NAG Library Function Document
nag_real_symm_general_eigenvalues (f02adc)

1 Purpose
nag_real_symm_general_eigenvalues (f02adc) calculates all the eigenvalues of \( Ax = \lambda Bx \), where \( A \) is a real symmetric matrix and \( B \) is a real symmetric positive definite matrix.

2 Specification
```c
#include <nag.h>
#include <nagf02.h>
void nag_real_symm_general_eigenvalues (Integer n, double a[], Integer tda,
                                      double b[], Integer tdb, double r[], NagError *fail)
```

3 Description
The problem is reduced to the standard symmetric eigenproblem using Cholesky’s method to decompose \( B \) into triangular matrices, \( B = LL^T \), where \( L \) is lower triangular. Then \( Ax = \lambda Bx \) implies \((L^{-1}AL^{-T})(L^Tx) = \lambda(L^Tx)\); hence the eigenvalues of \( Ax = \lambda Bx \) are those of \( Py = \lambda y \) where \( P \) is the symmetric matrix \( L^{-1}AL^{-T} \). Householder’s method is used to tridiagonalise the matrix \( P \) and the eigenvalues are then found using the QL algorithm.

4 References

5 Arguments
1: \( n \) – Integer
   \( \text{Input} \)
   On entry: \( n \), the order of the matrices \( A \) and \( B \).
   Constraint: \( n \geq 1 \).

2: \( a[n \times tda] \) – double
   \( \text{Input/Output} \)
   Note: the \((i,j)\)th element of the matrix \( A \) is stored in \( a[(i-1) \times tda + j - 1] \).
   On entry: the upper triangle of the \( n \) by \( n \) symmetric matrix \( A \). The elements of the array below the diagonal need not be set.
   On exit: the lower triangle of the array is overwritten. The rest of the array is unchanged.

3: \( tda \) – Integer
   \( \text{Input} \)
   On entry: the stride separating matrix column elements in the array \( a \).
   Constraint: \( tda \geq n \).

4: \( b[n \times tdb] \) – double
   \( \text{Input/Output} \)
   Note: the \((i,j)\)th element of the matrix \( B \) is stored in \( b[(i-1) \times tdb + j - 1] \).
   On entry: the upper triangle of the \( n \) by \( n \) symmetric positive definite matrix \( B \). The elements of the array below the diagonal need not be set.
On exit: the elements below the diagonal are overwritten. The rest of the array is unchanged.

5: $\text{tdb}$ – Integer

On entry: the stride separating matrix column elements in the array $b$.

Constraint: $\text{tdb} \geq n$.

6: $r[n]$ – double

On exit: the eigenvalues in ascending order.

7: $\text{fail}$ – NagError

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_2_INT_ARG_LT

On entry, $\text{tda} = \langle value \rangle$ while $n = \langle value \rangle$. These arguments must satisfy $\text{tda} \geq n$.

On entry, $\text{tdb} = \langle value \rangle$ while $n = \langle value \rangle$. These arguments must satisfy $\text{tdb} \geq n$.

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_INT_ARG_LT

On entry, $n = \langle value \rangle$.

Constraint: $n \geq 1$.

NE_NOT_POS_DEF

The matrix $B$ is not positive definite, possibly due to rounding errors.

NE_TOO_MANY_ITERATIONS

More than $\langle value \rangle$ iterations are required to isolate all the eigenvalues.

7 Accuracy

In general this function is very accurate. However, if $B$ is ill-conditioned with respect to inversion, the eigenvalues could be inaccurately determined. For a detailed error analysis see pages 310, 222 and 235 Wilkinson and Reinsch (1971).

8 Parallelism and Performance

Not applicable.

9 Further Comments

The time taken by nag_real_symm_general_eigenvalues ($f02adc$) is approximately proportional to $n^3$. 

f02adc

NAG Library Manual
10 Example

To calculate all the eigenvalues of the general symmetric eigenproblem $Ax = \lambda Bx$ where $A$ is the symmetric matrix

$$
\begin{pmatrix}
0.5 & 1.5 & 6.6 & 4.8 \\
1.5 & 6.5 & 16.2 & 8.6 \\
6.6 & 16.2 & 37.6 & 9.8 \\
4.8 & 8.6 & 9.8 & -17.1
\end{pmatrix}
$$

and $B$ is the symmetric positive definite matrix

$$
\begin{pmatrix}
1 & 3 & 4 & 1 \\
3 & 13 & 16 & 11 \\
4 & 16 & 24 & 18 \\
1 & 11 & 18 & 27
\end{pmatrix}
$$

10.1 Program Text

/* nag_real_symm_general_eigenvalues (f02adc) Example Program. *
 * Copyright 2014 Numerical Algorithms Group. *
 * Mark 1, 1990. *
 * Mark 8 revised, 2004. */
#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagf02.h>
#define A(I, J) a[(I) *tda + J]
#define B(I, J) b[(I) *tdb + J]
int main(void)
{
    Integer exit_status = 0, i, j, n, tda, tdb;
    NagError fail;
    double *a = 0, *b = 0, *r = 0;

    INIT_FAIL(fail);
    printf("nag_real_symm_general_eigenvalues (f02adc) Example Program"
           " Results\n");
    /* Skip heading in data file */
    #ifdef _WIN32
    scanf_s("%*[\n");
    #else
    scanf("%*[\n");
    #endif
    #ifdef _WIN32
    scanf_s("%"NAG_IFMT", &n);
    #else
    scanf("%"NAG_IFMT", &n);
    #endif
    if (n >= 1)
    {
        if (!(a = NAG_ALLOC(n*n, double))
            || !(b = NAG_ALLOC(n*n, double))
            || !(r = NAG_ALLOC(n, double)))
        {
            printf("Allocation failure\n");
            exit_status = -1;
            goto END;
        }
    }
    else
    {
        ...
printf("Invalid n.\n");
exit_status = 1;
return exit_status;
}

tda = n;
tdb = n;

for (i = 0; i < n; i++)
{
    for (j = 0; j < n; j++)
        scanf("%lf", &A(i, j));
}

for (j = 0; j < n; j++)
    scanf("%lf", &B(i, j));

/* nag_real_symm_general_eigenvalues (f02adc).
* All eigenvalues of generalized real symmetric-definite
* eigenproblem */

nag_real_symm_general_eigenvalues(n, a, tda, b, tdb, r, &fail);

if (fail.code != NE_NOERROR)
{
    printf(
        "Error from nag_real_symm_general_eigenvalues (f02adc)\n",
        fail.message);
    exit_status = 1;
    goto END;
}

printf("Eigenvalues\n");
for (i = 0; i < n; i++)
    printf("%9.4f\n", r[i], (i%8 == 7 || i == n-1)?"\n":" ");

END:
NAG_FREE(a);
NAG_FREE(b);
NAG_FREE(r);
return exit_status;

10.2 Program Data

nag_real_symm_general_eigenvalues (f02adc) Example Program Data

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10.3 Program Results

nag_real_symm_general_eigenvalues (f02adc) Example Program Results

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