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Chapters of the Library

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Random Number Generators
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NAG Numerical Routines for GPUs


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Linear Equations (LAPACK)

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1 Scope of the Chapter

This chapter provides functions for the solution of systems of simultaneous linear equations, and associated computations. It provides functions for matrix factorizations. Functions are provided for real data only. The functions in this chapter handle only dense matrices (not matrices with more specialized structures, or general sparse matrices).

2 Background to the Problems

This section is only a brief introduction to the numerical solution of systems of linear equations. Consult a standard textbook, for example Golub and Van Loan (1996) for a more thorough discussion.

2.1 Notation

We use the standard notation for a system of simultaneous linear equations:

\[ Ax = b \]  

where \( A \) is the coefficient matrix, \( b \) is the right-hand side, and \( x \) is the solution. \( A \) is assumed to be a square matrix of order \( n \).

If there are several right-hand sides, we write

\[ AX = B \]  

where the columns of \( B \) are the individual right-hand sides, and the columns of \( X \) are the corresponding solutions.

2.2 Matrix Factorizations

If \( A \) is upper or lower triangular, \( Ax = b \) can be solved by a straightforward process of backward or forward substitution.

Otherwise, the solution is obtained after first factorizing \( A \), as follows.

**General matrices (LU factorization with partial pivoting)**

\[ A = PLU \]

where \( P \) is a permutation matrix, \( L \) is lower-triangular with diagonal elements equal to 1, and \( U \) is upper-triangular; the permutation matrix \( P \) (which represents row interchanges) is needed to ensure numerical stability.

**Symmetric positive definite matrices (Cholesky factorization)**

\[ A = U^T U \quad \text{or} \quad A = LL^T \]

where \( U \) is upper triangular and \( L \) is lower triangular.

2.3 Solution of Systems of Equations

Given one of the above matrix factorizations, it is straightforward to compute a solution to \( Ax = b \) by solving two subproblems, as shown below, first for \( y \) and then for \( x \). Each subproblem consists essentially of solving a triangular system of equations by forward or backward substitution; the permutation matrix \( P \) introduces only a little extra complication:

**General matrices (LU factorization)**

\[ Ly = P^T b \]
\[ Ux = y \]

**Symmetric positive definite matrices (Cholesky factorization)**

\[ U^T y = b \quad \text{or} \quad Ly = b \]
\[ Ux = y \quad \text{or} \quad L^T x = y \]
3 Recommendations on Choice and Use of Available Functions

3.1 Available Functions

Functions are provided to perform Cholesky decomposition and LU factorization on dense, real-valued, double precision matrices:

- `naggpuDgetrfA` computes the LU factorization of a general matrix.
- `naggpuDpotrfA` computes the Cholesky decomposition of a symmetric positive definite matrix.

The initialization function `naggpuLinAlgInitA` must be called before the first call to any of the linear algebra functions. Once all calls to all linear algebra functions have completed, `naggpuLinAlgCleanupA` must be called to free allocated system resources.

3.2 Matrix Storage Schemes

In this chapter matrices must be stored in the conventional way: packed storage schemes are not supported. In the examples below, * indicates an array element which need not be set and is not referenced by the functions. The examples illustrate only the relevant part of the arrays; array arguments may of course have additional rows or columns.

3.2.1 Conventional storage

Matrices must be stored in column major order: a matrix $A$ is stored in a one-dimensional array $a$, with matrix element $a_{i,j}$ stored in array element $a((j-1) \times \text{pda} + i - 1)$ where pda is the principal dimension of the array (i.e., the stride separating row elements of the matrix).

Functions which handle symmetric matrices allow for either the upper or lower triangle of the matrix (as specified by `uplow`) to be stored in the corresponding elements of the array; the remaining elements of the array need not be set.

For example, when $n = 3$:

<table>
<thead>
<tr>
<th><code>uplow</code></th>
<th>Symmetric matrix $A$</th>
<th>Storage in array $a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAGGPUMATRIXUPLOW_UPPER</td>
<td>$\begin{pmatrix} a_{11} &amp; a_{12} &amp; a_{13} \ a_{12} &amp; a_{22} &amp; a_{23} \ a_{13} &amp; a_{23} &amp; a_{33} \end{pmatrix}$</td>
<td>$a_{11} \ast a_{13} a_{22} \ast a_{33}$</td>
</tr>
<tr>
<td>NAGGPUMATRIXUPLOW_LOWER</td>
<td>$\begin{pmatrix} a_{11} &amp; a_{21} &amp; a_{31} \ a_{21} &amp; a_{22} &amp; a_{32} \ a_{31} &amp; a_{32} &amp; a_{33} \end{pmatrix}$</td>
<td>$a_{11} a_{21} a_{31} \ast a_{22} a_{32} \ast a_{33}$</td>
</tr>
</tbody>
</table>

3.3 Argument Conventions

3.3.1 Problem dimensions

It is permissible for the problem dimensions (for example, $m$ in `naggpuDgetrfA`, $n$ in `naggpuDpotrfA`) to be passed as zero, in which case the computation is skipped. Negative dimensions are regarded as an error.

4 Functionality Index

4.1 Host-Callable Linear Equation (LAPACK) Functions

- Cholesky factorization of a real symmetric positive definite matrix $\ldots$ `naggpuDpotrfA`
- free system resources $\ldots$ `naggpuLinAlgCleanupA`
- initialise the linear equation functions $\ldots$ `naggpuLinAlgInitA`
- LU factorization of a real $m$ by $n$ matrix $\ldots$ `naggpuDgetrfA`
4.2 Serial CPU Functions

Linear Equations
  Cholesky factorization of a real symmetric positive definite matrix .......... nagCPUDpotrfA
  LU factorization of a real $m \times n$ matrix .................................................. nagCPUDgetrfA

5 References

1 Purpose

naggpuLinAlgInitA initializes the GPU linear algebra (LAPACK) suite of functions. This function must be called before any call to the linear algebra functions (such as naggpuDgetrfA) and must ultimately be followed by a call to the cleanup function naggpuLinAlgCleanupA to release system resources.

2 Specification

#include <nag_gpu.h>

extern "C"

cudaError_t naggpuLinAlgInitA(NagGpuLinAlgComm *comm, NagGpuError *error)

3 Description

3.1 Synchronization

This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.

3.2 Return Value

Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References

None.

5 Arguments

1: comm – NagGpuLinAlgComm *

NagGpuLinAlgComm is a structure which holds state and communication information and must not be modified in any way. The structure will be initialized and must be passed to the GPU linear algebra functions (such as naggpuDgetrfA). Once all calls to linear algebra functions have completed, comm must be passed to naggpuLinAlgCleanupA to free allocated system resources.

2: error – NagGpuError *

This parameter contains error information and should not be modified directly. Errors are indicated through the value of error → code which should be inspected after each call to this function. If error → code = 0 then no error occurred. If error → code ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.
6 Error Indicators and Warnings

**error → code = 1**

*On entry:* the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

**error → code = 2**

*During execution:* a CUDA runtime error was detected.

**error → code = 100**

*On entry:* the value of *comm* is NULL.

7 Example

There is no example program specifically for this function. For an example of how this function should be used, please see the example program for naggpuDgetrfA.
NAG Numerical Routines for GPUs Function Document

naggpuDgetrfA

1 Purpose

naggpuDgetrfA computes the \( LU \) factorization of a real \( m \) by \( n \) matrix.

The initialization function naggpuLinAlgInitA must be called prior to the first call to naggpuDgetrfA. Once all calls to all linear algebra functions have been completed, the function naggpuLinAlgCleanupA must be called to free allocated system resources.

Note: the matrix must be stored in column major order.

2 Specification

#include <nag_gpu.h>

extern "C"

cudaError_t naggpuDgetrfA(int m, int n, double *d_a, int pda, int *ipiv,
    NagGpuLinAlgComm *comm, NagGpuError *error)

3 Description

naggpuDgetrfA forms the \( LU \) factorization of a real \( m \) by \( n \) matrix \( A \) as \( A = PLU \), where \( P \) is a permutation matrix, \( L \) is lower triangular with unit diagonal elements (lower trapezoidal if \( m > n \)) and \( U \) is upper triangular (upper trapezoidal if \( m < n \)). Usually \( A \) is square (\( m = n \)), and both \( L \) and \( U \) are triangular.

Note: naggpuDgetrfA is currently not thread safe when used with multiple host threads and GPU devices.

3.1 Synchronization

This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.

3.2 Return Value

Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References

None.

5 Arguments

1: \( m \) – int  
   \textit{Input}
   
   On entry: \( m \), the number of rows of the matrix \( A \).
   
   Constraint: \( m \geq 0 \).

2: \( n \) – int  
   \textit{Input}
   
   On entry: \( n \), the number of columns of the matrix \( A \).
   
   Constraint: \( n \geq 0 \).
This buffer must reside in the GPU memory space.

**Note:** the dimension $d$ of the array $d_a$ must satisfy $d \geq \max(1, n \times pda)$.

**On entry:** the $m$ by $n$ matrix $A$. The $(i,j)$th element $a_{i,j}$ of the matrix $A$ is stored in $d_a[(j-1) \times pda + i - 1]$.

**On exit:** the factors $L$ and $U$ from the factorization $A = PLU$; the unit diagonal elements of $L$ are not stored. This means that

- $u_{i,j}$ is stored in $d_a[(j-1) \times pda + i - 1]$ for $j = 1, 2, \ldots, n$ and $i = 1, 2, \ldots, \min(j, m)$
- $l_{i,j}$ is stored in $d_a[(j-1) \times pda + i - 1]$ for $i = 2, 3, \ldots, m$ and $j = 1, 2, \ldots, \min(i-1, n)$

where $u_{i,j}$ denotes the $(i,j)$th element of the matrix $U$ and $l_{i,j}$ denotes the $(i,j)$th element of the matrix $L$.

**Input**

4: $pda$ – int

**Constraint:** $pda \geq \max(1, m)$.

**Output**

5: $ipiv[\min(m, n)]$ – int

**Output**

**On exit:** the pivot indices that define the permutation matrix. At the $i$th step, if $ipiv[i-1] > i$ then row $i$ of the matrix $A$ was interchanged with row $ipiv[i-1]$ for $i = 1, 2, \ldots, \min(m, n)$. $ipiv[i-1] \leq i$ indicates that at the $i$th step, a row interchange was not required.

**Communication Data**

6: $comm$ – NagGpuLinAlgComm

NagGpuLinAlgComm is a structure which holds state and communication information and must not be modified in any way. Once all calls to linear algebra functions have completed, $comm$ must be passed to naggpuLinAlgCleanupA to free allocated system resources.

**Error Reporting**

7: $error$ – NagGpuError

**Note:** on exit, $error \rightarrow subCodes[0]$ will contain the LAPACK info parameter which is traditionally used to indicate errors or warnings.

This parameter contains error information and should not be modified directly. Errors are indicated through the value of $error \rightarrow code$ which should be inspected after each call to this function. If $error \rightarrow code = 0$ then no error occurred. If $error \rightarrow code \neq 0$ then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

### 6 Error Indicators and Warnings

**error \rightarrow code = 1**

**On entry:** the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

**error \rightarrow code = 2**

**During execution:** a CUDA runtime error was detected.

**error \rightarrow code = 3**

**On entry:** an attempt was made to launch a double precision function on a GPU device that does not support double precision.

