NAG Library Function Document

nag_zgttrf (f07crc)

1 Purpose

nag_zgttrf (f07crc) computes the LU factorization of a complex n by n tridiagonal matrix A.

2 Specification

#include <nag.h>
#include <nagf07.h>

void nag_zgttrf (Integer n, Complex dl[], Complex d[], Complex du[],
                  Complex du2[], Integer ipiv[], NagError *fail)

3 Description

nag_zgttrf (f07crc) uses Gaussian elimination with partial pivoting and row interchanges to factorize the matrix A as

\[ A = PLU, \]

where P is a permutation matrix, L is unit lower triangular with at most one nonzero subdiagonal element in each column, and U is an upper triangular band matrix, with two superdiagonals.

4 References


5 Arguments

1:  n – Integer
    
    On entry: n, the order of the matrix A.

    Constraint: n ≥ 0.

2:  dl[dim] – Complex
    
    Note: the dimension, dim, of the array dl must be at least max(1, n – 1).

    On entry: must contain the (n – 1) subdiagonal elements of the matrix A.

    On exit: is overwritten by the (n – 1) multipliers that define the matrix L of the LU factorization of A.

3:  d[dim] – Complex
    
    Note: the dimension, dim, of the array d must be at least max(1, n).

    On entry: must contain the n diagonal elements of the matrix A.

    On exit: is overwritten by the n diagonal elements of the upper triangular matrix U from the LU factorization of A.

4:  du[dim] – Complex
    
    Note: the dimension, dim, of the array du must be at least max(1, n – 1).
On entry: must contain the \((n - 1)\) superdiagonal elements of the matrix \(A\).

On exit: is overwritten by the \((n - 1)\) elements of the first superdiagonal of \(U\).

5:  \texttt{du2}[n - 2] \textit{— Complex} \hspace{1cm} \textit{Output}

On exit: contains the \((n - 2)\) elements of the second superdiagonal of \(U\).

6:  \texttt{ipiv}[n] \textit{— Integer} \hspace{1cm} \textit{Output}

On exit: contains the \(n\) pivot indices that define the permutation matrix \(P\). At the \(i\)th step, row \(i\) of the matrix was interchanged with row \(\texttt{ipiv}[i - 1]\). \(\texttt{ipiv}[i - 1]\) will always be either \(i\) or \((i + 1)\), \(\texttt{ipiv}[i - 1] = i\) indicating that a row interchange was not performed.

7:  \texttt{fail} \textit{— NagError} * \hspace{1cm} \textit{Input/Output}

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 \hspace{1cm} \textbf{Error Indicators and Warnings}

\textbf{NE_ALLOC_FAIL}

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

\textbf{NE_BAD_PARAM}

On entry, argument \(\langle\text{value}\rangle\) had an illegal value.

\textbf{NE_INT}

On entry, \(n = \langle\text{value}\rangle\).
Constraint: \(n \geq 0\).

\textbf{NE_INTERNAL_ERROR}

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

\textbf{NE_NO_LICENCE}

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

\textbf{NE_SINGULAR}

Element \(\langle\text{value}\rangle\) of the diagonal is exactly zero. The factorization has been completed, but the factor \(U\) is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 \hspace{1cm} \textbf{Accuracy}

The computed factorization satisfies an equation of the form

\[ A + E = PLU, \]

where

\[ \|E\|_\infty = O(\epsilon)\|A\|_\infty \]

and \(\epsilon\) is the \textit{machine precision}. 

Following the use of this function, `nag_zgttrs (f07csc)` can be used to solve systems of equations $AX = B$ or $A^T X = B$ or $A^H X = B$, and `nag_zgtsv (f07cuc)` can be used to estimate the condition number of $A$.

8 Parallelism and Performance

Not applicable.

9 Further Comments

The total number of floating-point operations required to factorize the matrix $A$ is proportional to $n$.

The real analogue of this function is `nag_dgttrf (f07cdc)`.

10 Example

This example factorizes the tridiagonal matrix $A$ given by

$$
A = \begin{pmatrix}
-1.3 + 1.3i & 2.0 - 1.0i & 0 & 0 & 0 \\
1.0 - 2.0i & -1.3 + 1.3i & 2.0 + 1.0i & 0 & 0 \\
0 & 1.0 + 1.0i & -1.3 + 3.3i & -1.0 + 1.0i & 0 \\
0 & 0 & 2.0 - 3.0i & -0.3 + 4.3i & 1.0 - 1.0i \\
0 & 0 & 0 & 1.0 + 1.0i & -3.3 + 1.3i
\end{pmatrix}.
$$

10.1 Program Text

/* nag_zgttrf (f07crc) Example Program. */
/* Copyright 2014 Numerical Algorithms Group. */
/* Mark 23, 2011. */
*/
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>

int main(void)
{
    /* Scalars */
    Integer exit_status = 0, i, n;

