NAG Library Function Document
nag_dpprfs (f07ghc)

1 Purpose
nag_dpprfs (f07ghc) returns error bounds for the solution of a real symmetric positive definite system of linear equations with multiple right-hand sides, \( AX = B \), using packed storage. It improves the solution by iterative refinement, in order to reduce the backward error as much as possible.

2 Specification

```c
#include <nag.h>
#include <nagf07.h>

void nag_dpprfs (Nag_OrderType order, Nag_UploType uplo, Integer n,
                Integer nrhs, const double ap[], const double afp[], const double b[],
                Integer pdb, double x[], Integer pdx, double ferr[], double berr[],
                NagError *fail)
```

3 Description

nag_dpprfs (f07ghc) returns the backward errors and estimated bounds on the forward errors for the solution of a real symmetric positive definite system of linear equations with multiple right-hand sides \( AX = B \), using packed storage. The function handles each right-hand side vector (stored as a column of the matrix \( B \)) independently, so we describe the function of nag_dpprfs (f07ghc) in terms of a single right-hand side \( b \) and solution \( x \).

Given a computed solution \( x \), the function computes the component-wise backward error \( \beta \). This is the size of the smallest relative perturbation in each element of \( A \) and \( b \) such that \( x \) is the exact solution of a perturbed system

\[
(A + \delta A)x = b + \delta b
\]

\[
|\delta a_{ij}| \leq \beta |a_{ij}| \quad \text{and} \quad |\delta b_i| \leq \beta |b_i|.
\]

Then the function estimates a bound for the component-wise forward error in the computed solution, defined by:

\[
\max_i |x_i - \hat{x}_i| / \max_i |x_i|
\]

where \( \hat{x} \) is the true solution.

For details of the method, see the f07 Chapter Introduction.

4 References


5 Arguments

1: \( \textbf{order} \) – Nag_OrderType

\textit{Input}

\textit{On entry:} the \textbf{order} argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by \textbf{order} = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

\textit{Constraint:} \textbf{order} = Nag_RowMajor or Nag_ColMajor.
2: `uplo` – Nag_UploType  
   *Input*  
   *On entry*: specifies whether the upper or lower triangular part of $A$ is stored and how $A$ is to be factorized.
   
   `uplo = Nag_Upper`
   The upper triangular part of $A$ is stored and $A$ is factorized as $U^T$, where $U$ is upper triangular.
   
   `uplo = Nag_Lower`
   The lower triangular part of $A$ is stored and $A$ is factorized as $LL^T$, where $L$ is lower triangular.
   
   *Constraint*: `uplo = Nag_Upper` or `Nag_Lower`.

3: `n` – Integer  
   *Input*  
   *On entry*: $n$, the order of the matrix $A$.
   
   *Constraint*: $n \geq 0$.

4: `nrhs` – Integer  
   *Input*  
   *On entry*: $r$, the number of right-hand sides.
   
   *Constraint*: $nrhs \geq 0$.

5: `ap[dim]` – const double  
   *Input*  
   *Note*: the dimension, `dim`, of the array `ap` must be at least max(1, $n \times (n + 1)/2$).
   
   *On entry*: the $n$ by $n$ original symmetric positive definite matrix $A$ as supplied to nag_dpptrf (f07gdc).

6: `afp[dim]` – const double  
   *Input*  
   *Note*: the dimension, `dim`, of the array `afp` must be at least max(1, $n \times (n + 1)/2$).
   
   *On entry*: the Cholesky factor of $A$ stored in packed form, as returned by nag_dpptrf (f07gdc).

7: `b[dim]` – const double  
   *Input*  
   *Note*: the dimension, `dim`, of the array `b` must be at least max(1, $p$db $\times$ `nrhs`) when `order = Nag_ColMajor`; max(1, $n \times pdb$) when `order = Nag_RowMajor`.
   
   The $(i,j)$th element of the matrix $B$ is stored in
   
   $b[(j - 1) \times pdb + i - 1]$ when `order = Nag_ColMajor`;
   $b[(i - 1) \times pdb + j - 1]$ when `order = Nag_RowMajor`.
   
   *On entry*: the $n$ by $r$ right-hand side matrix $B$.

8: `pdb` – Integer  
   *Input*  
   *On entry*: the stride separating row or column elements (depending on the value of `order`) in the array `b`.
   
