaf}\) – Integer \hspace{1cm} \textit{Input}

   \textit{On entry:} the stride separating row or column elements (depending on the value of \texttt{order}) of the matrix in the array \(af\).

   \textit{Constraint:} \(\text{pdaf} \geq \max(1, n)\).

9: \(\text{ipiv}[\text{dim}]\) – const Integer \hspace{1cm} \textit{Input}

   \textit{Note:} the dimension, \(\text{dim}\), of the array \(ipiv\) must be at least \(\max(1, n)\).

   \textit{On entry:} details of the interchanges and the block structure of \(D\), as returned by nag_zsytrf (f07nrc).

10: \(b[\text{dim}]\) – const Complex \hspace{1cm} \textit{Input}

    \textit{Note:} the dimension, \(\text{dim}\), of the array \(b\) must be at least

    \(\max(1, \text{pdb} \times nrhs)\) when \texttt{order} = Nag_ColMajor;

    \(\max(1, n \times \text{pdb})\) when \texttt{order} = Nag_RowMajor.
The (i, j)th element of the matrix B is stored in
\[
bpdb + i - 1 \text{ when } \text{order} = \text{Nag\_ColMajor};
\]
\[
bpdb + j - 1 \text{ when } \text{order} = \text{Nag\_RowMajor}.
\]

On entry: the n by r right-hand side matrix B.

11: \textbf{pdb} – Integer

Input

On entry: the stride separating row or column elements (depending on the value of \text{order}) in the array b.

Constraints:
- if \text{order} = \text{Nag\_ColMajor}, \text{pdb} \geq \max(1, n);
- if \text{order} = \text{Nag\_RowMajor}, \text{pdb} \geq \max(1, nrhs).

12: \textbf{x}[\text{dim}] – Complex

Input/Output

Note: the dimension, \text{dim}, of the array \text{x} must be at least
\[
\max(1, \text{pdb} \times \text{nrhs}) \text{ when } \text{order} = \text{Nag\_ColMajor};
\]
\[
\max(1, n \times \text{pdb}) \text{ when } \text{order} = \text{Nag\_RowMajor}.
\]

The (i, j)th element of the matrix X is stored in
\[
xpdx + i - 1 \text{ when } \text{order} = \text{Nag\_ColMajor};
\]
\[
xpdx + j - 1 \text{ when } \text{order} = \text{Nag\_RowMajor}.
\]

On entry: the n by r solution matrix X, as returned by nag_zsytrs (f07nsc).

On exit: the improved solution matrix X.

13: \textbf{pdx} – Integer

Input

On entry: the stride separating row or column elements (depending on the value of \text{order}) in the array \text{x}.

Constraints:
- if \text{order} = \text{Nag\_ColMajor}, \text{pdx} \geq \max(1, n);
- if \text{order} = \text{Nag\_RowMajor}, \text{pdx} \geq \max(1, nrhs).

14: \textbf{ferr} [\text{nrhs}] – double

Output

On exit: \text{ferr}[j - 1] contains an estimated error bound for the jth solution vector, that is, the jth column of X, for j = 1, 2, \ldots, r.

15: \textbf{berr} [\text{nrhs}] – double

Output

On exit: \text{berr}[j - 1] contains the component-wise backward error bound \beta for the jth solution vector, that is, the jth column of X, for j = 1, 2, \ldots, r.

16: \textbf{fail} – \text{NagError}* 

Input/Output

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

\textbf{NE\_ALLOC\_FAIL}

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

\textbf{NE\_BAD\_PARAM}

On entry, argument \langle value \rangle had an illegal value.
**NE_INT**

On entry, \( n = \langle \text{value} \rangle \).
Constraint: \( n \geq 0 \).

On entry, \( nrhs = \langle \text{value} \rangle \).
Constraint: \( nrhs \geq 0 \).

On entry, \( pda = \langle \text{value} \rangle \).
Constraint: \( pda > 0 \).

On entry, \( pdaf = \langle \text{value} \rangle \).
Constraint: \( pdaf > 0 \).

On entry, \( pdb = \langle \text{value} \rangle \).
Constraint: \( pdb > 0 \).

On entry, \( pdx = \langle \text{value} \rangle \).
Constraint: \( pdx > 0 \).

**NE_INT_2**

On entry, \( pda = \langle \text{value} \rangle \) and \( n = \langle \text{value} \rangle \).
Constraint: \( pda \geq \max(1, n) \).

On entry, \( pdaf = \langle \text{value} \rangle \) and \( n = \langle \text{value} \rangle \).
Constraint: \( pdaf \geq \max(1, n) \).

On entry, \( pdb = \langle \text{value} \rangle \) and \( n = \langle \text{value} \rangle \).
Constraint: \( pdb \geq \max(1, n) \).

On entry, \( pdb = \langle \text{value} \rangle \) and \( nrhs = \langle \text{value} \rangle \).
Constraint: \( pdb \geq \max(1, nrhs) \).

On entry, \( pdx = \langle \text{value} \rangle \) and \( n = \langle \text{value} \rangle \).
Constraint: \( pdx \geq \max(1, n) \).

