NAG Library Function Document

gag_zposvx (f07fpc)

1 Purpose

nag_zposvx (f07fpc) uses the Cholesky factorization

\[ A = U^H U \quad \text{or} \quad A = L L^H \]

to compute the solution to a complex system of linear equations

\[ AX = B, \]

where \( A \) is an \( n \) by \( n \) Hermitian positive definite matrix and \( X \) and \( B \) are \( n \) by \( r \) matrices. Error bounds on the solution and a condition estimate are also provided.

2 Specification

```c
#include <nag.h>
#include <nagf07.h>

void nag_zposvx (Nag_OrderType order, Nag_FactoredFormType fact,
                Nag_UploType uplo, Integer n, Integer nrhs, Complex a[], Integer pda,
                Complex af[], Integer pdaf, Nag_EquilibrationType *equed, double s[],
                Complex b[], Integer pdb, Complex x[], Integer pdx, double *rcond,
                double ferr[], double berr[], NagError *fail)
```

3 Description

nag_zposvx (f07fpc) performs the following steps:

1. If \( \text{fact} = \text{Nag\_EquilibrateAndFactor} \), real diagonal scaling factors, \( D_S \), are computed to equilibrate the system:

\[ (D_S A D_S)^{-1} X = D_S B. \]

Whether or not the system will be equilibrated depends on the scaling of the matrix \( A \), but if equilibration is used, \( A \) is overwritten by \( D_S A D_S \) and \( B \) by \( D_S B \).

2. If \( \text{fact} = \text{Nag\_NotFactored} \) or \( \text{Nag\_EquilibrateAndFactor} \), the Cholesky decomposition is used to factor the matrix \( A \) (after equilibration if \( \text{fact} = \text{Nag\_EquilibrateAndFactor} \)) as \( A = U^H U \) if \( \text{uplo} = \text{Nag\_Upper} \) or \( A = L L^H \) if \( \text{uplo} = \text{Nag\_Lower} \), where \( U \) is an upper triangular matrix and \( L \) is a lower triangular matrix.

3. If the leading \( i \) by \( i \) principal minor of \( A \) is not positive definite, then the function returns with \( \text{fail.errnum} = i \) and \( \text{fail.code} = \text{NE\_MAT\_NOT\_POS\_DEF} \). Otherwise, the factored form of \( A \) is used to estimate the condition number of the matrix \( A \). If the reciprocal of the condition number is less than \( \text{machine precision} \), \( \text{fail.code} = \text{NE\_SINGULAR\_WP} \) is returned as a warning, but the function still goes on to solve for \( X \) and compute error bounds as described below.

4. The system of equations is solved for \( X \) using the factored form of \( A \).

5. Iterative refinement is applied to improve the computed solution matrix and to calculate error bounds and backward error estimates for it.

6. If equilibration was used, the matrix \( X \) is premultiplied by \( D_S \) so that it solves the original system before equilibration.
4 References


5 Arguments

1:  order – Nag_OrderType  
On entry: the order argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by order = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: order = Nag_RowMajor or Nag_ColMajor.

2:  fact – Nag_FactoredFormType  
On entry: specifies whether or not the factorized form of the matrix $A$ is supplied on entry, and if not, whether the matrix $A$ should be equilibrated before it is factorized.

fact = Nag_Factored
af contains the factorized form of $A$. If equed = Nag_Equilibrated, the matrix $A$ has been equilibrated with scaling factors given by $s$. $a$ and $af$ will not be modified.

fact = Nag_NotFactored
The matrix $A$ will be copied to af and factorized.

fact = Nag_EquilibrateAndFactor
The matrix $A$ will be equilibrated if necessary, then copied to af and factorized.

Constraint: fact = Nag_Factored, Nag_NotFactored or Nag_EquilibrateAndFactor.

3:  uplo – Nag_UploType  
On entry: if uplo = Nag_Upper, the upper triangle of $A$ is stored.
If uplo = Nag_Lower, the lower triangle of $A$ is stored.

Constraint: uplo = Nag_Upper or Nag_Lower.

4:  n – Integer  
On entry: $n$, the number of linear equations, i.e., the order of the matrix $A$.