   *Constraints*:
   
   if `order = Nag_ColMajor`, $pdb \geq \text{max}(1, n)$;
   if `order = Nag_RowMajor`, $pdb \geq \text{max}(1, nrhs)$.
9: \[ \mathbf{x}^{\text{dim}] = \text{double} \]
   \text{Input/Output}

   \textbf{Note:} the dimension, \( \textit{dim} \), of the array \( \mathbf{x} \) must be at least
   \[ \max(1, \text{pdx} \times \text{nrhs}) \text{ when } \text{order} = \text{Nag\_ColMajor;} \]
   \[ \max(1, \text{n} \times \text{pdx}) \text{ when } \text{order} = \text{Nag\_RowMajor}. \]

   The \((i,j)\)th element of the matrix \( \mathbf{X} \) is stored in
   \[ \mathbf{x}(j-1) \times \text{pdx} + i - 1] \text{ when } \text{order} = \text{Nag\_ColMajor;} \]
   \[ \mathbf{x}(i-1) \times \text{pdx} + j - 1] \text{ when } \text{order} = \text{Nag\_RowMajor}. \]

   \textit{On entry:} the \( n \) by \( r \) solution matrix \( \mathbf{X} \), as returned by nag_dpptrs (f07gec).

   \textit{On exit:} the improved solution matrix \( \mathbf{X} \).

10: \[ \text{pdx} = \text{Integer} \]
   \text{Input}

   \textit{On entry:} the stride separating row or column elements (depending on the value of \text{order}) in the
   array \( \mathbf{x} \).

   \textbf{Constraints:}
   \[ \begin{align*}
   & \text{if } \text{order} = \text{Nag\_ColMajor}, \text{pdx} \geq \max(1, \text{n}); \\
   & \text{if } \text{order} = \text{Nag\_RowMajor}, \text{pdx} \geq \max(1, \text{nrhs}).
   \end{align*} \]

11: \[ \text{ferr[nrhs]} = \text{double} \]
   \text{Output}

   \textit{On exit:} \( \text{ferr}^j - 1 \) contains an estimated error bound for the \( j \)th solution vector, that is, the \( j \)th
   column of \( \mathbf{X} \), for \( j = 1, 2, \ldots, r \).

12: \[ \text{berr[nrhs]} = \text{double} \]
   \text{Output}

   \textit{On exit:} \( \text{berr}^j - 1 \) contains the component-wise backward error bound \( \beta \) for the \( j \)th solution
   vector, that is, the \( j \)th column of \( \mathbf{X} \), for \( j = 1, 2, \ldots, r \).

13: \[ \text{fail} = \text{Nag\_Error*} \]
   \text{Input/Output}

   The NAG error argument (see Section 3.6 in the Essential Introduction).

6 \quad \textbf{Error Indicators and Warnings}

\textbf{NE\_ALLOC\_FAIL}

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

\textbf{NE\_BAD\_PARAM}

On entry, argument \( \langle \text{value} \rangle \) had an illegal value.

\textbf{NE\_INT}

On entry, \( n = \langle \text{value} \rangle \).
Constraint: \( n \geq 0 \).
On entry, \( \text{nrhs} = \langle \text{value} \rangle \).
Constraint: \( \text{nrhs} \geq 0 \).
On entry, \( \text{pdb} = \langle \text{value} \rangle \).
Constraint: \( \text{pdb} > 0 \).
On entry, \( \text{pdx} = \langle \text{value} \rangle \).
Constraint: \( \text{pdx} > 0 \).
NE_INT_2

On entry, $\text{pdb} = \langle\text{value}\rangle$ and $\text{n} = \langle\text{value}\rangle$.
Constraint: $\text{pdb} \geq \max(1, \text{n})$.

On entry, $\text{pdb} = \langle\text{value}\rangle$ and $\text{nrhs} = \langle\text{value}\rangle$.
Constraint: $\text{pdb} \geq \max(1, \text{nrhs})$.

On entry, $\text{pdx} = \langle\text{value}\rangle$ and $\text{n} = \langle\text{value}\rangle$.
Constraint: $\text{pdx} \geq \max(1, \text{n})$.

On entry, $\text{pdx} = \langle\text{value}\rangle$ and $\text{nrhs} = \langle\text{value}\rangle$.
Constraint: $\text{pdx} \geq \max(1, \text{nrhs})$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG. See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly. See Section 3.6.5 in the Essential Introduction for further information.

7 Accuracy

The bounds returned in $\text{ferr}$ are not rigorous, because they are estimated, not computed exactly; but in practice they almost always overestimate the actual error.

8 Parallelism and Performance

$nag_dpprfs$ ($f07ghc$) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

$nag_dpprfs$ ($f07ghc$) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users’ Note for your implementation for any additional implementation-specific information.