**error \rightarrow code = 100**

**On entry:** the value of $comm$ is NULL.
error \rightarrow code = 101

On entry: comm has not been initialized, or the internal state of comm is corrupted.

error \rightarrow code = 110

On entry: m < 0

error \rightarrow code = 111

On entry: n < 0

error \rightarrow code = 112

On entry: d_a is NULL.

error \rightarrow code = 113

On entry: pda does not satisfy the constraint listed above.

error \rightarrow code = 114

On entry: ipiv is NULL.

error \rightarrow code = 115

On exit: U(i, i) = 0 where i = error \rightarrow subCodes[0]. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Example

This example computes the LU factorization of the matrix \( A \), where

\[
A = \begin{pmatrix}
1.80 & 2.88 & 2.05 & -0.89 \\
5.25 & -2.95 & -0.95 & -3.80 \\
1.58 & -2.69 & -2.90 & -1.04 \\
-1.11 & -0.66 & -0.59 & 0.80
\end{pmatrix}
\]

7.1 Program Text

/**
 * Example Program: naggpuDgetrfA
 *
 * Copyright 2009, Numerical Algorithms Group Ltd, Oxford, UK.
 *
 * Version 0.4, 2011.
 *
 */

#include <stdio.h>
#include
using namespace std;
#include <nag_gpu.h>

void checkNagError(NagGpuError *error);
void checkCudaError(cudaError_t cuError);

int main(int argc, char **argv)
{
    // host (CPU) storage for the matrix
double *h_A = 0;
    // device (GPU) storage for the matrix
double *d_A = 0;
    // host (CPU) storage for ipiv array
    int *ipiv = 0;
}
const int m = 4;
const int n = 4;
const int lda = n;

// NAG GPU structures
NagGpuLinAlgComm comm;
NagGpuError error;
cudaError_t cuError;

// Print the title
cout << "NAG GPU Example Program: ";
cout << "naggpuDgetrfA";
cout << endl << endl;

// Allocate CPU and GPU memory
h_A = new double[m*n];
ipiv = new int[m];
cuError = cudaMalloc((void **)&d_A, sizeof(double)*m*n);
checkCudaError(cuError);

// Populate the matrix:
// NOTE: Column major order

// Copy data to GPU
cuError = cudaMemcpy(d_A, h_A, sizeof(double)*m*n,
cudaMemcpyHostToDevice);
checkCudaError(cuError);

// Initialise LAPACK
cout << "Initialising Linear Algebra routines ..." << endl << endl;
naggpuLinAlgInitA(&comm, &error);
checkNagError(&error);

// Do factorisation
naggpuDgetrfA(m, n, d_A, lda, ipiv, &comm, &error);
checkNagError(&error);

// Copy back values from the GPU for printing
cuError = cudaMemcpy(h_A, d_A, sizeof(double)*m*n,
cudaMemcpyDeviceToHost);
checkCudaError(cuError);

// Print output
cout << "The LU factorisation: h_A = " << endl;
cout.setf(ios::fixed,ios::floatfield);
cout.precision(4);
for(int row = 0; row < m; row++)
{
    for(int col = 0; col < n; col++)
    {
        printf(" % .4f", h_A[row + col*lda]);
    }
    cout << endl;
}
cout << endl;
cout << "Pivoting information: ipiv = " << endl;
for(int row=0; row < m; row++)
{
    cout << "\t" << ipiv[row];
}
cout << endl << endl;
// Call cleanup for the NAG routine
naggpuLinAlgCleanupA(&comm, &error);
checkNagError(&error);

    // Free CPU and GPU memory
    delete[] h_A;
    delete[] ipiv;
    if (d_A)
    {
        cuError = cudaFree(d_A);
        checkCudaError(cuError);
    }
    return 0;
}

void checkNagError(NagGpuError *error)
{
    if (error->code != 0)
    {
        char *buff;
        buff = new char[error->msgLength];
        naggpuErrorCopyMsg(buff, error);
        cout << buff << endl;
        delete[] buff;
        exit(1);
    }
}

void checkCudaError(cudaError_t cuError)
{
    if (cuError != cudaSuccess)
    {
        cout << cudaGetErrorString(cuError) << endl;
        exit(1);
    }
}

7.2 Program Data
None.

7.3 Program Results
NAG GPU Example Program: naggpuDgetrfA

Initialising Linear Algebra routines ...

The LU factorisation: h_A =
  5.2500 -2.9500 -0.9500 -3.8000
  0.3429  3.8914  2.3757  0.4129
  0.3010 -0.4631 -1.5139  0.2948
-0.2114 -0.3299  0.0047  0.1314

Pivoting information: ipiv =
  2 2 3 4
1 Purpose
naggpuDpotrfA computes the Cholesky factorization of a real symmetric positive definite matrix.

The initialization function naggpuLinAlgInitA must be called prior to the first call to naggpuDpotrfA. Once all calls to all linear algebra functions have been completed, the function naggpuLinAlgCleanupA must be called to free allocated system resources.

Note: the matrix must be stored in column major order.

2 Specification
#include <nag_gpu.h>
extern "C"
cudaError_t naggpuDpotrfA(NagGpuMatrixUpLow uplow, int n, double *d_a, int pda,
NagGpuLinAlgComm *comm, NagGpuError *error)

3 Description
naggpuDpotrfA forms the Cholesky factorization of a real symmetric positive definite matrix \( A \) either as \( A = U^T U \) if \( \text{uplow} = \text{NAGGPUMATRIXUPLOW\_UPPER} \) or \( A = LL^T \) if \( \text{uplow} = \text{NAGGPUMATRIXUPLOW\_LOWER} \), where \( U \) is an upper triangular matrix and \( L \) is lower triangular.

Note: naggpuDpotrfA is currently not thread safe when used with multiple host threads and GPU devices.

3.1 Synchronization
This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.

3.2 Return Value
Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References
None.

5 Arguments
1: 
   \( \text{uplow} \) – NagGpuMatrixUpLow

   Input

   \( \text{On entry:} \) specifies whether the upper or lower triangular part of \( A \) is stored and how \( A \) is to be factorized.

   \( \text{uplow} = \text{NAGGPUMATRIXUPLOW\_UPPER} \)

   The upper triangular part of \( A \) is stored and \( A \) is factorized as \( U^T U \), where \( U \) is upper triangular.
uplow = NAGGPUMATRIXUPLOW_LOWER
The lower triangular part of A is stored and A is factorized as \( LL^T \), where L is lower triangular.

Constraint: uplow = NAGGPUMATRIXUPLOW_UPPER or uplow = NAGGPUMATRIXUPLOW_LOWER.

2: n – int

On entry: n, the order of the matrix A.
Constraint: \( n \geq 0 \).

3: d_a[d] – double *

On entry: the \( n \) by \( n \) symmetric positive definite matrix A. Let \( a_{i,j} \) denote the \((i,j)\)th element of the matrix A:

if uplow = NAGGPUMATRIXUPLOW_UPPER, the upper triangular part \( a_{i,j} \) for \( j = 1, 2, \ldots, n \) and \( i = 1, 2, \ldots, j \) must be stored in \( d_a[(j-1) \times pda + i - 1] \) and the elements of the array below the diagonal are not referenced.

if uplow = NAGGPUMATRIXUPLOW_LOWER, the lower triangular part \( a_{i,j} \) for \( i = 1, 2, \ldots, n \) and \( j = 1, 2, \ldots, i \) must be stored in \( d_a[(j-1) \times pda + i - 1] \) and the elements of the array above the diagonal are not referenced.

On exit: the factor U or L from the Cholesky factorization \( A = U^T U \) or \( A = LL^T \). Let \( u_{i,j} \) denote the \((i,j)\)th element of the matrix \( U \):

if uplow = NAGGPUMATRIXUPLOW_UPPER, the element \( u_{i,j} \) for \( j = 1, 2, \ldots, n \) and \( i = 1, 2, \ldots, j \) is stored in \( d_a[(j-1) \times pda + i - 1] \)

if uplow = NAGGPUMATRIXUPLOW_LOWER, the element \( l_{i,j} \) for \( i = 1, 2, \ldots, n \) and \( j = 1, 2, \ldots, i \) is stored in \( d_a[(j-1) \times pda + i - 1] \)

4: pda – int

On entry: the stride separating row elements of the matrix A in the array d_a.
Constraint: \( pda \geq \max(1, n) \).

5: comm – NagGpuLinAlgComm *

Communication Data

NagGpuLinAlgComm is a structure which holds state and communication information and must not be modified in any way. Once all calls to linear algebra functions have completed, comm must be passed to naggpuLinAlgCleanupA to free allocated system resources.

6: error – NagGpuError *

Error Reporting

Note: on exit, error \( \rightarrow \) subCodes[0] will contain the LAPACK info parameter which is traditionally used to indicate errors or warnings.

This parameter contains error information and should not be modified directly. Errors are indicated through the value of error \( \rightarrow \) code which should be inspected after each call to this function. If error \( \rightarrow \) code = 0 then no error occurred. If error \( \rightarrow \) code \( \neq 0 \) then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.
6 Error Indicators and Warnings

error → code = 1

On entry: the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

error → code = 2

During execution: a CUDA runtime error was detected.

error → code = 3

On entry: an attempt was made to launch a double precision function on a GPU device that does not support double precision.

error → code = 100

On entry: the value of comm is NULL.

error → code = 101

On entry: comm has not been initialized, or the internal state of comm is corrupted.

error → code = 110

On entry: uplow does not satisfy the constraint listed above.

error → code = 111

On entry: n < 0

error → code = 112

On entry: d_a is NULL.

error → code = 113

On entry: pda does not satisfy the constraint listed above.

error → code = 114

On exit: the leading minor of order i where i = error → subCodes[0] is not positive definite, and the factorization could not be completed. Hence the matrix A itself is not positive definite. This may indicate an error in forming the matrix A.

7 Example

This example computes the Cholesky factorization of the matrix A, where

\[
A = \begin{pmatrix}
4.16 & -3.12 & 0.56 & -0.10 \\
-3.12 & 5.03 & -0.83 & 1.18 \\
0.56 & -0.83 & 0.76 & 0.34 \\
-0.10 & 1.18 & 0.34 & 1.18 \\
\end{pmatrix}
\]

7.1 Program Text

/*
 * Example Program: naggpuDpotrfA
 * Copyright 2009, Numerical Algorithms Group Ltd, Oxford, UK.
 * Version 0.4, 2011.
 */
#include <stdio.h>
#include
using namespace std;
#include <nag_gpu.h>

void checkNagError(NagGpuError *error);
void checkCudaError(cudaError_t cuError);

int main(int argc, char **argv) {
    // host (CPU) storage for the matrix
double *h_A = 0;
    // device (GPU) storage for the matrix
double *d_A = 0;
    // host (CPU) storage for ipiv array
int *ipiv = 0;

    const int n = 4;
    const int lda = n;

    // NAG GPU structures
NagGpuLinAlgComm comm;
NagGpuError error;

    cudaError_t cuError;

    // Print the title
    cout << "NAG GPU Example Program: ";
cout << "naggpuDpotrfA";
cout << endl << endl;

    // Allocate CPU and GPU memory
h_A = new double[n*n];
cuError = cudaMalloc((void **)&d_A, sizeof(double)*n*n);
checkCudaError(cuError);

    // Populate the matrix:
    // NOTE: Column major order
h_A[2]=0.56; h_A[6]=-0.83; h_A[10]=0.76; h_A[14]=0.34;

    // Copy data to GPU
    cuError = cudaMemcpy(d_A, h_A, sizeof(double)*n*n,
cudaMemcpyHostToDevice);
checkCudaError(cuError);

    // Initialising LAPACK
    cout << "Initialising Linear Algebra routines ..." << endl << endl;
naggpuLinAlgInitA(&comm, &error);
checkNagError(&error);

    // Do factorisation
naggpuDpotrfA(NAGGPUMATRIXUPLOW_LOWER, n, d_A, lda, &comm, &error);
checkNagError(&error);

    // Copy back values from the GPU for printing
    cuError = cudaMemcpy(h_A, d_A, sizeof(double)*n*n,
cudaMemcpyDeviceToHost);
checkCudaError(cuError);

    // Print output
    cout << "The Cholesky factorisation: h_A = " << endl;
cout.setf(ios::fixed,ios::floatfield);
cout.precision(4);
for(int row = 0; row < n; row++)
    {
        for(int col = 0; col < n; col++)
            {
            }
if (col <= row) printf(" % .4f", h_A[row + col*lda]);
else
    printf(" ");
}  
cout << endl;
}
cout << endl << endl;

// Call cleanup for the NAG routine
naggpuLinAlgCleanupA(&comm, &error);
checkNagError(&error);

// Free CPU and GPU memory
delete[] h_A;
if (d_A)
{
    cuError = cudaFree(d_A);
    checkCudaError(cuError);
}
return 0;

void checkNagError(NagGpuError *error)
{
    if (error->code != 0)
    {
        char *buff;
        buff = new char[error->msgLength];
        naggpuErrorCopyMsg(buff, error);
        cout << buff << endl;
        delete[] buff;
        exit(1);
    }
}

void checkCudaError(cudaError_t cuError)
{
    if (cuError != cudaSuccess)
    {
        cout << cudaGetErrorString(cuError) << endl;
        exit(1);
    }
}

7.2 Program Data
None.

7.3 Program Results
NAG GPU Example Program: naggpuDpotrfA

Initialising Linear Algebra routines ...

