1 Purpose

nag_zgeequ (f07atc) computes real diagonal scaling matrices $D_R$ and $D_C$ intended to equilibrate a complex $m$ by $n$ matrix $A$ and reduce its condition number.

2 Specification

```c
#include <nag.h>
#include <nagf07.h>

void nag_zgeequ (Nag_OrderType order, Integer m, Integer n,
      const Complex a[], Integer pda, double r[], double c[],
      double *rowcnd, double *colcnd, double *amax,
      NagError *fail)
```

3 Description

nag_zgeequ (f07atc) computes the diagonal scaling matrices. The diagonal scaling matrices are chosen to try to make the elements of largest absolute value in each row and column of the matrix $B$ given by

$$B = D_R A D_C$$

have absolute value 1. The diagonal elements of $D_R$ and $D_C$ are restricted to lie in the safe range $(\delta, 1/\delta)$, where $\delta$ is the value returned by function nag_real_safe_small_number (X02AMC). Use of these scaling factors is not guaranteed to reduce the condition number of $A$ but works well in practice.

4 References

None.

5 Arguments

1:  order – Nag_OrderType

   *Input*

   On entry: the order argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by order = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

   Constraint: order = Nag_RowMajor or Nag_ColMajor.

2:  m – Integer

   *Input*

   On entry: $m$, the number of rows of the matrix $A$.

   Constraint: $m \geq 0$.

3:  n – Integer

   *Input*

   On entry: $n$, the number of columns of the matrix $A$.

   Constraint: $n \geq 0$. 

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4: \texttt{a[\textit{dim}]} – const Complex \hspace{1cm} \textit{Input}

\textbf{Note:} the dimension, \textit{dim}, of the array \texttt{a} must be at least
\[
\max(1, \texttt{pda} \times \texttt{n}) \text{ when } \texttt{order} = \texttt{Nag\_ColMajor};
\max(1, \texttt{m} \times \texttt{pda}) \text{ when } \texttt{order} = \texttt{Nag\_RowMajor}.
\]
The \((i,j)\)th element of the matrix \(A\) is stored in
\[
\begin{align*}
\texttt{a[(j-1) \times pda + i - 1]} & \text{ when } \texttt{order} = \texttt{Nag\_ColMajor}; \\
\texttt{a[(i-1) \times pda + j - 1]} & \text{ when } \texttt{order} = \texttt{Nag\_RowMajor}.
\end{align*}
\]
\textit{On entry:} the matrix \(A\) whose scaling factors are to be computed.

5: \texttt{pda} – Integer \hspace{1cm} \textit{Input}

\textit{On entry:} the stride separating row or column elements (depending on the value of \texttt{order}) in the array \texttt{a}.

\textit{Constraints:}
\[
\begin{align*}
\text{if } \texttt{order} = \texttt{Nag\_ColMajor}, & \texttt{pda} \geq \max(1, \texttt{m}); \\
\text{if } \texttt{order} = \texttt{Nag\_RowMajor}, & \texttt{pda} \geq \max(1, \texttt{n}).
\end{align*}
\]

6: \texttt{r[m]} – double \hspace{1cm} \textit{Output}

\textit{On exit:} if \texttt{fail.code} = \texttt{NE\_NOERROR} or \texttt{fail.code} = \texttt{NE\_MAT\_COL\_ZERO}, \texttt{r} contains the row scale factors, the diagonal elements of \(DR\). The elements of \texttt{r} will be positive.

7: \texttt{c[n]} – double \hspace{1cm} \textit{Output}

\textit{On exit:} if \texttt{fail.code} = \texttt{NE\_NOERROR}, \texttt{c} contains the column scale factors, the diagonal elements of \(DC\). The elements of \texttt{c} will be positive.

8: \texttt{rowcnd} – double * \hspace{1cm} \textit{Output}

\textit{On exit:} if \texttt{fail.code} = \texttt{NE\_NOERROR} or \texttt{fail.code} = \texttt{NE\_MAT\_COL\_ZERO}, \texttt{rowcnd} contains the ratio of the smallest value of \(r[i-1]\) to the largest value of \(r[i-1]\). If \texttt{rowcnd} \geq 0.1 and \texttt{amax} is neither too large nor too small, it is not worth scaling by \(DR\).

9: \texttt{colcnd} – double * \hspace{1cm} \textit{Output}

\textit{On exit:} if \texttt{fail.code} = \texttt{NE\_NOERROR}, \texttt{colcnd} contains the ratio of the smallest value of \(c[i-1]\) to the largest value of \(c[i-1]\).

If \texttt{colcnd} \geq 0.1, it is not worth scaling by \(DC\).

