NAG Library Function Document

**nag_fft_multiple_qtr_cosine (c06hdc)**

1 **Purpose**

`nag_fft_multiple_qtr_cosine (c06hdc)` computes the discrete quarter-wave Fourier cosine transforms of \( m \) sequences of real data values.

2 **Specification**

```c
#include <nag.h>
#include <nagc06.h>

void nag_fft_multiple_qtr_cosine (Nag_TransformDirection direct, Integer m, Integer n, double x[], const double trig[], NagError *fail)
```

3 **Description**

Given \( m \) sequences of \( n \) real data values \( x_j^p \), for \( j = 0, 1, \ldots, n - 1 \) and \( p = 1, 2, \ldots, m \), this function simultaneously calculates the quarter-wave Fourier cosine transforms of all the sequences defined by

\[
\hat{x}_k^p = \frac{1}{\sqrt{n}} \left( \frac{1}{2} x_0^p + \sum_{j=1}^{n-1} x_j^p \cos \left( j(2k - 1)\frac{\pi}{2n} \right) \right)
\]

if \( \text{direct} \);

or its inverse

\[
x_k^p = \frac{2}{\sqrt{n}} \sum_{j=0}^{n-1} \hat{x}_j^p \cos \left( (2j - 1)k\frac{\pi}{2n} \right)
\]

if \( \text{direct} \);

for \( k = 0, 1, \ldots, n - 1 \) and \( p = 1, 2, \ldots, m \).

(Not the scale factor \( \frac{1}{\sqrt{n}} \) in this definition.)

A call of the function with \( \text{direct} = \text{Nag_ForwardTransform} \) followed by a call with \( \text{direct} = \text{Nag_BackwardTransform} \) will restore the original data (but see Section 9).

The transform calculated by this function can be used to solve Poisson’s equation when the derivative of the solution is specified at the left boundary, and the solution is specified at the right boundary (Swarztrauber (1977)).

The function uses a variant of the fast Fourier transform (FFT) algorithm (Brigham (1974)) known as the Stockham self-sorting algorithm, described in Temperton (1983), together with pre- and post-processing stages described in Swarztrauber (1982). Special coding is provided for the factors 2, 3, 4, 5 and 6.

4 **References**


5 Arguments

1: \textbf{direct} – Nag_TransformDirection \hspace{1cm} \textit{Input}

\textit{On entry}: if the forward transform as defined in Section 3 is to be computed, then \textbf{direct} must be set equal to Nag\_ForwardTransform. If the backward transform is to be computed, that is the inverse, then \textbf{direct} must be set equal to Nag\_BackwardTransform.

\textit{Constraint}: \textbf{direct} = Nag\_ForwardTransform or Nag\_BackwardTransform.

2: \textbf{m} – Integer \hspace{1cm} \textit{Input}

\textit{On entry}: the number of sequences to be transformed, \textit{m}.

\textit{Constraint}: \textbf{m} \geq 1.

3: \textbf{n} – Integer \hspace{1cm} \textit{Input}

\textit{On entry}: the number of real values in each sequence, \textit{n}.

\textit{Constraint}: \textbf{n} \geq 1.

4: \textbf{x}[m \times n] – double \hspace{1cm} \textit{Input/Output}

\textit{On entry}: the \textit{m} data sequences stored in \textbf{x} consecutively. If the data values of the \textit{p}th sequence to be transformed are denoted by \(x^p_j\), for \(j = 0,1, \ldots, n-1\) and \(p = 1,2, \ldots, m\), then the first \(mn\) elements of the array \textbf{x} must contain the values

\[
\begin{align*}
&x_0^1, x_1^1, \ldots, x_{n-1}^1, \\
&x_0^2, x_1^2, \ldots, x_{n-1}^2, \\
&\quad \vdots \\
&x_0^m, x_1^m, \ldots, x_{n-1}^m.
\end{align*}
\]

\textit{On exit}: the \textit{m} quarter-wave cosine transforms stored consecutively overwriting the corresponding original sequence.

5: \textbf{trig}[2 \times n] – const double \hspace{1cm} \textit{Input}

\textit{On entry}: trigonometric coefficients as returned by a call of nag\_fft\_init\_trig (c06gzc). nag\_fft\_multiple\_qtr\_cosine (c06hdc) makes a simple check to ensure that \textbf{trig} has been initialized and that the initialization is compatible with the value of \textbf{n}.

6: \textbf{fail} – NagError * \hspace{1cm} \textit{Input/Output}

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

\textbf{NE_ALLOC_FAIL}

Dynamic memory allocation failed.

\textbf{NE_BAD_PARAM}

On entry, argument \textbf{direct} had an illegal value.

\textbf{NE_C06_NOT_TRIG}

Value of \textbf{n} and \textbf{trig} array are incompatible or \textbf{trig} array not initialized.

\textbf{NE_INT_ARG_LT}

On entry, \textbf{m} = \langle value\rangle.

\textit{Constraint}: \textbf{m} \geq 1.

On entry, \textbf{n} = \langle value\rangle.