The Cholesky factorisation: h_A =

\[
\begin{bmatrix}
2.0396 & -1.5297 & 0.2746 & -0.0490 \\
-1.5297 & 1.6401 & -0.2500 & 0.6737 \\
0.2746 & -0.2500 & 0.7887 & 0.6617 \\
-0.0490 & 0.6737 & 0.7887 & 0.5347 \\
\end{bmatrix}
\]
1 Purpose
naggpuLinAlgCleanupA frees system resources that were allocated by a previous call to naggpuLinAlgInitA.

2 Specification
#include <nag_gpu.h>
extern "C"
cudaError_t naggpuLinAlgCleanupA(NagGpuLinAlgComm *comm, NagGpuError *error)

3 Description
3.1 Synchronization
This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.

3.2 Return Value
Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References
None.

5 Arguments
1: comm – NagGpuLinAlgComm *  
   Communication Data  
   *On entry*: the pointer that was passed to a previous call to naggpuLinAlgInitA.

2: error – NagGpuError *  
   Error Reporting  
   This parameter contains error information and should not be modified directly. Errors are indicated through the value of error → code which should be inspected after each call to this function. If error → code = 0 then no error occurred. If error → code ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings
error → code = 1
   *On entry*: the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

error → code = 2
   *During execution*: a CUDA runtime error was detected.
\begin{itemize}
\item \textbf{error → code} = 100
\hspace{1cm} \textit{On entry:} the value of \texttt{comm} is NULL.
\item \textbf{error → code} = 101
\hspace{1cm} \textit{On entry:} \texttt{comm} has not been initialized, or the internal state of \texttt{comm} is corrupted.
\end{itemize}

7 Example

There is no example program specifically for this function. For an example of how this function should be used, please see the example program for naggpuDgetrfA.
1 Purpose

nagCPUDgetrfA computes the LU factorization of a real \( m \) by \( n \) matrix.

Note: the matrix must be stored in column major order.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPUDgetrfA(int m, int n, double *a, int pda, int *ipiv, NagGpuError *error)
```

3 Description

nagCPUDgetrfA forms the LU factorization of a real \( m \) by \( n \) matrix \( A \) as \( A = PLU \), where \( P \) is a permutation matrix, \( L \) is lower triangular with unit diagonal elements (lower trapezoidal if \( m > n \)) and \( U \) is upper triangular (upper trapezoidal if \( m < n \)). Usually \( A \) is square \( (m = n) \), and both \( L \) and \( U \) are triangular.

3.1 Synchronization

As this function only uses the CPU there is no requirement for synchronization.

4 References

None.

5 Arguments

1: \( m \) – int

   **Input**

   On entry: \( m \), the number of rows of the matrix \( A \).

   Constraint: \( m \geq 0 \).

2: \( n \) – int

   **Input**

   On entry: \( n \), the number of columns of the matrix \( A \).

   Constraint: \( n \geq 0 \).

3: \( a[d] \) – double *

   **Input/Output**

   Note: the dimension \( d \) of the array \( a \) must satisfy \( d \geq \max(1, n \times pda) \).

   On entry: the \( m \) by \( n \) matrix \( A \). The \((i,j)\)th element \( a_{i,j} \) of the matrix \( A \) is stored in \( a[(j-1) \times pda + i - 1] \).

   On exit: the factors \( L \) and \( U \) from the factorization \( A = PLU \); the unit diagonal elements of \( L \) are not stored. This means that

   - \( u_{i,j} \) is stored in \( a[(j-1) \times pda + i - 1] \) for \( j = 1, 2, \ldots, n \) and \( i = 1, 2, \ldots, \min(j, m) \)
   - \( l_{i,j} \) is stored in \( a[(j-1) \times pda + i - 1] \) for \( i = 2, 3, \ldots, m \) and \( j = 1, 2, \ldots, \min(i - 1, n) \)

Mark 0.6

nagCPUDgetrfA.1
where \( u_{i,j} \) denotes the \((i,j)\)th element of the matrix \( U \) and \( l_{i,j} \) denotes the \((i,j)\)th element of the matrix \( L \).

4: \textbf{pda} – int  
\textit{Input}

\textit{On entry:} the stride separating row elements of the matrix \( A \) in the array \( a \).
\textit{Constraint:} \( pda \geq \max(1, m) \).

5: \textbf{ipiv[\min(m,n)]} – int *  
\textit{Output}

\textit{On exit:} the pivot indices that define the permutation matrix. At the \( i \)th step, if \( \text{ipiv}[i - 1] > i \) then row \( i \) of the matrix \( A \) was interchanged with row \( \text{ipiv}[i - 1] \), for \( i = 1, 2, \ldots, \min(m,n) \). \( \text{ipiv}[i - 1] \leq i \) indicates that at the \( i \)th step, a row interchange was not required.

6: \textbf{error} – NagGpuError *  
\textit{Error Reporting}

\textit{Note:} on exit, \( \text{error} \rightarrow \subCodes[0] \) will contain the LAPACK \texttt{info} parameter which is traditionally used to indicate errors or warnings.

This parameter contains error information and should not be modified directly. Errors are indicated through the value of \( \text{error} \rightarrow \text{code} \) which should be inspected after each call to this function. If \( \text{error} \rightarrow \text{code} = 0 \) then no error occurred. If \( \text{error} \rightarrow \text{code} \neq 0 \) then an error was detected and a call to \text{naggpuErrorCopyMsg} will retrieve a null terminated ANSI C string describing the error.

Please see the Error Handling Chapter Introduction for further details on error handling.

6 \textbf{Error Indicators and Warnings}

\textit{Error \rightarrow \text{code} = 110}
\textit{On entry:} \( m < 0 \)

\textit{error \rightarrow \text{code} = 111}
\textit{On entry:} \( n < 0 \)

\textit{error \rightarrow \text{code} = 112}
\textit{On entry:} \( a \) is NULL.

\textit{error \rightarrow \text{code} = 113}
\textit{On entry:} \( pda \) does not satisfy the constraint listed above.

\textit{error \rightarrow \text{code} = 114}
\textit{On entry:} \( \text{ipiv} \) is NULL.

\textit{error \rightarrow \text{code} = 115}
\textit{On exit:} \( U(i, i) = 0 \) where \( i = \text{error} \rightarrow \subCodes[0] \). The factorization has been completed, but the factor \( U \) is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 \textbf{Example}

None.
1 Purpose

nagCPUDpotrfA computes the Cholesky factorization of a real symmetric positive definite matrix.

Note: the matrix must be stored in column major order.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPUDpotrfA(NagGpuMatrixUpLow uplow, int n, double *a, int pda,
                     NagGpuError *error)
```

3 Description

nagCPUDpotrfA forms the Cholesky factorization of a real symmetric positive definite matrix $A$ either as $A = U^T U$ if $\text{uplow} = \text{NAGGPUMATRIXUPLOW_UPPER}$ or $A = LL^T$ if $\text{uplow} = \text{NAGGPUMATRIXUPLOW_LOWER}$, where $U$ is an upper triangular matrix and $L$ is lower triangular.

3.1 Synchronization

As this function only uses the CPU there is no requirement for synchronization.

4 References

None.

5 Arguments

1: $\text{uplow}$ – NagGpuMatrixUpLow

On entry: specifies whether the upper or lower triangular part of $A$ is stored and how $A$ is to be factorized.

$\text{uplow} = \text{NAGGPUMATRIXUPLOW_UPPER}$

The upper triangular part of $A$ is stored and $A$ is factorized as $U^T U$, where $U$ is upper triangular.

$\text{uplow} = \text{NAGGPUMATRIXUPLOW_LOWER}$

The lower triangular part of $A$ is stored and $A$ is factorized as $LL^T$, where $L$ is lower triangular.

Constraint: $\text{uplow} = \text{NAGGPUMATRIXUPLOW_UPPER}$ or $\text{uplow} = \text{NAGGPUMATRIXUPLOW_LOWER}$.

2: $n$ – int

On entry: $n$, the order of the matrix $A$.

Constraint: $n \geq 0$.

3: $a[d]$ – double *

On entry: $a$ must satisfy $d \geq \max(1, n \times \text{pda})$.
On entry: the $n$ by $n$ symmetric positive definite matrix $A$. Let $a_{i,j}$ denote the $(i,j)$th element of the matrix $A$:

- If $\text{uplow} = \text{NAGGPUMATRIXUPLOW}_\text{UPPER}$, the upper triangular part $a_{i,j}$ for $j = 1,2,\ldots,n$ and $i = 1,2,\ldots,j$ must be stored in $a[(j-1) \times \text{pda} + i - 1]$ and the elements of the array below the diagonal are not referenced.
- If $\text{uplow} = \text{NAGGPUMATRIXUPLOW}_\text{LOWER}$, the lower triangular part $a_{i,j}$ for $i = 1,2,\ldots,n$ and $j = 1,2,\ldots,i$ must be stored in $a[(j-1) \times \text{pda} + i - 1]$ and the elements of the array above the diagonal are not referenced.

On exit: the factor $U$ or $L$ from the Cholesky factorization $A = U^T U$ or $A = LL^T$. Let $u_{i,j}$ denote the $(i,j)$th element of the matrix $U$ and let $l_{i,j}$ denote the $(i,j)$th element of the matrix $L$:

- If $\text{uplow} = \text{NAGGPUMATRIXUPLOW}_\text{UPPER}$, the element $u_{i,j}$ for $j = 1,2,\ldots,n$ and $i = 1,2,\ldots,j$ is stored in $a[(j-1) \times \text{pda} + i - 1]$.
- If $\text{uplow} = \text{NAGGPUMATRIXUPLOW}_\text{LOWER}$, the element $l_{i,j}$ for $i = 1,2,\ldots,n$ and $j = 1,2,\ldots,i$ is stored in $a[(j-1) \times \text{pda} + i - 1]$.

4: \text{pda} – int

On entry: the stride separating row elements of the matrix $A$ in the array $a$.

Constraint: $\text{pda} \geq \text{max}(1,n)$.

5: \text{error} – NagGpuError *

Error Reporting

Note: on exit, \text{error} $\rightarrow$ subCodes[0] will contain the LAPACK info parameter which is traditionally used to indicate errors or warnings.