9 Further Comments

For each right-hand side, computation of the backward error involves a minimum of $4n^2$ floating-point operations. Each step of iterative refinement involves an additional $6n^2$ operations. At most five steps of iterative refinement are performed, but usually only 1 or 2 steps are required.

Estimating the forward error involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $2n^2$ operations.

The complex analogue of this function is $nag_zpprfs$ ($f07gvc$).
10 Example

This example solves the system of equations $AX = B$ using iterative refinement and to compute the forward and backward error bounds, where

$$A = \begin{pmatrix}
4.16 & -3.12 & 0.56 & -0.10 \\
-3.12 & 5.03 & -0.83 & 1.18 \\
0.56 & -0.83 & 0.76 & 0.34 \\
-0.10 & 1.18 & 0.34 & 1.18 \\
\end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix}
8.70 & 8.30 \\
-13.35 & 2.13 \\
1.89 & 1.61 \\
-4.14 & 5.00 \\
\end{pmatrix}.$$ 

Here $A$ is symmetric positive definite, stored in packed form, and must first be factorized by nag_dp.ptrf (f07gdc).

10.1 Program Text

```c
/* nag_dp.ptrs (f07ghc) Example Program. */

* Copyright 2014 Numerical Algorithms Group.
*
*/

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
    Integer i, j, n, nrhs, ap_len, afp_len, pdb, pdx, ferr_len, berr_len;
    Integer exit_status = 0;
    NagError fail;
    Nag_UploType uplo;
    Nag_OrderType order;
    /* Arrays */
    char nag_enum_arg[40];
    double *afp = 0, *ap = 0, *b = 0, *berr = 0, *ferr = 0, *x = 0;
    #ifdef NAG_COLUMN_MAJOR
    #define A_UPPER(I, J) ap[J*(J-1)/2 + I - 1]
    #define A_LOWER(I, J) ap[(2*n-J)*(J-1)/2 + I - 1]
    #define B(I, J) b[(J-1)*pdb+I-1]
    #define X(I, J) x[(J-1)*pdx+I-1]
    order = Nag_ColMajor;
    #else
    #define A_LOWER(I, J) ap[I*(I-1)/2 + J - 1]
    #define A_UPPER(I, J) ap[(2*n-I)*(J-1)/2 + J - 1]
    #define B(I, J) b[(J-1)*pdb+I-1]
    #define X(I, J) x[(I-1)*pdx+J-1]
    order = Nag_RowMajor;
    #endif

    INIT_FAIL(fail);
    printf("nag_dp.ptrs (f07ghc) Example Program Results\n\n");
    /* Skip heading in data file */
    #ifdef _WIN32
    scanf_s("%*[\n] ");
    #else
    scanf("%*[\n] ");
    #endif
    #ifdef _WIN32
    scanf_s("%"NAG_IFMT"%"NAG_IFMT"%*[\n] ", &n, &nrhs);
    #else
    scanf("%"NAG_IFMT"%"NAG_IFMT"%*[\n] ", &n, &nrhs);
    #endif
    ap_len = n * (n + 1)/2;
    ...
    ...
    ...

    ...
    ...
    ...
    return exit_status;
}
```
afp_len = n * (n + 1)/2;
#ifdef NAG_COLUMN_MAJOR
pdb = n;
pdx = n;
#else
pdb = nrhs;
pdx = nrhs;
#endif
ferr_len = nrhs;
berr_len = nrhs;