Constraint: $n \geq 0$.

5:  nrhs – Integer  
On entry: $r$, the number of right-hand sides, i.e., the number of columns of the matrix $B$.

Constraint: $nrhs \geq 0$.

6:  a[dim] – Complex  
Input/Output

Note: the dimension, dim, of the array a must be at least max(1,$pda \times n$).

On entry: the $n$ by $n$ Hermitian matrix $A$.

If fact = Nag_Factored and equed = Nag_Equilibrated, a must have been equilibrated by the scaling factor in s as $DSDAD_S$. 

If $\text{order} = \text{Nag\_ColMajor}$, $A_{ij}$ is stored in $a[(j - 1) \times \text{pda} + i - 1]$.

If $\text{order} = \text{Nag\_RowMajor}$, $A_{ij}$ is stored in $a[(i - 1) \times \text{pda} + j - 1]$.

If $\text{uplo} = \text{Nag\_Upper}$, the upper triangular part of $A$ must be stored and the elements of the array below the diagonal are not referenced.

If $\text{uplo} = \text{Nag\_Lower}$, the lower triangular part of $A$ must be stored and the elements of the array above the diagonal are not referenced.

On exit: if $\text{fact} = \text{Nag\_Factored}$ or $\text{Nag\_NotFactored}$, or if $\text{fact} = \text{Nag\_EquilibrateAndFactor}$ and $\text{equed} = \text{Nag\_NoEquilibration}$, $a$ is not modified.

If $\text{fact} = \text{Nag\_EquilibrateAndFactor}$ and $\text{equed} = \text{Nag\_Equilibrated}$, $a$ is overwritten by $D_S A D_S$.

7: $\text{pda}$ – Integer

Input

On entry: the stride separating row or column elements (depending on the value of $\text{order}$) of the matrix $A$ in the array $a$.

Constraint: $\text{pda} \geq \max(1, n)$.

8: $\text{af}[\text{dim}]$ – Complex

Input/Output

Note: the dimension, $\text{dim}$, of the array $\text{af}$ must be at least $\max(1, \text{pdaf} \times n)$.

The $(i, j)$th element of the matrix is stored in

$\text{af}[(j - 1) \times \text{pdaf} + i - 1]$ when $\text{order} = \text{Nag\_ColMajor};$

$\text{af}[(i - 1) \times \text{pdaf} + j - 1]$ when $\text{order} = \text{Nag\_RowMajor}$.

On entry: if $\text{fact} = \text{Nag\_Factored}$, $\text{af}$ contains the triangular factor $U$ or $L$ from the Cholesky factorization $A = U^H U$ or $A = L L^H$, in the same storage format as $a$. If $\text{equed} \neq \text{Nag\_NoEquilibration}$, $\text{af}$ is the factorized form of the equilibrated matrix $D_S A D_S$.

On exit: if $\text{fact} = \text{Nag\_NotFactored}$, $\text{af}$ returns the triangular factor $U$ or $L$ from the Cholesky factorization $A = U^H U$ or $A = L L^H$ of the original matrix $A$.

If $\text{fact} = \text{Nag\_EquilibrateAndFactor}$, $\text{af}$ returns the triangular factor $U$ or $L$ from the Cholesky factorization $A = U^H U$ or $A = L L^H$ of the equilibrated matrix $A$ (see the description of $a$ for the form of the equilibrated matrix).

9: $\text{pdaf}$ – Integer

Input

On entry: the stride separating row or column elements (depending on the value of $\text{order}$) of the matrix $A$ in the array $\text{af}$.

Constraint: $\text{pdaf} \geq \max(1, n)$.

10: $\text{equed}$ – Nag\_EquilibrationType *

Input/Output

On entry: if $\text{fact} = \text{Nag\_NotFactored}$ or $\text{Nag\_EquilibrateAndFactor}$, $\text{equed}$ need not be set.

If $\text{fact} = \text{Nag\_Factored}$, $\text{equed}$ must specify the form of the equilibration that was performed as follows:

- if $\text{equed} = \text{Nag\_NoEquilibration}$, no equilibration;
- if $\text{equed} = \text{Nag\_Equilibrated}$, equilibration was performed, i.e., $A$ has been replaced by $D_S A D_S$.