\textit{Constraint}: \textbf{n} \geq 1.
7 Accuracy

Some indication of accuracy can be obtained by performing a subsequent inverse transform and comparing the results with the original sequence (in exact arithmetic they would be identical).

8 Parallelism and Performance

Not applicable.

9 Further Comments

The time taken is approximately proportional to \(nm\log(n)\), but also depends on the factors of \(n\). The function is fastest if the only prime factors of \(n\) are 2, 3 and 5, and is particularly slow if \(n\) is a large prime, or has large prime factors.

10 Example

This program reads in sequences of real data values and prints their quarter-wave cosine transforms as computed by \texttt{nag_fft_multiple_qtr_cosine (c06hdc)} with \texttt{direct = Nag_ForwardTransform}. It then calls \texttt{nag_fft_multiple_qtr_cosine (c06hdc)} again with \texttt{direct = Nag_BackwardTransform} and prints the results which may be compared with the original data.

10.1 Program Text

```c
/* nag_fft_multiple_qtr_cosine (c06hdc) Example Program. */
/* Copyright 2014 Numerical Algorithms Group. */
/* Mark 2, 1991. */
/* Mark 8 revised, 2004. */
*
#include <nag.h>
#include <stdio.h>
#include <nag_stdtlb.h>
#include <nagc06.h>

#define X(I, J) x[(I) *n + (J)]

int main(void)
{
    Integer exit_status = 0, i, j, m, n;
    NagError fail;
    double *trig = 0, *x = 0;
    INIT_FAIL(fail);

    printf("nag_fft_multiple_qtr_cosine (c06hdc) Example Program Results\n");
    
    if (m >= 1 && n >= 1)
    {
        if (!(trig = NAG_ALLOC(2*n, double)) ||
            !(x = NAG_ALLOC(m*n, double)))
        {
            fail = NAG_HANDLE_ERROR("NAG_ALLOC failed in check for zero length\n");
            goto fail;
        }

        for (i = 0; i < m; i++)
            for (j = 0; j < n; j++)
                X(i, j) = x[i*n + j];

        for (i = 0; i < m; i++)
            for (j = 0; j < n; j++)
                x[i*n + j] = trig[2*j];

        printf("nag_fft_multiple_qtr_cosine (c06hdc) Example Program Results\n");
    }

    return exit_status;
}
```

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Mark 25
printf("Allocation failure\n");
exit_status = -1;
goto END;
}
else
{
    printf("Invalid m or n.\n");
    exit_status = 1;
    return exit_status;
}
#ifdef _WIN32
scanf_s(" %*[\n]"); /* Skip text in data file */
#else
scanf(" %*[\n]"); /* Skip text in data file */
#endif
#ifdef _WIN32
scanf_s(" %lf", &X(i, j));
#else
scanf(" %lf", &X(i, j));
#endif
for (i = 0; i < m; ++i)
    for (j = 0; j < n; ++j)
        printf("%10.4f%s", X(i, j),
            (j%7 == 6 && j != n-1?"\n":""));
printf("\nOriginal data values\n\n");
for (i = 0; i < m; ++i)
    for (j = 0; j < n; ++j)
        printf("%10.4f%s", X(i, j),
            (j%7 == 6 && j != n-1?"\n":""));
    printf("\n");
#endif
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_fft_init_trig (c06gzc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
#endif
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_fft_multiple_qtr_cosine (c06hdc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

printf("\nDiscrete quarter-wave Fourier cosine transforms\n\n");
for (i = 0; i < m; ++i)
    for (j = 0; j < n; ++j)
        printf("%10.4f%s", X(i, j),
            (j%7 == 6 && j != n-1?"\n":""));
printf("\n");
/* Compute inverse transform */
/* nag_fft_multiple_qtr_cosine (c06hdc), see above. */
nag_fft_multiple_qtr_cosine(Nag_BackwardTransform, m, n, x, trig, &fail);
if (fail.code != NE_NOERROR)
{
    printf(  
        "Error from nag_fft_multiple_qtr_cosine (c06hdc).
        fail.message);
    exit_status = 1;
    goto END;
}

printf("%nOriginal data as restored by inverse transform\n\n");
for (i = 0; i < m; ++i)
{
    for (j = 0; j < n; ++j)
    {
        printf(" %10.4f%s", X(i, j),
            (j%7 == 6 && j != n-1 ?"n":"
            ));
        printf("\n");
    }
}

END:
NAG_FREE(trig);
NAG_FREE(x);
return exit_status;

10.2 Program Data

nag_fft_multiple_qtr_cosine (c06hdc) Example Program Data

3 6 : Number of sequences, m, and number of values in each sequence, n
Real data sequences
0.3854 0.6772 0.1138 0.6751 0.6362 0.1424
0.5417 0.2983 0.1181 0.7255 0.8638 0.8723
0.9172 0.0644 0.6037 0.6430 0.0428 0.4815

10.3 Program Results

nag_fft_multiple_qtr_cosine (c06hdc) Example Program Results

Original data values

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Discrete quarter-wave Fourier cosine transforms

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Original data as restored by inverse transform

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