NAG Library Function Document

nag_dpttrf (f07jdc)

1 Purpose

nag_dpttrf (f07jdc) computes the modified Cholesky factorization of a real n by n symmetric positive definite tridiagonal matrix A.

2 Specification

```
#include <nag.h>
#include <nagf07.h>
```

```
void nag_dpttrf (Integer n, double d[], double e[], NagError *fail)
```

3 Description

nag_dpttrf (f07jdc) factorizes the matrix A as

$$A = LDL^{\mathrm{T}},$$

where L is a unit lower bidiagonal matrix and D is a diagonal matrix with positive diagonal elements. The factorization may also be regarded as having the form $U^{\mathrm{T}}DU$, where U is a unit upper bidiagonal matrix.

4 References

None.

5 Arguments

1: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **d**[*dim*] – double *Input/Output*

Note: the dimension, *dim*, of the array **d** must be at least $\max(1, \mathbf{n})$.

On entry: must contain the n diagonal elements of the matrix A.

On exit: is overwritten by the n diagonal elements of the diagonal matrix D from the LDL^{T} factorization of A.

3: **e**[*dim*] – double *Input/Output*

Note: the dimension, *dim*, of the array **e** must be at least $\max(1, \mathbf{n} - 1)$.

On entry: must contain the $(n - 1)$ subdiagonal elements of the matrix A.

On exit: is overwritten by the $(n - 1)$ subdiagonal elements of the lower bidiagonal matrix L. (**e** can also be regarded as containing the $(n - 1)$ superdiagonal elements of the upper bidiagonal matrix U.)

4: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_INT

On entry, $\mathbf{n} = \langle value \rangle$.
Constraint: $\mathbf{n} \geq 0$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_MAT_NOT_POS_DEF

The leading minor of order n is not positive definite, the factorization was completed, but $\mathbf{d}[\mathbf{n} - 1] \leq 0$.

The leading minor of order $\langle value \rangle$ is not positive definite, the factorization could not be completed.

7 Accuracy

The computed factorization satisfies an equation of the form

$$A + E = LDL^{\mathrm{T}},$$

where

$$\|E\|_\infty = O(\epsilon)\|A\|_\infty$$

and ϵ is the ***machine precision***.

Following the use of this function, nag_dpttrs (f07jec) can be used to solve systems of equations $AX = B$, and nag_dptcon (f07jgc) can be used to estimate the condition number of A.

8 Parallelism and Performance

Not applicable.

9 Further Comments

The total number of floating-point operations required to factorize the matrix A is proportional to n.

The complex analogue of this function is nag_zpttrf (f07jrc).

10 Example

This example factorizes the symmetric positive definite tridiagonal matrix A given by

$$A = \begin{pmatrix} 4.0 & -2.0 & 0 & 0 & 0 \\ -2.0 & 10.0 & -6.0 & 0 & 0 \\ 0 & -6.0 & 29.0 & 15.0 & 0 \\ 0 & 0 & 15.0 & 25.0 & 8.0 \\ 0 & 0 & 0 & 8.0 & 5.0 \end{pmatrix}.$$

10.1 Program Text

```
/* nag_dpttrf (f07jdc) Example Program.
 *
 * Copyright 2004 Numerical Algorithms Group.
 *
 * Mark 23, 2011.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>

int main(void)
{

  /* Scalars */
  Integer  exit_status = 0, i, n;

  /* Arrays */
  double   *d = 0, *e = 0;

  /* Nag Types */
  NagError fail;

  INIT_FAIL(fail);

  printf("nag_dpttrf (f07jdc) Example Program Results\n\n");

  /* Skip heading in data file */
  scanf("%*[^\n]");
  scanf("%ld%*[^\n]", &n);
  if (n < 0)
    {
      printf("Invalid n\n");
      exit_status = 1;
      goto END;
    }
  /* Allocate memory */
  if (!(d = NAG_ALLOC(n, double)) ||
      !(e = NAG_ALLOC(n-1, double)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read the lower bidiagonal part of the tridiagonal matrix A from
   * data file
   */
  for (i = 0; i < n; ++i) scanf("%lf", &d[i]);
  scanf("%*[^\n]");
  for (i = 0; i < n - 1; ++i) scanf("%lf", &e[i]);
  scanf("%*[^\n]");

  /* Factorize the tridiagonal matrix A using nag_dpttrf (f07jdc). */
  nag_dpttrf(n, d, e, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_dpttrf (f07jdc).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  /* Print details of the factorization */
  printf("Details of factorization\n\n");
  printf(" The diagonal elements of D\n");
  for (i = 0; i < n; ++i) printf("%9.4f%s", d[i], i%8 == 7?"\n":" ");

  printf("\n\n Sub-diagonal elements of the Cholesky factor L\n");
```

```
  for (i = 0; i < n-1; ++i) printf("%9.4f%s", e[i], i%8 == 7?"\n":" ");
  printf("\n");

 END:
  NAG_FREE(d);
  NAG_FREE(e);

  return exit_status;
}
```

10.2 Program Data

```
nag_dpttrf (f07jdc) Example Program Data
  5                             : n
  4.0  10.0  29.0  25.0   5.0 : diagonal D
 -2.0  -6.0  15.0   8.0       : sub-diagonal E
```

10.3 Program Results

```
nag_dpttrf (f07jdc) Example Program Results

Details of factorization

 The diagonal elements of D
   4.0000    9.0000   25.0000   16.0000    1.0000

 Sub-diagonal elements of the Cholesky factor L
  -0.5000   -0.6667    0.6000    0.5000
```