On exit: if $\text{fact} = \text{Nag\_Factored}$, $\text{equed}$ is unchanged from entry.

Otherwise, if no constraints are violated, $\text{equed}$ specifies the form of the equilibration that was performed as specified above.

Constraint: if $\text{fact} = \text{Nag\_Factored}$, $\text{equed} = \text{Nag\_NoEquilibration}$ or $\text{Nag\_Equilibrated}$.
11: \( s[dim] \) – double

**Input/Output**

**Note:** the dimension, \( dim \), of the array \( s \) must be at least \( \max(1, n) \).

**On entry:** if \( fact = \text{Nag\_NotFactored} \) or \( \text{Nag\_EquilibrateAndFactor} \), \( s \) need not be set.

If \( fact = \text{Nag\_Factored} \) and \( equed = \text{Nag\_Equilibrated} \), \( s \) must contain the scale factors, \( D_S \), for \( A \); each element of \( s \) must be positive.

**On exit:** if \( fact = \text{Nag\_Factored} \), \( s \) is unchanged from entry.

Otherwise, if no constraints are violated and \( equed = \text{Nag\_Equilibrated} \), \( s \) contains the scale factors, \( D_S \), for \( A \); each element of \( s \) is positive.

12: \( b[dim] \) – Complex

**Input/Output**

**Note:** the dimension, \( dim \), of the array \( b \) must be at least \( \max(1, \text{pdb} \times \text{nrhs}) \) when \( \text{order} = \text{Nag\_ColMajor} \);

\( \max(1, n \times \text{pdb}) \) when \( \text{order} = \text{Nag\_RowMajor} \).

The \((i, j)\)th element of the matrix \( B \) is stored in

\[ b[(j - 1) \times \text{pdb} + i - 1] \] when \( \text{order} = \text{Nag\_ColMajor} \);

\[ b[(i - 1) \times \text{pdb} + j - 1] \] when \( \text{order} = \text{Nag\_RowMajor} \).

**On entry:** the \( n \) by \( r \) right-hand side matrix \( B \).

**On exit:** if \( equed = \text{Nag\_NoEquilibration} \), \( b \) is not modified.

If \( equed = \text{Nag\_Equilibrated} \), \( b \) is overwritten by \( D_S B \).

13: \( \text{pdb} \) – Integer

**Input**

**On entry:** the stride separating row or column elements (depending on the value of \( \text{order} \)) in the array \( b \).

**Constraints:**

\[
\begin{align*}
\text{if } \text{order} = \text{Nag\_ColMajor}, & \quad \text{pdb} \geq \max(1, n); \\
\text{if } \text{order} = \text{Nag\_RowMajor}, & \quad \text{pdb} \geq \max(1, \text{nrhs}).
\end{align*}
\]

14: \( x[dim] \) – Complex

**Output**

**Note:** the dimension, \( dim \), of the array \( x \) must be at least \( \max(1, \text{pdx} \times \text{nrhs}) \) when \( \text{order} = \text{Nag\_ColMajor} \);

\( \max(1, n \times \text{pdx}) \) when \( \text{order} = \text{Nag\_RowMajor} \).

The \((i, j)\)th element of the matrix \( X \) is stored in

\[ x[(j - 1) \times \text{pdx} + i - 1] \] when \( \text{order} = \text{Nag\_ColMajor} \);

\[ x[(i - 1) \times \text{pdx} + j - 1] \] when \( \text{order} = \text{Nag\_RowMajor} \).

**On exit:** if \( \text{fail\_code} = \text{NE\_NOERROR} \) or \( \text{NE\_SINGULAR\_WP} \), the \( n \) by \( r \) solution matrix \( X \) to the original system of equations. Note that the arrays \( A \) and \( B \) are modified on exit if \( equed = \text{Nag\_Equilibrated} \), and the solution to the equilibrated system is \( D_S^{-1} X \).

15: \( \text{pdx} \) – Integer

**Input**

**On entry:** the stride separating row or column elements (depending on the value of \( \text{order} \)) in the array \( x \).