This parameter contains error information and should not be modified directly. Errors are indicated through the value of \text{error} $\rightarrow$ code which should be inspected after each call to this function. If \text{error} $\rightarrow$ code $= 0$ then no error occurred. If \text{error} $\rightarrow$ code $\neq 0$ then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

- \text{error} $\rightarrow$ code $= 110$

On entry: uplow does not satisfy the constraint listed above.

- \text{error} $\rightarrow$ code $= 111$

On entry: $n < 0$

- \text{error} $\rightarrow$ code $= 112$

On entry: $a$ is NULL.

- \text{error} $\rightarrow$ code $= 113$

On entry: pda does not satisfy the constraint listed above.

- \text{error} $\rightarrow$ code $= 114$

On exit: the leading minor of order $i$ where $i = \text{error} \rightarrow$ subCodes[0] is not positive definite, and the factorization could not be completed. Hence the matrix $A$ itself is not positive definite. This may indicate an error in forming the matrix $A$.

7 Example

None.
NAG Numerical Routines for GPUs Data Type Document

NagGpuLinAlgComm

1 Purpose
NagGpuLinAlgComm is used by the library for communication between the GPU linear algebra (LAPACK) functions. It is for internal library use and should not be modified by the user in any way.

2 Specification
#include <nag_gpu.h>
struct NagGpuLinAlgComm {
    NagGpuTuneOrigin param1;
    void *param2;
    int param3;
    void *param4;
}

3 Description

4 References
None.

5 Members
The full structure definition is provided so that the library can be called from languages other than C/C++. The members of this structure not documented below are private to the library and must not be modified in any way.
NAG Numerical Routines for GPUs Data Type Document
NagGpuMatrixUpLow

1 Purpose
An enumeration to identify the upper or lower triangular parts of a matrix.

2 Specification
#include <nag_gpu.h>
enum NagGpuMatrixUpLow {
    NAGGPUMATRIXUPLOW_UPPER = 700,
    NAGGPUMATRIXUPLOW_LOWER
}

3 Description

4 References
None.

5 Symbols
1: NAGGPUMATRIXUPLOW_UPPER
   Identifies the upper triangular part of a matrix

2: NAGGPUMATRIXUPLOW_LOWER
   Identifies the lower triangular part of a matrix
Chapter Introduction

Host-Callable Generator Functions

naggpuRandInitA
naggpuRandExpA
naggpuRandGammaA
naggpuRandNormalA
naggpuRandUniformA
naggpuRandCleanupA

naggpuQuasiRandInitA
naggpuQuasiRandExpA
naggpuQuasiRandNormalA
naggpuQuasiRandUniformA
naggpuQuasiRandCleanupA

naggpuBBInitA
naggpuBBA
naggpuBBCleanupA
naggpuBBIncInitA
naggpuBBIncA
naggpuBBIncCleanupA
naggpuMakeBridgeOrderA

naggpuDepthBBInitA
naggpuDepthBBA
naggpuDepthBBIncInitA
naggpuDepthBBIncA
naggpuDepthBBCleanupA

Inline Device Function Generators

naggpuMrg32k3aDeviceInitA
naggpudevMrg32k3aInitA
naggpudevMrg32k3aExpA
naggpudevMrg32k3aGammaA
naggpudevMrg32k3aGammaSetParamsA
naggpudevMrg32k3aNormalA
naggpudevMrg32k3aUniformA
naggpuMrg32k3aDeviceCleanupA

naggpuSobolDeviceInitA
naggpudevSobolInitA
naggpudevSobolExpA
naggpudevSobolNormalA
naggpudevSobolUniformA
naggpuSobolDeviceCleanupA
Serial CPU Functions

nagCPURandInitA
nagCPURandExpA
nagCPURandGammaA
nagCPURandNormalA
nagCPURandUniformA
nagCPURandCleanupA
nagCPUQuasiRandInitA
nagCPUQuasiRandExpA
nagCPUQuasiRandNormalA
nagCPUQuasiRandUniformA
nagCPUQuasiRandCleanupA
nagCPUBBInitA
nagCPUBBA
nagCPUBBCleanupA
nagCPUBBIncInitA
nagCPUBBIncA
nagCPUBBIncCleanupA
nagCPUDepthBBInitA
nagCPUDepthBBA
nagCPUDepthBBIncInitA
nagCPUDepthBBIncA
nagCPUDepthBBCleanupA

List of Structures

NagGpuBBComm
NagGpuBBIncComm
NagGpuBBTune
NagGpuDepthBBComm
NagGpuMrg32k3aDeviceComm
NagGpuQuasiRandComm
NagGpuQuasiRandTune
NagGpuRandComm
NagGpuRandTune
NagGpuSobolDeviceComm
NagCPUBBComm
NagCPUBBIncComm
NagCPUDepthBBComm
NagCPUQuasiRandComm
NagCPURandComm

List of Enumerations

NagGpuBridgeOrder
NagGpuQuasiGen
NagGpuQuasiOrient
NagGpuRandGen
NagGpuRandOrder
NagGpuScramTypes
NagGpuTuneOrigin
NAG Numerical Routines for GPUs Chapter Introduction

Random Number Generators

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1 Scope of the Chapter

This chapter contains functions for the generation of sequences of pseudorandom and quasi-random numbers from various distributions.

2 Background to the Problems

2.1 Pseudorandom Numbers

A pseudorandom sequence is a sequence of numbers generated in some systematic way such that they are independent and statistically indistinguishable from a truly random sequence. A pseudorandom number generator (PRNG) is a mathematical algorithm that, given an initial state, produces a sequence of pseudorandom numbers. A PRNG has several advantages over a true random number generator in that the generated sequence is repeatable, has known mathematical properties and can be implemented without needing any specialist hardware. Many books on statistics and computer science have good introductions to PRNGs, for example Knuth (1981) or Banks (1998).

Suppose for a given seed, the PRNG produces the sequence of random numbers \(X_0, X_1, X_2, X_3, \ldots\). The library allows users to advance, or skip ahead, the seed by an amount

\[
s = a_1 2^b_1 + a_2 2^b_2 + c
\]

so that the generator will now produce the sequence of random numbers \(X_s, X_{s+1}, X_{s+2}, X_{s+3}, \ldots\). In applications where \(T\) separate copies of a generator are to operate independently, this technique is often used in one of two ways:

1. To split a sample of \(N\) consecutive values into \(T\) adjacent, non-overlapping blocks where each block typically has length \(N/T\). This can only be done if \(N\) is known, i.e. if it is known in advance how many values are to be generated, and in this case the \(i\)-th generator simply skips the seed ahead by \((i - 1)N/T\) steps for \(1 \leq i \leq T\).

2. To produce \(T\) independent substreams. This is done if it is not known in advance how many random values are needed, however now the task of determining where each generator should start generating is non-trivial. Merely ensuring that the generators (or ‘substreams’) do not overlap (e.g., by choosing a very large \(s\)) does not ensure statistical independence between substreams. In general, the task of determining good values of \(a_1, b_1, a_2, b_2\) and \(c\) for use in these computations is not a simple matter, and will depend heavily on the particular generator.

Note: when using multiple streams and substreams to create independent generators, care should be taken that the skip aheads \(s\) do not exceed the period of the generator and result in two streams inadvertently starting at the same point (or close enough to overlap). Therefore it is recommended that \(s\) never exceed the period of the generator being used. The periods of the generators are listed below.

Independent generators will most likely only be used by applications which use multiple GPUs simultaneously. In this case, each generator will have its own communication structure (i.e. NagGpuRandComm) which encapsulates all the information the generator needs to function. An array of communication structures with judiciously chosen skip aheads represents an array of independent generators. Note that the initialization function must access the same GPU device context on which the generate call will be made. Please consult the CUDA documentation for details on how to achieve this and how to access multiple GPUs simultaneously from a single application.

For the most common task of generating a block of pseudorandom numbers on a single GPU, users will typically only have a single GPU generator (i.e. only one communication structure) and the skip ahead \(s\) above can be set to zero.

2.1.1 MRG32k3a Generator

The Multiple Recursive Generator MRG32k3a is described in L’Ecuyer (1999), where L’Ecuyer gives a very efficient serial implementation. The period of the generator is approximately \(2^{191}\). Parallelization relies on the fact that the two recurrences defining this generator are of the same form, and can be represented as simple matrices leading to an efficient means of computing skip ahead points. The GPU implementation is fairly straightforward. The matrices defining the skip ahead can be pre-computed on the
host and copied to device memory. Each thread uses the skip ahead algorithm to compute the initial point and then uses the standard algorithm to compute a contiguous block of pseudorandom variates.

In a later paper L’Ecuyer et al. (2002), the authors consider the problem of partitioning the generator’s period into independent substreams. They consider various values of $b_1$ and $b_2$ in (1) above, and use a spectral test to search for values where the resulting streams of numbers have good statistical properties. They conclude that good statistical properties are obtained when $b_1$ or $b_2$ are equal to 76 or 127. The $i$th independent stream of the generator is selected by setting $s = i2^{127}$ for $i = 0, 1, \ldots$, and within the $i$th stream the $j$th independent substream is selected by setting $s = i2^{127} + j2^{76}$ for $j = 0, 1, \ldots$. The reader is referred to L’Ecuyer et al. (2002) for further details. The library has been optimized for values of $b_1$ or $b_2$ equal to 76 or 127: skipping ahead with these parameters is significantly faster than skipping ahead with different values of $b_1$ and $b_2$.

2.1.1.1 Implementation Changes Since Release 0.3

The MRG32k3a generator interface has changed significantly since release 0.3. The main changes are as follows:

1. The function names and argument lists have changed.
2. The seed has been unified into a single array. Previously, the generator was initialized via a call to `nag_gnu_mrg32k3a_init(v1, v2, offset)` where $v1$ and $v2$ were arrays of three unsigned integers. Now (see `nag_gpuRandInitA`) there is a single array `seed` which corresponds to $v1$ and $v2$ as follows: $\text{seed}[i] = v1[i]$ and $\text{seed}[i + 3] = v2[i]$ for $i = 0, 1, 2$.
3. Previously, the input seed was advanced by one step before the first point was generated. Now the input seed is used as-is to generate the first point.

To match the CPU generators in release 0.3, call `nagCPURandInitA` with

1. `genid = NAGGPURANDGEN_MRG32K3A`
2. `a1 = b1 = a2 = b2 = 0`
3. `c = offset + 1`
4. `seed` initialized from $v1, v2$ as shown above

Then call the generator functions (such as `nagCPURandUniformA`) as usual, passing in the desired distribution parameters.

To match the GPU generators in release 0.3, call `nag_gpuRandInitA` in the same way as just described for `nagCPURandInitA`, and then call one of the generator functions (e.g. `nag_gpuRandUniformA`) and

1. set `order = NAGGPURANDORDER_OPTIMAL`
2. create a NagGpuRandTune structure `compat`, set `compat.mrgOptAThdsPerBlk = nt` and `compat.mrgOptAPtsPerThd = np`, and set `tune = compat`
3. set `n = nb * nt * np` where $np$, $nt$ and $nb$ were the generator parameters release 0.3 of the library.

Note: if the GPU generators in release 0.3 were forced to match the serial ordering (i.e. by specifying $nt = 1$), then call `nag_gpuRandInitA` as described above, but call the generator function (e.g. `nag_gpuRandUniformA`) with `order = NAGGPURANDORDERCONSISTENT`, $n = nb * np$ and `tune = NULL`.

