1 Purpose

nag_quad_md_numth_coeff_prime (d01gyc) calculates the optimal coefficients for use by nag_quad_md_numth_vec (d01gdc), for prime numbers of points.

2 Specification

```c
#include <nag.h>
#include <nagd01.h>
void nag_quad_md_numth_coeff_prime (Integer ndim, Integer npts, double vk[],
NagError *fail)
```

3 Description

The Korobov (1963) procedure for calculating the optimal coefficients $a_1, a_2, \ldots, a_n$ for $p$-point integration over the $n$-cube $[0,1]^n$ imposes the constraint that

$$a_1 = 1 \quad \text{and} \quad a_i = a_{i-1} \pmod{p}, \quad i = 1, 2, \ldots, n$$

(1)

where $p$ is a prime number and $a$ is an adjustable argument. This argument is computed to minimize the error in the integral

$$3^n \int_0^1 dx_1 \cdots \int_0^1 dx_n \prod_{i=1}^n (1 - 2x_i)^2,$$

(2)

when computed using the number theoretic rule, and the resulting coefficients can be shown to fit the Korobov definition of optimality.

The computation for large values of $p$ is extremely time consuming (the number of elementary operations varying as $p^2$) and there is a practical upper limit to the number of points that can be used. Function nag_quad_md_numth_coeff_2prime (d01gzc) is computationally more economical in this respect but the associated error is likely to be larger.

4 References

Korobov N M (1963) Number Theoretic Methods in Approximate Analysis Fizmatgiz, Moscow

5 Arguments

1: ndim – Integer

   On entry: $n$, the number of dimensions of the integral.

   Constraint: ndim ≥ 1.

2: npts – Integer

   On entry: $p$, the number of points to be used.

   Constraint: npts must be a prime number ≥ 5.

3: vk[ndim] – double

   On exit: the $n$ optimal coefficients.
6 Error Indicators and Warnings

**NE_ACCURACY**

The *machine precision* is insufficient to perform the computation exactly. Try reducing **npts**: 
**npts** = (**value**).

**NE_ALLOC_FAIL**

Dynamic memory allocation failed. 
See Section 3.2.1.2 in the Essential Introduction for further information.

**NE_BAD_PARAM**

On entry, argument (**value**) had an illegal value.

**NE_INT**

On entry, **ndim** = (**value**). 
Constraint: **ndim** ≥ 1.

On entry, **npts** = (**value**). 
Constraint: **npts** must be a prime number.

On entry, **npts** = (**value**). 
Constraint: **npts** ≥ 5.

**NE_INTERNAL_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG. 
See Section 3.6.6 in the Essential Introduction for further information.

**NE_NO_LICENCE**

Your licence key may have expired or may not have been installed correctly. 
See Section 3.6.5 in the Essential Introduction for further information.

7 Accuracy

The optimal coefficients are returned as exact integers (though stored in a double array).

8 Parallelism and Performance

Not applicable.

9 Further Comments

The time taken is approximately proportional to \( p^2 \) (see Section 3).

10 Example

This example calculates the Korobov optimal coefficients where the number of dimensions is 4 and the number of points is 631.
10.1 Program Text

/* nag_quad_md_numth_coeff_prime (d01gyc) Example Program. */
* Copyright 2014 Numerical Algorithms Group.
* Mark 23, 2011.
*/

#include <stdio.h>
#include <nag.h>
#include <nagd01.h>

int main(void)
{
    Integer exit_status = 0;
    Integer i, ndim, npts;
    double *vk = 0;
    NagError fail;

    INIT_FAIL(fail);
    printf("nag_quad_md_numth_coeff_prime (d01gyc) Example Program Results\n");
    /* Skip heading in data file */
    #ifdef _WIN32
        scanf_s("%*[\n] ");
    #else
        scanf("%*[\n] ");
    #endif
    #ifdef _WIN32
        scanf_s("%"NAG_IFMT"", &ndim);
    #else
        scanf("%"NAG_IFMT"", &ndim);
    #endif
    #ifdef _WIN32
        scanf_s("%"NAG_IFMT"%*[\n] ", &npts);
    #else
        scanf("%"NAG_IFMT"%*[\n] ", &npts);
    #endif
    if (!(vk = NAG_ALLOC(ndim, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* nag_quad_md_numth_coeff_prime (d01gyc).
    * Korobov optimal coefficients for use in nag_quad_md_numth_vec (d01gdc),
    * when number of points is prime.
    */
    nag_quad_md_numth_coeff_prime(ndim, npts, vk, &fail);
    if (fail.code != NE_NOERROR)
    {
        printf("Error from nag_quad_md_numth_coeff_prime (d01gyc).\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }

    printf("ndim = %3"NAG_IFMT" npts = %6"NAG_IFMT"\n", ndim, npts);
    printf("Coefficients =\"");
    for (i = 0; i < ndim; i++)
    {
        printf("%4.0f ", vk[i]);
    }
    printf("n");
}

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10.2 Program Data

None.

10.3 Program Results

nag_quad_md_numth_coeff_prime (d01gyc) Example Program Results

ndim = 4  npts = 631

Coefficients = 1  198  82  461