10: \texttt{amax} – double * \hspace{1cm} \textit{Output}

\textit{On exit:} \max|\texttt{a}_{ij}|. If \texttt{amax} is very close to overflow or underflow, the matrix \(A\) should be scaled.

11: \texttt{fail} – \texttt{NagError} * \hspace{1cm} \textit{Input/Output}

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 \hspace{0.5cm} \textbf{Error Indicators and Warnings}

\textbf{NE\_ALLOC\_FAIL}

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

\textbf{NE\_BAD\_PARAM}

On entry, argument \textlangle value\rangle had an illegal value.
NE_INT
On entry, \( m = \langle \text{value} \rangle \).
Constraint: \( m \geq 0 \).

On entry, \( n = \langle \text{value} \rangle \).
Constraint: \( n \geq 0 \).

On entry, \( pda = \langle \text{value} \rangle \).
Constraint: \( pda > 0 \).

NE_INT_2
On entry, \( pda = \langle \text{value} \rangle \) and \( m = \langle \text{value} \rangle \).
Constraint: \( pda \geq \max(1, m) \).

On entry, \( pda = \langle \text{value} \rangle \) and \( n = \langle \text{value} \rangle \).
Constraint: \( pda \geq \max(1, n) \).

NE_INTERNAL_ERROR
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_MAT_COL_ZERO
Column \( \langle \text{value} \rangle \) of \( A \) is exactly zero.

NE_MAT_ROW_ZERO
Row \( \langle \text{value} \rangle \) of \( A \) is exactly zero.

NE_NO LICENCE
Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

7 Accuracy
The computed scale factors will be close to the exact scale factors.

8 Parallelism and Performance
Not applicable.

9 Further Comments
The real analogue of this function is nag_dgeequ (f07afc).

10 Example
This example equilibrates the general matrix \( A \) given by
\[
A = \begin{pmatrix}
-1.34 + 2.55i & (0.28 + 3.17i) \times 10^{10} & -6.39 - 2.20i \\
-1.70 - 1.41i & (3.31 - 0.15i) \times 10^{10} & -0.15 + 1.34i \\
(2.41 + 0.39i) \times 10^{-10} & -0.56 + 1.47i & (-0.83 - 0.69i) \times 10^{-10}
\end{pmatrix}.
\]

Details of the scaling factors, and the scaled matrix are output.
10.1 Program Text

/* nag_zgeequ (f07atc) Example Program.*/
/* Copyright 2014 Numerical Algorithms Group.*/
/* Mark 23, 2011.*/
*/
#include <stdio.h>
#include <nag.h>
#include <nagx04.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx02.h>

int main(void)
{
    /* Scalars */
    double amax, big, colcnd, rowcnd, small;
    Integer i, j, m, n, pda;
    Integer exit_status = 0;

    /* Arrays */
    Complex *a = 0;
    double *c = 0, *r = 0;

    /* Nag Types */
    NagError fail;
    Nag_OrderType order;
    Nag_Boolean scaled = Nag_FALSE;

#ifdef NAG_COLUMN_MAJOR
    #define A(I, J) a[(J-1)*pda +I-1 ]
    order = Nag_ColMajor;
#else
    #define A(I, J) a[(I-1)*pda+J-1 ]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_zgeequ (f07atc) Example Program Results\n\n");

    /* Skip heading in data file */
    #ifdef _WIN32
        scanf_s("%*[\n ]");
    #else
        scanf("%*[\n ]");
    #endif
    #ifdef _WIN32
        scanf("%"NAG_IFMT"%*[\n ]", &n);
    #else
        scanf("%"NAG_IFMT"%*[\n ]", &n);
    #endif
    if (n < 0)
    {
        printf("Invalid n\n");
        exit_status = 1;
        return exit_status;
    }

    m = n;
    pda = n;

    /* Allocate memory */
    if (!a = NAG_ALLOC(m*n, Complex)) ||
        !(c = NAG_ALLOC(n, double)) ||
        !(r = NAG_ALLOC(m, double))
    {
        printf("Allocation failure\n");
    }
exit_status = -1;
goto END;
}

/* Read the n by n matrix A from data file */
for (i = 1; i <= n; ++i)
  for (j = 1; j <= n; ++j)
    #ifdef _WIN32
      scanf_s(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
    #else
      scanf(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
    #endif
    #ifdef _WIN32
      scanf_s("%*[^
"]);
    #else
      scanf("%*[^
"]);
    #endif
  