2.1.2 Mersenne Twister MT19937 Generator

The Mersenne Twister is described in Matsumoto and Nishimura (1998). It is a twisted generalized feedback shift register generator with very good multidimensional statistical properties and a period approximately equal to $2^{19937}$. The library implementation follows the implementation given in Matsumoto and Nishimura (1998). Parallelization uses the polynomial methods outlined in Haramoto et al. (2008).

The library allows skip aheads of the form (1) above for the Mersenne Twister, so that independent streams and substreams can be constructed. However there is currently no literature on recommended values for $b_1$.
and \(b_2\). Users are urged to exercise some caution in this matter. Note also that for large values of \(b_1\) and \(b_2\), the skip ahead calculation can take several seconds.

### 2.2 Quasi-random Numbers

Quasi-random numbers are intended primarily for use in Monte Carlo integration. Like pseudorandom numbers they are evenly distributed, but whereas pseudorandom sequences aim for statistical independence, quasi-random sequences aim for low discrepancy. Discrepancy is a measure of how evenly a sequence fills an area of multidimensional space. In a low discrepancy sequence, each point will tend to lie an equal distance from all its neighbouring points. This is typically not true for pseudorandom sequences, where one usually observes some form of clustering. For this reason quasi-random sequences are often more efficient in multidimensional Monte Carlo methods.

The low discrepancy sequence due to Sobol is provided here, based on the extension to higher dimensions described in Joe and Kuo (2003) and Joe and Kuo (2008). The digital scrambling described in Hong and Hickernell (2003) is provided. Please see NagGpuScramTypes for further details. Digital scrambling is an attempt to eliminate the bias inherent in a quasi-random sequence while retaining its low-discrepancy properties. It can be used to obtain error estimates of Monte Carlo integrals and can also alleviate systematic dependencies between dimensions, caused e.g. by poor choices of direction numbers.

### 2.3 Brownian Bridge

The term ‘Brownian bridge’ can mean one of two things: either it refers to a particular stochastic process which resembles a standard Brownian motion, but is forced to be zero at some time \(T > 0\); or it refers to a particular algorithm used to construct Brownian sample paths. The bridge algorithm constructs a Brownian sample path by first simulating its final value and then recursively filling in intermediate values through an interpolation formula. When compared with standard path construction methods, the bridge algorithm often displays advantages when solving stochastic differential equations or when pricing path dependent options. It is also possible to construct sample paths of the Brownian bridge process via the Brownian bridge algorithm, and this could be useful in certain stochastic models.

The Brownian bridge can be constructed in many different orderings, and two suites of functions are provided to address this. The first suite consists of functions with names containing ‘DepthBB’ (e.g. naggpuDepthBBInitA and naggpuDepthBBA). This suite uses a modified depth-ordered algorithm. The discretized Brownian motion \(X_t\), for \(k = 0, 1, \ldots, N\), on the time interval \([0, T]\) is generated in the order \(X_0, X_T, X_{T/2}, X_{T/4}, X_{T/8}, \ldots, X_{1/2} \ldots\), where for simplicity the case of equal time increments with \(N\) a power of 2 is shown. The algorithm provided allows for unequal time increments and does not restrict \(N\) to be a power of 2. However, for the purpose of introducing the samples from a Normal distribution, the Brownian bridge is structured internally by levels proceeding from the coarsest subdivisions of the time interval \([0, T]\) to the finest. This ensures the largest part of the variance of the Brownian path is concentrated in the coarser levels which can be matched to the lowest dimensions of a low discrepancy sequence such as that of Sobol when these appear as the earliest entries in the input array. For further details please see the documentation for naggpuDepthBBA.

The second suite consists of functions with similar names but without the ‘Depth’ token (e.g. naggpuBInitA and naggpuBBA). This suite allows the user to specify an arbitrary bridge construction order. To illustrate, fix an interval \([0, T]\) for some \(T > 0\) and suppose we wish to sample a Brownian motion \(X\) at a set of times \(0 < t_1 < t_2 < \ldots < t_{12} < T\) using the Brownian bridge algorithm. Suppose the construction order

\[
T \quad t_2 \quad t_4 \quad t_5 \quad t_9 \quad t_1 \quad t_7 \quad t_{12} \quad t_5 \quad t_{10} \quad t_6 \quad t_{11} \quad t_8
\]

is specified. This is interpreted as follows. The point \(X_T\) is constructed first (this must always be the case). Thereafter \(X_{t_9}\) is interpolated between \(X_0\) and \(X_T\); \(X_{t_4}\) is interpolated between \(X_{t_5}\) and \(X_T\); \(X_{t_1}\) is interpolated between \(X_{t_2}\) and \(X_{t_4}\); \(X_{t_9}\) is interpolated between \(X_{t_8}\) and \(X_{t_2}\); and so on. The order in which the interpolation times are listed is the order in which the bridge is constructed. Moreover, when the bridge is paired with Normal random variates generated from a quasi-random generator, the first dimension of each 13 dimensional quasi-random point is used to construct \(X_T\), the second dimension is used to construct \(X_{t_1}\), the third dimension is used to construct \(X_{t_5}\), the fourth dimension is used to construct \(X_{t_9}\), and so on. Successive dimensions are used to construct successive Brownian points as listed in the construction order. This straightforward mapping allows a user to specify which dimensions of each
multidimensional quasi-random point to use when constructing each part of the bridge. For further details please see the documentation for nagppuBBInitA. To ease the process of using these Brownian bridge generators, the function nagppuMakeBridgeOrderA is provided. It takes as input a set of time points (in increasing order) at which the Brownian motion is to be sampled, and provides as output the same set of times permuted into a bridge construction order such as (2). The function supports several popular bridge construction orders (see NagGpuBridgeOrder). A user may therefore take an array of time points (in increasing order) at which the Brownian motion is to be sampled, pass it to nagppuMakeBridgeOrderA to obtain a (pre-defined) construction order, pass this construction order to nagppuBBInitA to initialize the generator, and then call nagppuBBA to generate the Brownian sample paths. Finally the user should call nagppuBBCleanupA to free system resources.

3 Recommendations on Choice and Use of Available Functions

3.1 Host-Callable Generator Functions

A suite of pre-written CUDA kernels are provided which users can call from their CPU programs. These kernels will launch computations on the GPU device through the CUDA runtime. The available functionality is described below.

Prior to generating any pseudorandom variates the base generator being used must be initialized by calling the function nagppuRandInitA. Once initialized, a distributional generator can be called to obtain the variates required.

The random numbers computed can be chosen to be either double or single precision, however, double precision is recommended since the algorithms are designed for double precision storage and in single precision the sequences cannot be guaranteed to pass all statistical tests. If a sequence of random variates from a uniform distribution on the interval \([a, b]\) is required, then the uniform distribution function nagppuRandUniformA should be called. Functions for Normal (nagppuRandNormalA), exponential (nagppuRandExpA) and gamma (nagppuRandGammaA) distributions are also included. Once all required random variates have been obtained, nagppuRandCleanupA should be called to free allocated system resources.

nagppuDepthBBIncInitA and nagppuBBIncInitA initialize the Brownian bridge increments generators nagppuDepthBBIncA and nagppuBBIncA respectively. These generate scaled Brownian increments of the form \(X_{t_{i+1}} - X_{t_i}/(t_{i+1} - t_i)\) taking as input a previously computed array of pseudo or quasi-random numbers sampled from the standard Normal distribution. nagppuDepthBBCleanupA or nagppuBBIncCleanupA must be called to release system resources following generation of the Brownian bridge increments.

The Brownian sample paths generated by nagppuDepthBBIncA or nagppuBBIncA can be used to solve a stochastic differential equation (SDE) driven by a Wiener process

\[
dy(t) = f(t, y)dt + g(t, y)dW(t)
\]

for

\[
0 \leq t \leq T \quad \text{with} \quad y(0) = y_0,
\]

where \(f(t, y)\) is the drift coefficient, \(g(t, y)\) is the diffusion coefficient and \(W(t)\) is the standard Wiener process whose increment is

\[
\Delta W(t) = W(t + \Delta t) - W(t) = \sqrt{\Delta t} Z
\]

and \(Z \sim \mathcal{N}(0, 1)\). For numerical solution, the given SDE can be discretized using the Euler-Maruyama method to give

\[
y_{n+1} = y_n + f(t_n, y_n)\Delta t + g(t_n, y_n)\Delta W_n,
\]

with \(t_n = n\Delta t\). The output values from the generators nagppuDepthBBIncA and nagppuBBIncA are of the form

\[
\hat{W}_n = \frac{\Delta W_n}{\Delta t} = \frac{Z_n}{\sqrt{\Delta t}}
\]

so that the discretized SDE is
\[ y_{n+1} = y_n + \Delta t \left( f(t_n, y_n) + g(t_n, y_n)\dot{W}_n \right). \]

For other applications, naggpuDepthBBA and naggpuBBA are provided. They return the values of a Brownian path sampled at specified interpolation points rather than the scaled increments supplied by the bridge increments generators. A call to the initialization functions naggpuDepthBBInitA or naggpuBBInitA must be made before calling naggpuDepthBBA or naggpuBBA respectively to generate the Brownian sample paths, and should be followed by a call to naggpuDepthBBCleanupA or naggpuBBCleanupA respectively to free resources.

The Brownian bridge construction is often used with a low discrepancy sequence, such as the Sobol sequence, to supply the Normal variates. The quasi-random sequence generator must be initialized by a call to naggpuQuasiRandInitA. The sequence of Normal variates can then be generated by naggpuQuasiRandNormalA and resources freed by naggpuQuasiRandCleanupA.

Uniform and exponential distributions for the Sobol sequence can be computed by naggpuQuasiRandUniformA and naggpuQuasiRandExpA respectively.

### 3.2 Inline Device Function Generators

A small set of inline GPU device functions written using PTX assembly language are provided. These are functions which can be called from a user’s own CUDA kernel, in other words these functions live entirely on the GPU and can be used when writing CUDA kernels. Embedding a pseudorandom generator directly in a user’s kernel has a number of advantages: most notably, it can reduce memory traffic by avoiding writing numbers to global memory only to read them back in again. However inlining a generator will typically require quite a few registers, and this can lead to register pressure in some applications. As a guideline, if a kernel does a large number of floating point calculations, inline generators may not be necessary since there is enough computation to hide the memory traffic. Inline device generators are provided for the MRG32k3a and the Sobol generators only.

The use of device functions called from a program executing on the GPU is described in NVIDIA CUDA (2011). The distribution functions provided here pass successive variates, as they are generated, to the user’s CUDA threads as they execute on the GPU: each CUDA thread generates a contiguous segment of the quasi and pseudorandom number sequence. The functions are identified by the type qualifier \_\_device\_. The output of the distribution functions can be declared as double or float through the template argument FP.

Prior to generating any pseudorandom variates, the MRG32k3a inline generator must be initialized by a call to the host function naggpuMr32k3aDeviceInitA followed by a call (in the user’s CUDA kernel) to the GPU device function naggpudevMr32k3aInitA. Successive variates from either uniform, Normal, exponential or gamma distributions are generated by each thread through calls to naggpudevMr32k3aUniformA, naggpudevMr32k3aNormalA, naggpudevMr32k3aExpA or naggpudevMr32k3aGammaA, respectively. Resources used in the initialization are freed by a final call to the host function naggpuMr32k3aDeviceCleanupA after the user’s kernel exits.

Before using the inline device functions for generating a Sobol sequence, the host initialization function naggpuSobolDeviceInitA must be called followed by a call (in the user’s CUDA kernel) to the GPU device function naggpudevSobolInitA. The individual points in the Sobol sequence are then computed by repeated calls to naggpudevSobolUniformA, naggpudevSobolNormalA or naggpudevSobolExpA. Resources used in the initialization are freed by a final call to host function naggpuSobolDeviceCleanupA after the user’s kernel exits. Note that generating of a set of Sobol numbers with the inline generators is slower than using the host generators such as naggpuQuasiRandUniformA.