**Constraints:**

\[
\begin{align*}
\text{if } \text{order} = \text{Nag\_ColMajor}, & \quad \text{pdx} \geq \max(1, n); \\
\text{if } \text{order} = \text{Nag\_RowMajor}, & \quad \text{pdx} \geq \max(1, \text{nrhs})
\end{align*}
\]
rcond – double * 

*Output*

On exit: if no constraints are violated, an estimate of the reciprocal condition number of the matrix A (after equilibration if that is performed), computed as \( rcond = 1.0 / \left( \|A\|_1 \|A^{-1}\|_1 \right) \).

ferr[nrhs] – double 

*Output*

On exit: if fail.code = NE_NOERROR or NE_SINGULAR_WP, an estimate of the forward error bound for each computed solution vector, such that \( \|\hat{x}_j - x_j\|_\infty / \|x_j\|_\infty \leq ferr[j - 1] \) where \( \hat{x}_j \) is the \( j \)th column of the computed solution returned in the array x and \( x_j \) is the corresponding column of the exact solution \( X \). The estimate is as reliable as the estimate for rcond, and is almost always a slight overestimate of the true error.

berr[nrhs] – double 

*Output*

On exit: if fail.code = NE_NOERROR or NE_SINGULAR_WP, an estimate of the component-wise relative backward error of each computed solution vector \( \hat{x}_j \) (i.e., the smallest relative change in any element of \( A \) or \( B \) that makes \( \hat{x}_j \) an exact solution).

fail – NagError *

*Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

**NE_ALLOC_FAIL**

Dynamic memory allocation failed.

See Section 3.2.1.2 in the Essential Introduction for further information.

**NE_BAD_PARAM**

On entry, argument \( \langle \text{value} \rangle \) had an illegal value.

**NE_INT**

On entry, \( n = \langle \text{value} \rangle \).

Constraint: \( n \geq 0 \).

On entry, \( nrhs = \langle \text{value} \rangle \).

Constraint: \( nrhs \geq 0 \).

On entry, \( pda = \langle \text{value} \rangle \).

Constraint: \( pda > 0 \).

On entry, \( pdaf = \langle \text{value} \rangle \).

Constraint: \( pdaf > 0 \).

On entry, \( pdb = \langle \text{value} \rangle \).

Constraint: \( pdb > 0 \).

On entry, \( pdx = \langle \text{value} \rangle \).

Constraint: \( pdx > 0 \).

**NE_INT_2**

On entry, \( pda = \langle \text{value} \rangle \) and \( n = \langle \text{value} \rangle \).

Constraint: \( pda \geq \max(1, n) \).

On entry, \( pdaf = \langle \text{value} \rangle \) and \( n = \langle \text{value} \rangle \).

Constraint: \( pdaf \geq \max(1, n) \).

On entry, \( pdb = \langle \text{value} \rangle \) and \( n = \langle \text{value} \rangle \).

Constraint: \( pdb \geq \max(1, n) \).
On entry, pdb = \(\text{value}\) and nrhs = \(\text{value}\).
Constraint: \(pdb \geq \max(1, \text{nrhs})\).

On entry, pdx = \(\text{value}\) and n = \(\text{value}\).
Constraint: \(pdx \geq \max(1, n)\).

On entry, pdx = \(\text{value}\) and nrhs = \(\text{value}\).
Constraint: \(pdx \geq \max(1, \text{nrhs})\).

**NE_INTERNAL_ERROR**
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.
An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

**NE_MAT_NOT_POS_DEF**
The leading minor of order \(\text{value}\) of \(A\) is not positive definite, so the factorization could not be completed, and the solution has not been computed. \(rcond = 0.0\) is returned.

**NE_NO_LICENCE**
Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

**NE_SINGULAR_WP**
\(U\) (or \(L\)) is nonsingular, but \(rcond\) is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of \(rcond\) would suggest.