On entry, \( pdx = \langle \text{value} \rangle \) and \( nrhs = \langle \text{value} \rangle \).
Constraint: \( pdx \geq \max(1, nrhs) \).

**NE_INTERNAL_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG. See Section 3.6.6 in the Essential Introduction for further information.

**NE_NO_LICENCE**

Your licence key may have expired or may not have been installed correctly. See Section 3.6.5 in the Essential Introduction for further information.

7 Accuracy

The bounds returned in \( \text{ferr} \) are not rigorous, because they are estimated, not computed exactly; but in practice they almost always overestimate the actual error.

8 Parallelism and Performance

\text{nag_zsyrfs (f07nvc)} is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

\text{nag_zsyrfs (f07nvc)} makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.
Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users’ Note for your implementation for any additional implementation-specific information.

9 Further Comments

For each right-hand side, computation of the backward error involves a minimum of $16n^2$ real floating-point operations. Each step of iterative refinement involves an additional $24n^2$ real operations. At most five steps of iterative refinement are performed, but usually only one or two steps are required.

Estimating the forward error involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8n^2$ real operations.

The real analogue of this function is nag_dsytrs (f07mhc).

10 Example

This example solves the system of equations $AX = B$ using iterative refinement and to compute the forward and backward error bounds, where

$$A = \begin{pmatrix}
-0.39 - 0.71i & 5.14 - 0.64i & -7.86 - 2.96i & 3.80 + 0.92i \\
5.14 - 0.64i & 8.86 + 1.81i & -3.52 + 0.58i & 5.32 - 1.59i \\
-7.86 - 2.96i & -3.52 + 0.58i & -2.83 - 0.03i & -1.54 - 2.86i \\
3.80 + 0.92i & 5.32 - 1.59i & -1.54 - 2.86i & -0.56 + 0.12i
\end{pmatrix}$$

and

$$B = \begin{pmatrix}
-55.64 + 41.22i & -19.09 - 35.97i \\
-48.18 + 66.00i & -12.08 - 27.02i \\
-0.49 - 1.47i & 6.95 + 20.49i \\
-6.43 + 19.24i & -4.59 - 35.53i
\end{pmatrix}$$

Here $A$ is symmetric and must first be factorized by nag_zsymm (f07nvc).
#define A(I, J) a[(J-1)*pda + I - 1]
#define AF(I, J) af[(J-1)*pdaf + I - 1]
#define B(I, J) b[(J-1)*pdb + I - 1]
#define X(I, J) x[(J-1)*pdx + I - 1]

order = Nag_ColMajor;
#else
#define A(I, J) a[(I-1)*pda + J - 1]
#define AF(I, J) af[(I-1)*pdaf + J - 1]
#define B(I, J) b[(I-1)*pdb + J - 1]
#define X(I, J) x[(I-1)*pdx + J - 1]
#endif

INIT_FAIL(fail);

printf("nag_zsyrfs (f07nvc) Example Program Results\n\n");
/* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n\] ");
#else
    scanf("%*[\n\] ");
#endif
#ifdef _WIN32
    scanf_s("%"NAG_IFMT"%"NAG_IFMT"%*[\n\] ", &n, &nrhs);
#else
    scanf("%"NAG_IFMT"%"NAG_IFMT"%*[\n\] ", &n, &nrhs);
#endif
#ifdef NAG_COLUMN_MAJOR
    pda = n;
    pdaf = n;
    pdb = n;
    pdx = n;
#else
    pda = n;
    pdaf = n;
    pdb = nrhs;
    pdx = nrhs;
#endif

ferr_len = nrhs;
berr_len = nrhs;

/* Allocate memory */
if(!(ipiv = NAG_ALLOC(n, Integer)) ||
   !(a = NAG_ALLOC(n * n, Complex)) ||
   !(af = NAG_ALLOC(n * n, Complex)) ||
   !(b = NAG_ALLOC(n * nrhs, Complex)) ||
   !(x = NAG_ALLOC(n * nrhs, Complex)) ||
   !(berr = NAG_ALLOC(berr_len, double)) ||
   !(ferr = NAG_ALLOC(ferr_len, double)))
{
    printf("Allocation failure\n\n");
    exit_status = -1;
    goto END;
}

/* Read A and B from data file, and copy A to AF and B to X */
#ifdef _WIN32
    scanf_s(" %39s%*[\n\] ", nag_enum_arg, _countof(nag_enum_arg));
#else
    scanf(" %39s%*[\n\] ", nag_enum_arg);
#endif