There are currently no inline device functions for the MT19937 generator.

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5 References

naggpuRandInitA

1 Purpose
naggpuRandInitA initializes a GPU pseudorandom number generator to a give a repeatable sequence of pseudorandom numbers. This function must be called before any call to the GPU generator functions (such as naggpuRandUniformA) and must ultimately be followed by a call to the cleanup function naggpuRandCleanupA to release system resources. A base generator is selected through the genid parameter and initialized with the values given in the seed array.

2 Specification
#include <nag_gpu.h>
extern "C"
cudaError_t naggpuRandInitA(NagGpuRandGen genid, int a1, int b1, int a2, int b2,
   long long c, const unsigned int *seed, NagGpuRandComm *comm,
   NagGpuError *error)

3 Description
For different values of seed, a given generator will yield different sequences of random numbers. Alternatively, the same sequence of random numbers will be generated if the same value of seed is used. In general there is no guarantee of statistical properties between sequences, only within sequences. This is important when generators are used in parallel. This function can ‘skip ahead’ or advance the seed by an amount

\[ s = a_12^{b_1} + a_22^{b_2} + c \] (1)

so that the generator will produce the sequence of random numbers \( X_s, X_{s+1}, X_{s+2}, \ldots \) instead of the original sequence \( X_0, X_1, X_2, \ldots \). This technique is useful to produce independent generators, often also called independent streams and substreams. Please see the Random Number Generators Chapter Introduction for further information.

Independent generators will be important mostly to applications which use multiple GPUs simultaneously. In this case, each generator will have its own NagGpuRandComm structure which encapsulates all the information the generator requires. An array of NagGpuRandComm structures with judiciously chosen skip aheads (see the Random Number Generators Chapter Introduction) represents an array of independent generators. Each structure must be initialized by a call to naggpuRandInitA. Note that when initializing a given communication structure, the call to naggpuRandInitA must access the same GPU device context which will be used when generating numbers with that communication structure. Please consult the CUDA documentation for details on how to achieve this and how to access multiple GPUs simultaneously from a single application.

For the most common task of generating a block of pseudorandom numbers on a single GPU, users will typically only have a single GPU generator (i.e. only one communication structure) and the skip ahead \( s \) above can be set to zero. The comments about arrays of communication structures and multiple GPU contexts can be ignored.

3.1 Synchronization
This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.
3.2 Return Value

Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References

None.

5 Arguments

1: genid – NagGpuRandGen

*Input

On entry: the type of generator to be used:

genid = NAGGPRANDGEN_MRG32K3A

genid = NAGGPRANDGEN_MT19937

Please see the Random Number Generators Chapter Introduction for details about each of these base generators.

Constraint: genid = NAGGPRANDGEN_MRG32K3A or NAGGPRANDGEN_MT19937.

2: a1 – int

*Input

On entry: the value of $a_1$ in the skip ahead equation (1) above.

Constraint: $a_1 \geq 0$.

3: b1 – int

*Input

On entry: the value of $b_1$ in the skip ahead equation (1) above.

Constraints:

if genid = NAGGPRANDGEN_MRG32K3A, $0 \leq b_1 \leq 191$;

if genid = NAGGPRANDGEN_MT19937, $0 \leq b_1 \leq 19937$.

4: a2 – int

*Input

On entry: the value of $a_2$ in the skip ahead equation (1) above.

Constraint: $a_2 \geq 0$.

5: b2 – int

*Input

On entry: the value of $b_2$ in the skip ahead equation (1) above.

Constraints:

if genid = NAGGPRANDGEN_MRG32K3A, $0 \leq b_2 \leq 191$;

if genid = NAGGPRANDGEN_MT19937, $0 \leq b_2 \leq 19937$.

6: c – long long

*Input

On entry: the value of $c$ in the skip ahead equation (1) above.

Constraint: $c \geq 0$.

7: seed[n] – const unsigned int *

*Input

On entry: an array of $n$ 32-bit unsigned integers to initialize the generator.

Constraints:

if genid = NAGGPRANDGEN_MRG32K3A,
\[ n = 6 \]
for \( i = 0, 1, 2, \) \( \text{seed}[i] < 2^{32} - 209 \) and \( \text{seed}(i) \neq 0 \) for at least one \( i \)

for \( i = 3, 4, 5, \) \( \text{seed}[i] < 2^{32} - 22853 \) and \( \text{seed}(i) \neq 0 \) for at least one \( i \); if \( \text{genid} = \text{NAGGPRANDGEN\_MT19937}, \)

\[ n = 624 \]
for \( i = 0, 1, 2, \ldots, 623, \) \( \text{seed}(i) \neq 0 \) for at least one \( i \).

8: \( \text{comm} - \text{NagGpuRandComm} \) *

Communication Data
NagGpuRandComm is a structure which holds state and communication information and must not be modified in any way. The structure will be initialized and must be passed to the generator functions (such as naggpuRandUniformA). Once all required points have been obtained, \text{comm} must be passed to naggpuRandCleanupA to free allocated system resources.

9: \( \text{error} - \text{NagGpuError} \) *

Error Reporting
This parameter contains error information and should not be modified directly. Errors are indicated through the value of \( \text{error} \rightarrow \text{code} \) which should be inspected after each call to this function. If \( \text{error} \rightarrow \text{code} = 0 \) then no error occurred. If \( \text{error} \rightarrow \text{code} \neq 0 \) then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

error \rightarrow \text{code} = 1

On entry: the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

error \rightarrow \text{code} = 2

During execution: a CUDA runtime error was detected.

error \rightarrow \text{code} = 100

On entry: the value of \text{comm} is NULL.

error \rightarrow \text{code} = 110

On entry: \text{genid} does not specify a valid pseudorandom number generator. See NagGpuRandGen for permitted values.

error \rightarrow \text{code} = 111

On entry: the value of \text{a1} is negative.

error \rightarrow \text{code} = 112

On entry: the value of \text{b1} does not satisfy the constraints listed above.

error \rightarrow \text{code} = 113

On entry: the value of \text{a2} is negative.

error \rightarrow \text{code} = 114

On entry: the value of \text{b2} does not satisfy the constraints listed above.

error \rightarrow \text{code} = 115

On entry: the value of \text{c} is negative.
error → code = 116

On entry: the value of seed is NULL.

error → code = 117

On entry: the values in the seed array do not satisfy the constraints listed above.

7 Example

There is no example program specifically for this function. For examples of how this function should be used, please see the example program for naggpuRandUniformA.
naggpuRandExpA

1 Purpose
naggpuRandExpA generates \( n \) values \( X_i \) from an exponential distribution with mean \( \lambda \).

The initialization function naggpuRandInitA must be called prior to the first call to naggpuRandExpA. Thereafter, this function may be called repeatedly to generate additional sets of random values. Once all desired values have been obtained, the function naggpuRandCleanupA must be called to free allocated system resources.

Note: To obtain the same values from naggpuRandExpA as from the function nag_gpu_mrg32-k3a_exp(nb, nt, np, d_P) in release 0.3 of the library, please see Section 2.1.1.1 in the Random Number Generators Chapter Introduction.

2 Specification

```c
#include <nag_gpu.h>
extern "C"
cudaError_t naggpuRandExpA_sp(int n, NagGpuRandOrder order, float lambda,
    float *d_buff, const NagGpuRandTune *tune, cudaStream_t custream,
    NagGpuRandComm *comm, NagGpuError *error)

extern "C"
cudaError_t naggpuRandExpA(int n, NagGpuRandOrder order, double lambda,
    double *d_buff, const NagGpuRandTune *tune, cudaStream_t custream,
    NagGpuRandComm *comm, NagGpuError *error)
```

3 Description

The exponential distribution has probability density function given by

\[
f(x) = \begin{cases} \frac{1}{\lambda} e^{-x/\lambda} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}
\]

where \( \lambda > 0 \). This function returns

\[X_i = -\lambda \ln Y_i\]

where \( Y_i \) are the next \( n \) values generated by the underlying uniform \([0,1]\) generator.

3.1 Synchronization

This function is non-blocking. Control will return immediately to the calling program while the computation is executed on the GPU. The user is responsible for synchronization between host and GPU code. Please see the synchronization chapter in the CUDA Programming Guide for further details in this direction. For example, a call to cudaMemcpy in the CUDA runtime library is enough to force the host to wait for the GPU to finish, and then copy the results from the GPU to the host.

3.2 Return Value

Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References

None.
5 Arguments

1: \( n \) – int  
   \( \text{Input} \)
   
   \text{On entry: the number of random values to be generated.}

   \text{Constraint: } n \geq 1.

2: \( \text{order} \) – NagGpuRandOrder  
   \( \text{Input} \)
   
   \text{On entry: the ordering to be observed by the underlying GPU generator:}

   \text{order} = \text{NAGGPRANDORDER}\_\text{OPTIMAL}

   \text{order} = \text{NAGGPRANDORDER}\_\text{CONSISTENT}

   \text{See NagGpuRandOrder for further details.}

   \text{Constraint:}

   \text{order} = \text{NAGGPRANDORDER}\_\text{OPTIMAL} \text{ or } \text{NAGGPRANDORDER}\_\text{CONSISTENT}.

3: \( \lambda \) – float  
   \( \text{Input} \)
   
   \text{This parameter has type float or double depending on whether the single or double precision version}

   \text{of this function is called.}

   \text{On entry: the mean, } \lambda, \text{ of the distribution.}

   \text{Constraint: } \lambda > 0.

4: \( \lambda \) – double  
   \( \text{Input} \)

5: \( \text{d\_buff\_n} \) – float *  
   \( \text{Output} \)

6: \( \text{d\_buff\_n} \) – double *  
   \( \text{Output} \)

   \text{This parameter has type float or double depending on whether the single or double precision version}

   \text{of this function is called.}

   \text{This buffer must reside in the GPU memory space.}

   \text{On exit: the } n \text{ pseudorandom numbers from the specified distribution. The output tuning structure}

   \text{comm} \to \text{tuneParamsUsed} \text{ will contain the parameters used to launch the kernel. If}

   \text{order} = \text{NAGGPRANDORDER}\_\text{OPTIMAL}, \text{ these parameters may determine the output}

   \text{ordering (see NagGpuRandTune for details).}

7: \( \text{tune} \) – const NagGpuRandTune *  
   \( \text{Input} \)

   \text{This parameter is optional and may be set to NULL.}

   \text{On entry: if specified, points to a NagGpuRandTune structure containing launch parameters for the}

   \text{selected GPU kernel. Upon a successful return from this function, the relevant data will be copied}

   \text{to the output tuning structure comm} \to \text{tuneParamsUsed}. \text{ Please see NagGpuRandTune for}

   \text{additional information about performance tuning.}

8: \( \text{custream} \) – cudaStream_t  
   \( \text{Input} \)

   \text{On entry: specifies the CUDA stream on which to launch the selected GPU kernel. If no streams}

   \text{are used, set this parameter to 0. Please see the chapter on Streams in the CUDA Programming}

   \text{Guide for further details.}

9: \( \text{comm} \) – NagGpuRandComm *  
   \( \text{Communication Data} \)

   \text{NagGpuRandComm is a structure which holds state and communication information and must not}

   \text{be modified in any way. Once all required points have been obtained, } \text{comm} \text{ must be passed to}

   \text{naggpuRandCleanupA to free allocated system resources.}

   \text{ Upon successful return from this function, the launch configurations applied to the underlying GPU}

   \text{kernels may be observed through } \text{comm}. \text{ This will typically only be of interest to users wanting to}

   \text{fine tune the performance of this function. Please see NagGpuRandTune for details on performance}
tuning, and consult the NagGpuRandComm documentation for how to observe the launch parameters. Note that these parameters are no longer observable after calling naggpuRandCleanupA.