7 Accuracy
For each right-hand side vector \(b\), the computed solution \(x\) is the exact solution of a perturbed system of equations \((A + E)x = b\), where

if uplo = Nag_Upper, \(|\mathbf{E}| \leq c(n)\epsilon\|\mathbf{U}\|\|\mathbf{U}\|\);
if uplo = Nag_Lower, \(|\mathbf{E}| \leq c(n)\epsilon\|\mathbf{L}\|\|\mathbf{L}\|\),

c(n) is a modest linear function of \(n\), and \(\epsilon\) is the machine precision. See Section 10.1 of Higham (2002) for further details.
If \(\hat{x}\) is the true solution, then the computed solution \(x\) satisfies a forward error bound of the form

\[
\frac{\|x - \hat{x}\|_{\infty}}{\|\hat{x}\|_{\infty}} \leq w_c \text{cond}(A, \hat{x}, b)
\]

where \(\text{cond}(A, \hat{x}, b) = \|A^{-1}\|\|A\|\|\hat{x}\|_{\infty} / \|\hat{x}\|_{\infty} \leq \text{cond}(A) = \|A^{-1}\|\|A\|_{\infty} \leq \kappa_{\infty}(A)\). If \(\hat{x}\) is the \(j\)th column of \(X\), then \(w_c\) is returned in \(\text{berr}[j - 1]\) and a bound on \(\|x - \hat{x}\|_{\infty} / \|\hat{x}\|_{\infty}\) is returned in \(\text{ferr}[j - 1]\). See Section 4.4 of Anderson et al. (1999) for further details.

8 Parallelism and Performance
\text{nag_zposvx} (f07fpc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.
\text{nag_zposvx} (f07fpc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.
Further Comments
The factorization of $A$ requires approximately $\frac{4}{3}n^3$ floating-point operations.

For each right-hand side, computation of the backward error involves a minimum of $16n^2$ floating-point operations. Each step of iterative refinement involves an additional $24n^2$ operations. At most five steps of iterative refinement are performed, but usually only one or two steps are required. Estimating the forward error involves solving a number of systems of equations of the form $Ax = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $8n^2$ operations.

The real analogue of this function is nag_dposvx (f07fbc).

Example
This example solves the equations

$$AX = B,$$

where $A$ is the Hermitian positive definite matrix

$$A = \begin{pmatrix} 3.23 & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 3.93 - 6.14i & 1.48 + 6.58i \\ 6.17 + 9.42i & 4.64 - 4.75i \\ -7.17 - 21.83i & -4.91 + 2.29i \\ 1.99 - 14.38i & 7.64 - 10.79i \end{pmatrix}.$$}

Error estimates for the solutions, information on equilibration and an estimate of the reciprocal of the condition number of the scaled matrix $A$ are also output.

10.1 Program Text
/* nag_zposvx (f07fpc) Example Program. */
* Copyright 2014 Numerical Algorithms Group.
* Mark 23, 2011.
*/
#include <stdio.h>
#include <nag.h>
#include <nagx04.h>
#include <nagf07.h>
int main(void)
{
    /* Scalars */
    double rcond;
    Integer exit_status = 0, i, j, n, nrhs, pda, pdaf, pdb, pdx;
    /* Arrays */
    Complex *a = 0, *af = 0, *b = 0, *x = 0;
    double *berr = 0, *ferr = 0, *s = 0;
    /* Nag Types */
    NagError fail;
Nag_OrderType order;
Nag_EquilibrationType equed;

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J-1)*pda + I - 1]
#define B(I, J) b[(J-1)*pdb + I - 1]
order = Nag_ColMajor;
#else
define A(I, J) a[(I-1)*pda+J-1]
define B(I, J) b[(I-1)*pdb + J - 1]
order = Nag_RowMajor;
#endif

INIT_FAIL(fail);
printf("nag_zposvx (f07fpc) Example Program Results\n\n");

.getOrElseSkip_heading_in_data_file */
#endif _WIN32
scanf_s("%*[\n"]);
#else
scanf("%*[\n"]);
#endif

.timedelta Win
	scanf_s("%"NAG_IFMT"%"NAG_IFMT"%*[\n"]", &n, &nrhs);
#else
	scanf("%"NAG_IFMT"%"NAG_IFMT"%*[\n"]", &n, &nrhs);
#endif
if (n < 0 || nrhs < 0)
{
  printf("Invalid n or nrhs\n");
extit_status = 1;
goto END;
}

pda = n;
pdaf = n;
#ifdef NAG_COLUMN_MAJOR
pdb = n;
pdx = n;
#else
pdb = nrhs;
pdx = nrhs;
#endif