    /* Arrays */
    Complex *d = 0, *dl = 0, *du = 0, *du2 = 0;
    Integer *ipiv = 0;

    /* Nag Types */
    NagError fail;

    INIT_FAIL(fail);

    printf("nag_zgttrf (f07crc) Example Program Results\n\n");
    /* Skip heading in data file */
    #ifdef _WIN32
    scanf_s("%*[\n]");
    #else
    scanf("%*[\n]");
    #endif

    #ifdef _WIN32
    scanf_s(""%NAG_IFMT"%*[\n]", &n);
    #else
    scanf(""%NAG_IFMT"%*[\n]", &n);
    #endif
if (n < 0)
{
    printf("Invalid n\n");
    exit_status = 1;
    goto END;
}

/* Allocate memory */
if (!(d = NAG_ALLOC(n, Complex)) ||
    !(dl = NAG_ALLOC(n-1, Complex)) ||
    !(du = NAG_ALLOC(n-1, Complex)) ||
    !(du2 = NAG_ALLOC(n-2, Complex)) ||
    !(ipiv = NAG_ALLOC(n, Integer)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read the tridiagonal matrix A from data file */
#ifdef _WIN32
    for (i=0; i<n-1; ++i) scanf_s(" ( %lf , %lf )", &du[i].re, &du[i].im);
#else
    for (i=0; i<n-1; ++i) scanf(" ( %lf , %lf )", &du[i].re, &du[i].im);
#endif
#ifdef _WIN32
    scanf_s("%*[\n");
#else
    scanf("%*[\n");
#endif
#ifdef _WIN32
    for (i = 0; i < n; ++i) scanf_s(" ( %lf , %lf )", &d[i].re, &d[i].im);
#else
    for (i = 0; i < n; ++i) scanf(" ( %lf , %lf )", &d[i].re, &d[i].im);
#endif
#ifdef _WIN32
    scanf_s("%*[\n");
#else
    scanf("%*[\n");
#endif
#ifdef _WIN32
    for (i=0; i<n-1; ++i) scanf_s(" ( %lf , %lf )", &dl[i].re, &dl[i].im);
#else
    for (i=0; i<n-1; ++i) scanf(" ( %lf , %lf )", &dl[i].re, &dl[i].im);
#endif
#ifdef _WIN32
    scanf_s("%*[\n");
#else
    scanf("%*[\n");
#endif

/* Factorize the tridiagonal matrix A using nag_zgttrf (f07crc). */
ng_zgttrf(n, dl, d, du, du2, ipiv, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_zgttrf (f07crc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Print details of the factorization */
printf("Details of factorization (U)\n");
printf("Main diagonal", "First super-diagonal", "Second super-diagonal");
for (i = 0; i < n; ++i)
{
    printf("(%8.4f, %8.4f)\n", d[i].re, d[i].im);
    if (i < n-1) printf(" (%8.4f, %8.4f)\n", du[i].re, du[i].im);
    if (i < n-2) printf(" (%8.4f, %8.4f)\n", du2[i].re, du2[i].im);
    printf("\n");
printf("\n Multipliers\n");
for (i = 0; i < n - 1; ++i) printf("(%8.4f, %8.4f)\n", dl[i].re, dl[i].im);

printf("\n Vector of interchanges\n");
for (i = 0; i < n; ++i) printf("%7"NAG_IFMT"%s", ipiv[i], i%5 == 4?"\n":" ");
printf("\n");
END:
NAG_FREE(d);
NAG_FREE(dl);
NAG_FREE(du);
NAG_FREE(du2);
NAG_FREE(ipiv);
return exit_status;
}

10.2 Program Data

nag_zgttrf (f07crc) Example Program Data

5 : n
( 2.0,-1.0) ( 2.0, 1.0) (-1.0, 1.0) ( 1.0,-1.0) : du
(-1.3, 1.3) (-1.3, 1.3) (-1.3, 3.3) (-0.3, 4.3) (-3.3, 1.3) : d
( 1.0,-2.0) ( 1.0, 1.0) ( 2.0,-3.0) ( 1.0, 1.0) : dl

10.3 Program Results

nag_zgttrf (f07crc) Example Program Results

Details of factorization (U)

Main diagonal  First super-diagonal Second super-diagonal
( 1.0000, -2.0000) ( -1.3000, 1.3000) ( 2.0000, 1.0000)
( 1.0000, 1.0000) ( -1.3000, 3.3000) ( -1.0000, 1.0000)
( 2.0000, -3.0000) ( -0.3000, 4.3000) ( 1.0000, -1.0000)
( 1.0000, 1.0000) ( -3.3000, 1.3000)
( -1.3399, 0.2875)

Multipliers
( -0.7800, -0.2600)
( 0.1620, -0.4860)
( -0.0452, -0.0010)
( -0.3979, -0.0562)

Vector of interchanges
2 3 4 5 5