10: `error` – NagGpuError *

Error Reporting

This parameter contains error information and should not be modified directly. Errors are indicated through the value of `error → code` which should be inspected after each call to this function. If `error → code = 0` then no error occurred. If `error → code ≠ 0` then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

- **error → code = 1**
  - *On entry:* the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call `cudaGetLastError()` in the CUDA runtime library to clear the runtime error status.

- **error → code = 2**
  - *During execution:* a CUDA runtime error was detected.

- **error → code = 3**
  - *On entry:* an attempt was made to launch a double precision function on a GPU device that does not support double precision.

- **error → code = 100**
  - *On entry:* the value of `comm` is NULL.

- **error → code = 101**
  - *On entry:* `comm` has not been initialized, or the internal state of `comm` is corrupted.

- **error → code = 110**
  - *On entry:* `n ≤ 0`.

- **error → code = 111**
  - *On entry:* `order` does not specify a valid ordering. See NagGpuRandOrder for permitted values.

- **error → code = 112**
  - *On entry:* `d_buff` is NULL.

- **error → code = 114**
  - *On entry:* `lambda ≤ 0`.

- **error → code = 200**
  - *On entry:* the MRG32k3a base generator is selected (see naggpuRandInitA for details), `order = NAGGPURANDORDER_CONSISTENT`, `tune ≠ NULL` and the kernel launch requires too much shared memory. Try reducing the value of `tune → mrgConAThdsPerBlk`.

- **error → code = 250**
  - *On entry:* the MRG32k3a base generator is selected (see naggpuRandInitA for details), `order = NAGGPURANDORDER_OPTIMAL`, `tune ≠ NULL` and `tune → mrgOptAThdsPerBlk` is out of bounds. See NagGpuRandTune for further details.
error → code = 251

On entry: the MRG32k3a base generator is selected (see naggpuRandInitA for details),
order = NAGGPRANDORDER_OPTIMAL, tune ≠ NULL and
tune → mrgOptAPtsPerThd is out of bounds. See NagGpuRandTune for further details.

error → code = 252

On entry: the MRG32k3a base generator is selected (see naggpuRandInitA for details),
order = NAGGPRANDORDER_CONSISTENT, tune ≠ NULL and
tune → mrgConAThdsPerBlk is out of bounds. See NagGpuRandTune for further details.

error → code = 253

On entry: the MRG32k3a base generator is selected (see naggpuRandInitA for details),
order = NAGGPRANDORDER_CONSISTENT, tune ≠ NULL and
tune → mrgConAThdsPerBlk is not divisible by W where W = 16 on devices of compute
capability 1.3 or lower and W = 32 otherwise. See NagGpuRandTune for further details.

error → code = 254

On entry: the MRG32k3a base generator is selected (see naggpuRandInitA for details),
order = NAGGPRANDORDER_CONSISTENT, tune ≠ NULL and
tune → mrgConANumLoops is out of bounds. See NagGpuRandTune for further details.

error → code = 300

On entry: the MT19937 base generator is selected (see naggpuRandInitA for details),
tune ≠ NULL and tune → mtANumBlks is out of bounds. See NagGpuRandTune for further details.

error → code = 301

On entry: the MT19937 base generator is selected (see naggpuRandInitA for details),
tune ≠ NULL and tune → mtAGen is NULL.

error → code = 302

On entry: the MT19937 base generator is selected (see naggpuRandInitA for details),
tune ≠ NULL and tune → mtAGen contains negative entries. See NagGpuRandTune for further details.

error → code = 303

On entry: the MT19937 base generator is selected (see naggpuRandInitA for details),
tune ≠ NULL and the entries in tune → mtAGen do not sum to n. See NagGpuRandTune for further details.

7 Example

This example program uses naggpuRandExpA to print 50 pseudorandom numbers from an exponential
distribution using the MRG32k3a generator. For usage of the MT19937 generator, including tuning
aspects, please see the example program for naggpuRandNormalA.

7.1 Program Text

/*
 * Example Program: naggpu_rand_expA
 * *
 * Copyright 2009, Numerical Algorithms Group Ltd, Oxford, UK.
 * *
 * Version 0.4, 2011.
 * *
 */
#include
using namespace std;
#include <nag_gpu.h>

// precision defined by Makefile
#define SINGLEPRECISION
#define FP float
#else
#define FP double
#endif

callable checkNagError(NagGpuError *error);
callable printNagTuningParamsUsed(NagGpuRandComm *comm);
callable checkCudaError(cudaError_t cuError);

int main(int argc, char **argv)
{
  // host (CPU) storage for generated GPU random numbers
  FP *h_buff = 0;
  // device (GPU) storage for generated random numbers
  FP *d_buff = 0;
  // total number of points to generate
  int N = 1000000;
  // seed variables
  const int seed_length = 6;
  unsigned int seed[seed_length];
  // skip ahead variables
  int a1, a2, b1, b2;
  long long c;
  // distribution parameters
  FP lambda = 1.0;
  // NAG GPU structures
  NagGpuRandComm comm;
  NagGpuRandTune tune;
  NagGpuError error;
  cudaError_t cuError;
  // Print the title
  cout << "NAG GPU Example Program: ";
  cout << "naggpuRandExpA";
  if (sizeof(FP)==sizeof(float)) cout << "_sp";
  cout << endl << endl;

  // Allocate CPU and GPU memory
  h_buff = new FP[N];
  cuError = cudaMalloc((void **)&d_buff, sizeof(FP)*N);
  checkCudaError(cuError);

  // Initialise the generator only once
  cout << "Initialising generator..." << endl << endl;

  // arbitrary seed and skip ahead parameters
  for (int i = 0; i < seed_length; i++) seed[i] = i;
  a1 = 14;
  b1 = 34;
  a2 = 2;
  b2 = 21;
  c = 123;

  naggpuRandInitA(NAGGPURANDGEN_MRG32K3A, a1, b1, a2, b2, c, seed, &comm, &error);
  checkNagError(&error);
// Generate N/2 numbers using default tuning parameters
cout << "Generate with default tuning parameters..." << endl;
#ifdef SINGLEPRECISION
    naggpuRandExpA_sp(N/2, NAGGPURANDORDER_CONSISTENT, lambda,
                      d_buff, NULL, 0, &comm, &error);
#else
    naggpuRandExpA(N/2, NAGGPURANDORDER_CONSISTENT, lambda,
                   d_buff, NULL, 0, &comm, &error);
#endif
checkNagError(&error);

// Print out the tuning parameters used
cout << "Tuning Parameters used: " << endl;
printNagTuningParamsUsed(&comm);

// Generate (N - N/2) numbers using specified tuning parameters
cout << "Generate with user supplied tuning parameters..." << endl;
tune.mrgConAThdsPerBlk = 96;
tune.mrgConANumLoops = 30;
cout << "Tuning Parameters supplied: " << endl;
cout << " tune.mrgConAThdsPerBlk = " << tune.mrgConAThdsPerBlk << endl;
cout << " tune.mrgConANumLoops = " << tune.mrgConANumLoops << endl;
#ifdef SINGLEPRECISION
    naggpuRandExpA_sp(N - N/2, NAGGPURANDORDER_CONSISTENT, lambda,
                      d_buff + N/2, &tune, 0, &comm, &error);
#else
    naggpuRandExpA(N - N/2, NAGGPURANDORDER_CONSISTENT, lambda,
                   d_buff + N/2, &tune, 0, &comm, &error);
#endif
checkNagError(&error);

// Print out the tuning parameters used
cout << "Tuning Parameters used: " << endl;
printNagTuningParamsUsed(&comm);

// Copy back values from the GPU for printing
cuError = cudaMemcpy(h_buff, d_buff, sizeof(FP)*N,
c                      cudaMemcpyDeviceToHost);
checkCudaError(cuError);

// Print random numbers
cout << "The first 50 GPU random numbers:" << endl;
cout.setf(ios::fixed,ios::floatfield);
cout.precision(3);
for(int row = 0; row < 10; row++)
{
    for(int col = 0; col < 5; col++)
    {
        cout << h_buff[row*10 + col] << "\t";
    }
    cout << endl;
}
// Call cleanup for the NAG routine
naggpuRandCleanupA(&comm, &error);
checkNagError(&error);

// Free CPU and GPU memory
delete[] h_buff;
if (d_buff)
{
    cuError = cudaFree(d_buff);
    checkCudaError(cuError);
}
void checkNagError(NagGpuError *error)
{
    if (error->code != 0)
    {
        char *buff;
        buff = new char[error->msgLength];
        naggpuErrorCopyMsg(buff, error);
        cout << buff << endl;
        delete[] buff;
        exit(1);
    }
}

void printNagTuningParamsUsed(NagGpuRandComm *comm)
{
    cout << " comm.tuneOrigin = ";
    switch (comm->tuneOrigin)
    {
    case NAGGPUTUNEORIGIN_NA:
        cout << "NAGGPUTUNEORIGIN_NA";
        break;
    case NAGGPUTUNEORIGIN_DEFAULT:
        cout << "NAGGPUTUNEORIGIN_DEFAULT";
        break;
    case NAGGPUTUNEORIGIN_USER:
        cout << "NAGGPUTUNEORIGIN_USER";
        break;
    case NAGGPUTUNEORIGIN_AUTO:
        cout << "NAGGPUTUNEORIGIN_AUTO";
        break;
    default:
        cout << "Unrecognised tuneOrigin";
    }
    cout << endl;
    cout << " comm.tuneParamsUsed->mrgConAThdsPerBlk = ";
    cout << comm->tuneParamsUsed->mrgConAThdsPerBlk << endl;
    cout << " comm.tuneParamsUsed->mrgConANumLoops = ";
    cout << comm->tuneParamsUsed->mrgConANumLoops << endl;
    cout << endl;
}

void checkCudaError(cudaError_t cuError)
{
    if (cuError != cudaSuccess)
    {
        cout << cudaGetErrorString(cuError) << endl;
        exit(1);
    }
}

7.2 Program Data
None.
7.3 Program Results

NAG GPU Example Program: naggpuRandExpA_sp

Initialising generator...

Generate with default tuning parameters...
Tuning Parameters used:
  comm.tuneOrigin = NAGGPUTUNEORIGIN_DEFAULT
  comm.tuneParamsUsed->mrgConAThdsPerBlk = 32
  comm.tuneParamsUsed->mrgConANumLoops = 8

Generate with user supplied tuning parameters...
Tuning Parameters supplied:
  tune.mrgConAThdsPerBlk = 96
  tune.mrgConANumLoops = 30
Tuning Parameters used:
  comm.tuneOrigin = NAGGPUTUNEORIGIN_USER
  comm.tuneParamsUsed->mrgConAThdsPerBlk = 96
  comm.tuneParamsUsed->mrgConANumLoops = 30

The first 50 GPU random numbers:
1.030 0.043 1.340 0.433 0.352
1.427 0.664 1.182 2.276 0.313
0.019 1.246 0.671 0.111 1.340
0.352 0.921 0.456 0.658 0.581
0.941 0.235 0.400 0.353 0.506
1.579 0.439 2.200 0.518 0.064
0.274 0.110 2.678 0.597 0.903
0.725 0.427 1.912 1.212 0.340
0.151 2.020 0.992 0.347 5.351
0.975 1.483 2.598 0.150 1.632
1 Purpose

naggpuRandGammaA generates \( n \) values \( X_i \) from a gamma distribution with shape parameter \( \alpha \) and scale parameter \( \beta \).