/* Print the matrix A using nag_gen_complx_mat_print_comp (x04dbc). */
fflush(stdout);
nag_gen_complx_mat_print_comp(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, n, a, pda, Nag_BracketForm, "%11.2e",
  "Matrix A", Nag_IntegerLabels, 0, Nag_IntegerLabels, 0, 80, 0, 0, &fail);
if (fail.code != NE_NOERROR)
{
  printf("Error from nag_gen_complx_mat_print_comp (x04dbc).
    %s
", fail.message);
  exit_status = 1;
goto END;
}

/* Compute row and column scaling factors */
nag_zgeequ(order, n, n, a, pda, r, c, &rowcnd, &colcnd, &amax, &fail);
if (fail.code != NE_NOERROR)
{
  printf("Error from nag_zgeequ (f07atc).\n%sn", fail.message);
  exit_status = 1;
goto END;
}

/* Print rowcnd, colcnd, amax and the scale factors */
printf("rowcnd = %10.1e, colcnd = %10.1e, amax = %10.1e\n", rowcnd, colcnd, amax);
printf("Row scale factors\n");
for (i = 1; i <= n; ++i) printf("%11.2e\n", r[i-1]);
printf("Column scale factors\n");
for (i = 1; i <= n; ++i) printf("%11.2e\n", c[i-1]);

/* Compute values close to underflow and overflow using
 * nag_real_safe_small_number (x02amc), nag_machine_precision (x02ajc) and
 * nag_real_base (x02bhc)
 */
small = nag_real_safe_small_number / (nag_machine_precision * nag_real_base);
big = 1.0 / small;
if (colcnd < 0.1)
{
  /* column scale A */
  scaled = Nag_TRUE;
  for (j = 1; j <= n; ++j)
    for (i = 1; i <= n; ++i) {
      A(i, j).re *= c[j - 1];
      A(i, j).im *= c[j - 1];
    }
}
if (rowcnd < 0.1 || amax < small || amax > big)
{
  /* row scale A */
  scaled = Nag_TRUE;
for (j = 1; j <= n; ++j)
for (i = 1; i <= n; ++i) {
    A(i, j).re *= r[i - 1];
    A(i, j).im *= r[i - 1];
}

if (scaled)
{
    /* Print the scaled matrix using nag_gen_complx_mat_print_comp (x04dbc) */
    fflush(stdout);
    nag_gen_complx_mat_print_comp(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, n, a, pda, Nag_BracketForm, 0,
    "Scaled matrix", Nag_IntegerLabels, 0,
    Nag_IntegerLabels, 0, 80, 0, 0, &fail);
    if (fail.code != NE_NOERROR)
    {
        printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n\n", fail.message);
        exit_status = 1;
        goto END;
    }
}

END:
NAG_FREE(a);
NAG_FREE(c);
NAG_FREE(r);
return exit_status;

10.2 Program Data

nag_zgeequ (f07atc) Example Program Data

\begin{verbatim}
n : n

( -1.34e+00, 2.55e+00) ( 0.28e+10, 3.17e+10) ( -6.39e+00, -2.20e+00)
(-1.70e+00,-1.41e+00) ( 3.31e+10,-0.15e+00) ( 3.31e+10,-0.15e+00)
( 2.41e-10, 0.39e-10) (-0.56e+00, 1.47e+00) (-0.83e-10,-0.69e-10)
\end{verbatim}

: matrix A

10.3 Program Results

nag_zgeequ (f07atc) Example Program Results

\begin{verbatim}
Matrix A

Matrix A

1 2
1 ( -1.34e+00, 2.55e+00) ( 2.80e+09, 3.17e+10)
2 ( -1.70e+00, -1.41e+00) ( 3.31e+10, -1.50e+09)
3 ( 2.41e-10, 3.90e-11) ( -5.60e-01, 1.47e+00)

3
1 ( -6.39e+00, -2.20e+00)
2 ( -1.50e-01, 1.34e+00)
3 ( -8.30e-11, -6.90e-11)

rowcnd = 5.9e-11, colcnd = 1.4e-10, amax = 3.5e+10

Row scale factors
2.90e-11 2.89e-11 4.93e-01

Column scale factors
7.25e+09 1.00e+00 4.02e+09
\end{verbatim}
## Scaled matrix

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(-0.2816, 0.5359)</td>
<td>(0.0812, 0.9188)</td>
<td>(-0.7439, -0.2561)</td>
</tr>
<tr>
<td>2</td>
<td>(-0.3562, -0.2954)</td>
<td>(0.9566, -0.0434)</td>
<td>(-0.0174, 0.1555)</td>
</tr>
<tr>
<td>3</td>
<td>(0.8607, 0.1393)</td>
<td>(-0.2759, 0.7241)</td>
<td>(-0.1642, -0.1365)</td>
</tr>
</tbody>
</table>