/* Allocate memory */
if (!(afp = NAG_ALLOC(ap_len, double)) ||
    !(ap = NAG_ALLOC(afp_len, double)) ||
    !(b = NAG_ALLOC(n * nrhs, double)) ||
    !(berr = NAG_ALLOC(berr_len, double)) ||
    !(ferr = NAG_ALLOC(ferr_len, double)) ||
    !(x = NAG_ALLOC(n * nrhs, double)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read A and B from data file, and copy A to AFP and B to X */
#ifdef _WIN32
    scanf_s(" %39s\n", nag_enum_arg, _countof(nag_enum_arg));
#else
    scanf(" %39s\n", nag_enum_arg);
#endif
/* nag_enum_name_to_value (x04nac).
* Converts NAG enum member name to value *
*/
uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);
if (uplo == Nag_Upper)
{
    for (i = 1; i <= n; ++i)
    {
        for (j = i; j <= n; ++j)
        {
            #ifdef _WIN32
            scanf_s("%lf", &A_UPPER(i, j));
            #else
            scanf("%lf", &A_UPPER(i, j));
            #endif
        }
        #ifdef _WIN32
        scanf_s("\n");
        #else
        scanf("\n");
        #endif
    }
    else
    {
        for (i = 1; i <= n; ++i)
        {
            for (j = 1; j <= i; ++j)
            {
                #ifdef _WIN32
                scanf_s("%lf", &A_LOWER(i, j));
                #else
                scanf("%lf", &A_LOWER(i, j));
                #endif
            }
            #ifdef _WIN32
            scanf_s("\n");
            #else
            scanf("\n");
            #endif
        }
    }
}
for (i = 1; i <= n; ++i)
    for (j = 1; j <= nrhs; ++j)
#ifdef _WIN32
    scanf_s("%lf", &B(i, j));
#else
    scanf("%lf", &B(i, j));
#endif
#endif
        scanf("%*[\n"]");
#endif
        scanf("%*[\n"]");
#endif
for (i = 0; i < n * (n + 1) / 2; ++i)
    afp[i] = ap[i];
for (i = 1; i <= n; ++i)
    {for (j = 1; j <= nrhs; ++j)
        X(i, j) = B(i, j);
    }
/* Factorize A in the array AFP */
/* nag_dpptrf (f07gdc). */
/* Cholesky factorization of real symmetric */
/* positive-definite matrix, packed storage */
/* nag_dpptrf(order, uplo, n, afp, &fail); */
if (fail.code != NE_NOERROR)
{ printf("Error from nag_dpptrf (f07gdc).\n%s\n", fail.message);
  exit_status = 1;
  goto END;
}
/* Compute solution in the array X */
/* nag_dpptrs (f07gec). */
/* Solution of real symmetric positive-definite system of */
/* linear equations, multiple right-hand sides, matrix */
/* already factorized by nag_dpptrf (f07gdc), packed storage */
/* nag_dpptrs(order, uplo, n, nrhs, afp, x, pdx, &fail); */
if (fail.code != NE_NOERROR)
{ printf("Error from nag_dpptrs (f07gec).\n%s\n", fail.message);
  exit_status = 1;
  goto END;
}
/* Improve solution, and compute backward errors and */
/* estimated bounds on the forward errors */
/* nag_dpprfs (f07ghc). */
/* Refined solution with error bounds of real symmetric */
/* positive-definite system of linear equations, multiple */
/* right-hand sides, packed storage */
/* nag_dpprfs(order, uplo, n, nrhs, ap, afp, b, pdb, x, pdx, ferr, berr, 
&fail); */
if (fail.code != NE_NOERROR)
{ printf("Error from nag_dpprfs (f07ghc).\n%s\n", fail.message);
  exit_status = 1;
  goto END;
}
/* Print solution */
/* nag_gen_real_mat_print (x04cac). */
/* Print real general matrix (easy-to-use) */
fflush(stdout);
if (fail.code != NE_NOERROR)
{ printf("Error from nag_gen_real_mat_print (x04cac).\n%s\n", fail.message);
  exit_status = 1;
  goto END;
}
fail.message);
    exit_status = 1;
    goto END;
}

printf("\nBackward errors (machine-dependent)\n");
for (j = 1; j <= nrhs; ++j)
    printf("%11.1e%s", berr[j-1], j%7 == 0?"\n": "");
printf("\nEstimated forward error bounds (machine-dependent)\n");
for (j = 1; j <= nrhs; ++j)
    printf("%11.1e%s", ferr[j-1], j%7 == 0?"\n": "");
printf("\n");
END:
NAG_FREE(afp);
NAG_FREE(ap);
NAG_FREE(b);
NAG_FREE(berr);
NAG_FREE(ferr);
NAG_FREE(x);
return exit_status;
}

10.2 Program Data

nag_dpprfs (f07ghc) Example Program Data

Values of n and nrhs
Nag_Lower
4
2
-3.12  5.03
0.56  -0.83  0.76
-0.10  1.18  0.34  1.18
8.70  8.30
-13.35  2.13
1.89  1.61
-4.14  5.00

Values of uplo
:End of matrix A

:End of matrix B

10.3 Program Results

nag_dpprfs (f07ghc) Example Program Results

Solution(s)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>2</td>
<td>-1.0000</td>
<td>3.0000</td>
</tr>
<tr>
<td>3</td>
<td>2.0000</td>
<td>2.0000</td>
</tr>
<tr>
<td>4</td>
<td>-3.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Backward errors (machine-dependent)
6.7e-17    7.9e-17

Estimated forward error bounds (machine-dependent)
2.3e-14    2.3e-14