	/* Allocate memory */
if (! (a = NAG_ALLOC(n * n, Complex)) ||
    (af = NAG_ALLOC(n * n, Complex)) ||
    (b = NAG_ALLOC(n * nrhs, Complex)) ||
    (x = NAG_ALLOC(n * nrhs, Complex)) ||
    (!berr = NAG_ALLOC(nrhs, double)) ||
    (!ferr = NAG_ALLOC(nrhs, double)) ||
    (!s = NAG_ALLOC(n, double)))
{
  printf("Allocation failure\n");
extit_status = -1;
goto END;
}
	/* Read the upper triangular part of A from data file */
for (i = 1; i <= n; ++i)
  for (j = i; j <= n; ++j)
  #ifdef _WIN32
		scanf_s(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
  #else
		scanf(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
  #endif
  #ifdef _WIN32
		scanf_s("%*[\n"]);
  #else

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/* Read B from data file */
for (i = 1; i <= n; ++i)
    for (j = 1; j <= nrhs; ++j)
#ifdef _WIN32
    scanf_s(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
#else
    scanf(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
#endif
#endif

/* Solve the equations AX = B for X using nag_zposvx (f07fpc). */
if (fail.code != NE_NOERROR && fail.code != NE_SINGULAR)
{
    printf("Error from nag_zposvx (f07fpc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Print solution using nag_gen_complx_mat_print_comp (x04dbc). */
fflush(stdout);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Print error bounds, condition number and the form of equilibration */
for (j = 0; j < nrhs; ++j) printf("%11.1e%s", berr[j], j%7 == 6?"\n":" ");
for (j = 0; j < nrhs; ++j) printf("%11.1e%s", ferr[j], j%7 == 6?"\n":" ");
printf("%11.1e\n", rcond);
if (equed == Nag_NoEquilibration)
    printf("A has not been equilibrated\n");
else if (equed == Nag_RowAndColumnEquilibration)
    printf("A has been row and column scaled as diag(S)*A*diag(S)\n");
if (fail.code == NE_SINGULAR)
{
    printf("Error from nag_zposvx (f07fpc).\n%s\n", fail.message);
    exit_status = 1;
}

END:
NAG_FREE(a);
NAG_FREE(af);
NAG_FREE(b);
NAG_FREE(x);
NAG_FREE(berr);
NAG_FREE(ferr);
NAG_FREE(s);
return exit_status;
#undef B
#undef A

10.2 Program Data

nag_zposvx (f07fpc) Example Program Data

\[
\begin{align*}
\text{matrix A:} & \\
(3.23, 0.00) & (1.51, -1.92) & (1.90, 0.84) & (0.42, 2.50) \\
(3.58, 0.00) & (-0.23, 1.11) & (-1.18, 1.37) & \\
(4.09, 0.00) & (2.33, -0.14) & & (4.29, 0.00)
\end{align*}
\]

\[
\begin{align*}
\text{matrix B:} & \\
(3.93, -6.14) & (1.48, 6.58) \\
(6.17, 9.42) & (4.65, -4.75) \\
(-7.17, -21.83) & (-4.91, 2.29) \\
(1.99, -14.38) & (7.64, -10.79)
\end{align*}
\]

10.3 Program Results

nag_zposvx (f07fpc) Example Program Results

Solution(s)

\[
\begin{align*}
1 & \\
& (1.0000, -1.0000) \quad (-1.0000, 2.0000) \\
2 & (-0.0000, 3.0000) \quad (3.0000, -4.0000) \\
3 & (-4.0000, -5.0000) \quad (-2.0000, 3.0000) \\
4 & (2.0000, 1.0000) \quad (4.0000, -5.0000)
\end{align*}
\]

Backward errors (machine-dependent)

5.9e-17 4.8e-17

Estimated forward error bounds (machine-dependent)

6.0e-14 7.2e-14

Estimate of reciprocal condition number

6.6e-03

A has not been equilibrated