/* nag_enum_name_to_value (x04nac).
 * Converts NAG enum member name to value */
uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);

if (uplo == Nag_Upper)
{
for (i = 1; i <= n; ++i) 
  { 
    for (j = i; j <= n; ++j) 
      #ifdef _WIN32
        scanf_s(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
      #else
        scanf(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
      #endif 
      if (uplo == Nag_Upper) 
        { 
          for (j = 1; j <= i; ++j) 
            #ifdef _WIN32
              scanf_s(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
            #else
              scanf(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
            #endif 
        } 
      for (i = 1; i <= n; ++i) 
        { 
          for (j = 1; j <= nrhs; ++j) 
            #ifdef _WIN32
              scanf_s(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
            #else
              scanf(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
            #endif 
        } 
    } 
  }
/* Copy A to AF and B to X */ if (uplo == Nag_Upper) 
  { 
    for (i = 1; i <= n; ++i) 
      { 
        for (j = i; j <= n; ++j) 
          
            AF(i, j).re = A(i, j).re;
            AF(i, j).im = A(i, j).im;
        } 
      } 
else 
  { 
    for (i = 1; i <= n; ++i) 
      { 
        for (j = 1; j <= i; ++j) 
          
            AF(i, j).re = A(i, j).re;
            AF(i, j).im = A(i, j).im;
        } 
    } 
  for (i = 1; i <= n; ++i)
for (j = 1; j <= nrhs; ++j)
{
X(i, j).re = B(i, j).re;
X(i, j).im = B(i, j).im;
}

/* Factorize A in the array AF */
/* nag_zsytrf (f07nrc). *
* Bunch-Kaufman factorization of complex symmetric matrix *
*/
nag_zsytrf(order, uplo, n, af, pdaf, ipiv, &fail);
if (fail.code != NE_NOERROR)
{
printf("Error from nag_zsytrf (f07nrc).\n", fail.message);
exit_status = 1;
goto END;
}

/* Compute solution in the array X */
/* nag_zsytrs (f07nsc). *
* Solution of complex symmetric system of linear equations, *
* multiple right-hand sides, matrix already factorized by *
* nag_zsytrf (f07nrc) *
*/
nag_zsytrs(order, uplo, n, nrhs, af, pdaf, x, pdx,
&fail);
if (fail.code != NE_NOERROR)
{
printf("Error from nag_zsytrs (f07nsc).\n", fail.message);
exit_status = 1;
goto END;
}

/* Improve solution, and compute backward errors and */
/* estimated bounds on the forward errors */
/* nag_zsyrfs (f07nvc). *
* Refined solution with error bounds of complex symmetric *
* system of linear equations, multiple right-hand sides *
*/
nag_zsyrfs(order, uplo, n, nrhs, a, pda, af, pdaf, ipiv,
b, pdb, x, pdx, ferr, berr, &fail);
if (fail.code != NE_NOERROR)
{
printf("Error from nag_zsyrfs (f07nvc).\n", fail.message);
exit_status = 1;
goto END;
}

/* Print solution */
/* nag_gen_complx_mat_print_comp (x04dbc). *
* Print complex general matrix (comprehensive) *
*/
flush(stdout);
nag_gen_complx_mat_print_comp(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n,
rhs, x, pdx, Nag_BracketForm, "%7.4f",
"Solution(s)", Nag_IntegerLabels,
0, Nag_IntegerLabels, 0, 80, 0, 0, &fail);
if (fail.code != NE_NOERROR)
{
printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n", fail.message);
exit_status = 1;
goto END;
}

printf("\nBackward errors (machine-dependent)\n");
for (j = 1; j <= nrhs; ++j)
{
printf("%11.1e\n", berr[j-1], j%4 == 0?"\n":" ");
}
printf("\nEstimated forward error bounds "
"(machine-dependent)\n");
for (j = 1; j <= nrhs; ++j)
{
printf("%11.1e\n", ferr[j-1], j%4 == 0?"\n":" ");
}

END:
NAG_FREE(ipiv);
NAG_FREE(a);
NAG_FREE(af);
NAG_FREE(b);
NAG_FREE(x);
NAG_FREE(berr);
NAG_FREE(ferr);
return exit_status;
}

10.2 Program Data

nag_zsyrfs (f07nvc) Example Program Data

4 2:Values of n and nrhs
Nag_Lower:Value of uplo

(-0.39, -0.71)
( 5.14, -0.64) ( 8.86, 1.81)
(-7.86, -2.96) (-3.52, 0.58) (-2.83, -0.03)
( 3.80, 0.92) ( 5.32, -1.59) (-1.54, -2.86) (-0.56, 0.12):End of matrix A
(-55.64, 41.22) (-19.09, -35.97)
(-48.18, 66.00) (-12.08, -27.02)
(-0.49, -1.47) ( 6.95, 20.49)
(-6.43, 19.24) (-4.59, -35.53):End of matrix B

10.3 Program Results

nag_zsyrfs (f07nvc) Example Program Results

Solution(s)

1 2
1 ( 1.0000, -1.0000) (-2.0000, -1.0000)
2 (-2.0000, 5.0000) ( 1.0000, -3.0000)
3 ( 3.0000, -2.0000) ( 3.0000, 2.0000)
4 (-4.0000, 3.0000) (-1.0000, 1.0000)

Backward errors (machine-dependent)
8.2e-17 4.9e-17
Estimated forward error bounds (machine-dependent)
1.2e-14 1.2e-14