NAG Library Function Document

\textbf{nag_ode_bvp_ps_lin_cgl_vals (d02ubc)}

1 Purpose

\texttt{nag_ode_bvp_ps_lin_cgl_vals (d02ubc)} evaluates a function, or one of its lower order derivatives, from its Chebyshev series representation at Chebyshev Gauss–Lobatto points on \([a, b]\). The coefficients of the Chebyshev series representation required are usually derived from those returned by \texttt{nag_ode_bvp_ps_lin_coeffs (d02uac)} or \texttt{nag_ode_bvp_ps_lin_solve (d02uec)}.

2 Specification

\begin{verbatim}
#include <nag.h>
#include <nagd02.h>
void nag_ode_bvp_ps_lin_cgl_vals (Integer n, double a, double b, Integer q,
                           const double c[], double f[], NagError *fail)
\end{verbatim}

3 Description

\texttt{nag_ode_bvp_ps_lin_cgl_vals (d02ubc)} evaluates the Chebyshev series
\[ S(\bar{x}) = \frac{1}{2}c_1T_0(\bar{x}) + c_2T_1(\bar{x}) + c_3T_2(\bar{x}) + \cdots + c_{n+1}T_n(\bar{x}), \]
or its derivative (up to fourth order) at the Chebyshev Gauss–Lobatto points on \([a, b]\). Here \(T_j(\bar{x})\) denotes the Chebyshev polynomial of the first kind of degree \(j\) with argument \(\bar{x}\) defined on \([-1, 1]\). In terms of your original variable, \(x\) say, the input values at which the function values are to be provided are
\[ x_r = -\frac{1}{2}(b-a)\cos(\pi(r-1)/n) + \frac{1}{2}(b+a), \quad r = 1, 2, \ldots, n + 1, \]
where \(b\) and \(a\) are respectively the upper and lower ends of the range of \(x\) over which the function is required.

The calculation is implemented by a forward one-dimensional discrete Fast Fourier Transform (DFT).

4 References


5 Arguments

1: \hspace{1cm} \textbf{n} – Integer \hspace{1cm} \textit{Input}

\textit{On entry:} \(n\), where the number of grid points is \(n + 1\). This is also the largest order of Chebyshev polynomial in the Chebyshev series to be computed.

\textit{Constraint:} \(n > 0\) and \(n\) is even.

2: \hspace{1cm} \textbf{a} – double \hspace{1cm} \textit{Input}

\textit{On entry:} \(a\), the lower bound of domain \([a, b]\).

\textit{Constraint:} \(a < b\).
3: \( b \) – double \hspace{1cm} \text{Input} \\
   On entry: \( b \), the upper bound of domain \([a, b]\). 
   Constraint: \( b > a \).

4: \( q \) – Integer \hspace{1cm} \text{Input} \\
   On entry: the order, \( q \), of the derivative to evaluate. 
   Constraint: \( 0 \leq q \leq 4 \).

5: \( c[n + 1] \) – const double \hspace{1cm} \text{Input} \\
   On entry: the Chebyshev coefficients, \( c_i \), for \( i = 1, 2, \ldots, n + 1 \).

6: \( f[n + 1] \) – double \hspace{1cm} \text{Output} \\
   On exit: the derivatives \( S^{(q)}(x_i) \), for \( i = 1, 2, \ldots, n + 1 \), of the Chebyshev series, \( S \).

7: \( \text{fail} \) – NagError * \hspace{1cm} \text{Input/Output} \\
   The NAG error argument (see Section 3.6 in the Essential Introduction).

6 \hspace{1cm} \textbf{Error Indicators and Warnings}

**NE_ALLOC_FAIL**
Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

**NE_BAD_PARAM**
On entry, argument \( \langle \text{value} \rangle \) had an illegal value.

**NE_INT**
On entry, \( n = \langle \text{value} \rangle \). 
Constraint: \( n > 0 \). 
On entry, \( n = \langle \text{value} \rangle \). 
Constraint: \( n \) is even. 
On entry, \( q = \langle \text{value} \rangle \). 
Constraint: \( 0 \leq q \leq 4 \).

**NE_INTERNAL_ERROR**
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.
An unexpected error has been triggered by this function. Please contact NAG. 
See Section 3.6.6 in the Essential Introduction for further information.

**NE_NO_LICENCE**
Your licence key may have expired or may not have been installed correctly. 
See Section 3.6.5 in the Essential Introduction for further information.

**NE_REAL_2**
On entry, \( a = \langle \text{value} \rangle \) and \( b = \langle \text{value} \rangle \). 
Constraint: \( a < b \).
7 Accuracy
Evaluations of DFT to obtain function or derivative values should be an order $n$ multiple of machine precision assuming full accuracy to machine precision in the given Chebyshev series representation.

8 Parallelism and Performance

nag_ode_bvp_ps_lin_cgl_vals (d02ubc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag_ode_bvp_ps_lin_cgl_vals (d02ubc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users’ Note for your implementation for any additional implementation-specific information.

9 Further Comments
The number of operations is of the order $n \log(n)$ and the memory requirements are $O(n)$; thus the computation remains efficient and practical for very fine discretizations (very large values of $n$).

10 Example
See Section 10 in nag_ode_bvp_ps_lin_solve (d02uec).