The initialization function naggpuRandInitA must be called prior to the first call to naggpuRandGammaA. Thereafter, this function may be called repeatedly to generate additional sets of random values. Once all desired values have been obtained, the function naggpuRandCleanupA must be called to free allocated system resources.

Note: currently only the MRG32k3a base generator is supported. Support for MT19937 will be added in future releases.

2 Specification

```c
#include <nag_gpu.h>
extern "C"

cudaError_t naggpuRandGammaA_sp(int n, NagGpuRandOrder order, float alpha,
                                   float beta, float *d_buff, const NagGpuRandTune *tune,
                                   cudaStream_t custream, NagGpuRandComm *comm, NagGpuError *error)

extern "C"

cudaError_t naggpuRandGammaA(int n, NagGpuRandOrder order, double alpha,
                              double beta, double *d_buff, const NagGpuRandTune *tune,
                              cudaStream_t custream, NagGpuRandComm *comm, NagGpuError *error)
```

3 Description

The gamma distribution has probability density function given by

\[
    f(x) = \begin{cases} 
        \frac{1}{\beta \Gamma(\alpha)} x^{\alpha - 1} e^{-x/\beta} & \text{if } x \geq 0 \\
        0 & \text{otherwise} 
    \end{cases}
\]

where \( \alpha, \beta > 0 \). The rejection algorithm described in Marsaglia and Tsang (2000) is used to generate the gamma pseudorandom variates when \( \alpha \geq 1 \). When \( 0 < \alpha < 1 \), the scaling

\[
    \gamma_{\alpha} = \gamma_{1+\epsilon} U^\alpha
\]

is used where \( U \) denotes a uniform random variable in the interval \([0, 1]\) and \( \gamma_{\alpha} \) denotes a gamma random variable with shape parameter \( \alpha \) and scale parameter \( \beta = 1 \). Please see the members of the NagGpuRandTune structure which pertain to this function for further details about the implementation. Note that currently only the MRG32k3a base generator is supported. In addition, when \( \text{order} = \text{NAGGPURANDORDER\_OPTIMAL} \), the sequences produced by GPUs with compute capability 1.3 and lower will differ from those produced by GPUs with compute capability 2.0 and higher, even when identical launch configurations are specified (via \( \text{tune} \)). This is due to different optimal orderings being used for the different GPU architectures.

Note: rejection algorithms are extremely sensitive to computational accuracy. When a variate is generated close to the rejection envelope, small differences in numerical values can lead to it being accepted in double precision while it is rejected in single precision (or vice versa). From this point on, the single and double precision sequences will be different. The same behaviour is seen when comparing single precision sequences generated on the CPU and the GPU: differences in the floating point calculations will lead to the sequences diverging after a certain number of variates. In double precision, the CPU and GPU sequences will take much longer (on average) before they diverge, agreeing to tens or even hundreds of millions of variates before numerical differences cause a variate to be accepted on one platform while it is rejected on the other.
3.1 Synchronization

When \( \text{order} = \text{NAGGPURANDORDER\_CONSISTENT} \), this function is blocking. Control will not return to the calling program until the function has completed. An iterative procedure is followed to generate all \( n \) pseudorandom variates in consistent order.

When \( \text{order} = \text{NAGGPURANDORDER\_OPTIMAL} \), this function is non-blocking. Control will return immediately to the calling program while the computation is executed on the GPU. Independent substreams are used to avoid the blocking required to produce pseudorandom variates in consistent order. The user is responsible for synchronization between host and GPU code. Please see the synchronization chapter in the CUDA Programming Guide for further details in this direction. For example, a call to \text{cudaMemcpy} in the CUDA runtime library is enough to force the host to wait for the GPU to finish, and then copy the results from the GPU to the host.

3.2 Return Value

Any CUDA runtime errors that were encountered, or \text{cudaSuccess} if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References


5 Arguments

1: \( n \) – int

\textit{Input}

\textit{On entry:} the number of random values to be generated.

\textit{Constraint:} \( n \geq 1 \).

2: \( \text{order} \) – NagGpuRandOrder

\textit{Input}

\textit{On entry:} the ordering to be observed by the underlying GPU generator:

\( \text{order} = \text{NAGGPURANDORDER\_OPTIMAL} \)

\( \text{order} = \text{NAGGPURANDORDER\_CONSISTENT} \)

See NagGpuRandOrder for further details.

\textit{Constraint:}

\( \text{order} = \text{NAGGPURANDORDER\_OPTIMAL} \) or \( \text{NAGGPURANDORDER\_CONSISTENT} \).

3: \( \alpha \) – float

\textit{Input}

\textit{On entry:} the shape parameter, \( \alpha \), of the distribution.

\textit{Constraint:} \( \alpha > 0 \).

4: \( \alpha \) – double

\textit{Input}

\textit{Constraint:} \( \alpha > 0 \).

5: \( \beta \) – float

\textit{Input}

\textit{On entry:} the scale parameter, \( \beta \), of the distribution.

\textit{Constraint:} \( \beta > 0 \).

6: \( \beta \) – double

\textit{Input}

\textit{Constraint:} \( \beta > 0 \).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>d_buff[n]</code></td>
<td>float * Output</td>
<td>This parameter has type float or double depending on whether the single or double precision version of this function is called. This buffer must reside in the GPU memory space.</td>
</tr>
<tr>
<td><code>d_buff[n]</code></td>
<td>double * Output</td>
<td></td>
</tr>
<tr>
<td><code>n</code></td>
<td></td>
<td></td>
</tr>
<tr>
<td><code>comm</code></td>
<td>NagGpuRandComm *</td>
<td>Communication Data</td>
</tr>
<tr>
<td><code>order</code></td>
<td></td>
<td></td>
</tr>
<tr>
<td><code>tune</code></td>
<td>const NagGpuRandTune *</td>
<td>Input</td>
</tr>
<tr>
<td><code>custream</code></td>
<td>cudaStream_t</td>
<td>Input</td>
</tr>
<tr>
<td><code>error</code></td>
<td>NagGpuError *</td>
<td>Error Reporting</td>
</tr>
</tbody>
</table>

**6 Error Indicators and Warnings**

- **error → code = 1**
  
  *On entry:* the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call `cudaGetLastError()` in the CUDA runtime library to clear the runtime error status.

- **error → code = 2**
  
  *During execution:* a CUDA runtime error was detected.
error → code = 3

On entry: an attempt was made to launch a double precision function on a GPU device that does not support double precision.

error → code = 100

On entry: the value of comm is NULL.

error → code = 101

On entry: comm has not been initialized, or the internal state of comm is corrupted.

error → code = 110

On entry: n ≤ 0.

error → code = 111

On entry: order does not specify a valid ordering. See NagGpuRandOrder for permitted values.

error → code = 112

On entry: d_buff is NULL.

error → code = 113

On entry: alpha ≤ 0

error → code = 114

On entry: beta ≤ 0

error → code = 115

On entry: the MT19937 base generator is selected (see naggpuRandInitA for further details). Currently only the MRG32k3a base generator is supported.

error → code = 200

On entry: order = NAGGPRANDORDER_CONSISTENT, tune ≠ NULL and the kernel launch requires too much shared memory. Try reducing the number of threads per block. The runtime error message obtained from naggpuErrorCopyMsg will contain additional diagnostic information.

error → code = 201

On entry: order = NAGGPRANDORDER_OPTIMAL, tune ≠ NULL and the kernel launch requires too much shared memory. Try reducing the value of tune → mrgRejOptAThdsPerBlk.

error → code = 250

On entry: order = NAGGPRANDORDER_OPTIMAL, tune ≠ NULL and tune → mrgRejOptAThdsPerBlk is out of bounds. See NagGpuRandTune for further details.

error → code = 251

On entry: order = NAGGPRANDORDER_OPTIMAL, tune ≠ NULL and tune → mrgRejOptANumBlks is not divisible by W where W = 16 on devices of compute capability 1.3 or lower and W = 32 otherwise. See NagGpuRandTune for further details.

error → code = 252

On entry: order = NAGGPRANDORDER_OPTIMAL, tune ≠ NULL and tune → mrgRejOptANumBlks is out of bounds. See NagGpuRandTune for further details.
error → code = 253

On entry: order = NAGGPURANDORDER_CONSISTENT, tune ≠ NULL and
tune → mrgRejConA is NULL. See NagGpuRandTune for further details.

error → code = 254

On entry: order = NAGGPURANDORDER_CONSISTENT, tune ≠ NULL and the
number of threads per block returned by tune → mrgRejConA is out of bounds. See
NagGpuRandTune for further details. The runtime error message obtained from
naggpuErrorCopyMsg will contain additional diagnostic information.

error → code = 255

On entry: order = NAGGPURANDORDER_CONSISTENT, tune ≠ NULL and the
number of threads per block returned by tune → mrgRejConA is not divisible by W
where W = 16 on devices of compute capability 1.3 or lower and W = 32 otherwise. See
NagGpuRandTune for further details. The runtime error message obtained from
naggpuErrorCopyMsg will contain additional diagnostic information.

error → code = 256

On entry: order = NAGGPURANDORDER_CONSISTENT, tune ≠ NULL and the
number of blocks returned by tune → mrgRejConA is out of bounds. See
NagGpuRandTune for further details. The runtime error message obtained from
naggpuErrorCopyMsg will contain additional diagnostic information.

7 Example

This example program uses naggpuRandGammaA to print 50 pseudorandom numbers from a gamma
distribution using the MRG32k3a generator.

7.1 Program Text

/*
 * Example Program: naggpuRandGammaA
 *
 * Copyright 2009, Numerical Algorithms Group Ltd, Oxford, UK.
 *
 * Version 0.4, 2011.
 *
 */

#include
using namespace std;
#include <nag_gpu.h>

// precision defined by Makefile
#ifdef SINGLEPRECISION
#define FP float
#else
#define FP double
#endif

void checkNagError(NagGpuError *error);
void printNagTuningParamsUsed(NagGpuRandComm *comm);
void checkCudaError(cudaError_t cuError);

// This function is for illustrative purposes only.
// Realistic tuning functions would have finer subdivisions
// of the npts range and would probably use higher values
// of nthds.
static void tuneFunc(int npts, int *nthds, int *nblks)
{
    if(npts < 10000) {
        (*nthds) = 128;
    } else
        (*nthds) = 128;
}
naggpuRandGammaA

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(*nblks) = 10;
} else {
  (*nthds) = 64;
  (*nblks) = 60;
}

int main(int argc, char **argv)
{
  // host (CPU) storage for generated GPU random numbers
  FP *h_buff = 0;
  // device (GPU) storage for generated random numbers
  FP *d_buff = 0;
  // total number of points to generate
  int N = 1000000;

  // seed variables
  const int seed_length = 6;
  unsigned int seed[seed_length];

  // skip ahead variables
  int a1, a2, b1, b2;
  long long c;

  // distribution parameters
  FP alpha = 1.7;
  FP beta = 1.3;

  // NAG GPU structures
  NagGpuRandComm comm;
  NagGpuRandTune tune;
  NagGpuError error;
  cudaError_t cuError;

  // Print the title
  cout << "NAG GPU Example Program: ";
  cout << "naggpuRandGammaA";
  if (sizeof(FP)==sizeof(float)) cout << "_sp";
  cout << endl << endl;

  // Allocate CPU and GPU memory
  h_buff = new FP[N];
  cuError = cudaMalloc((void**)&d_buff, sizeof(FP)*N);
  checkCudaError(cuError);

  // Initialise the generator only once
  cout << "Initialising generator..." << endl << endl;

  // arbitrary seed and skip ahead parameters
  for (int i = 0; i < seed_length; i++) seed[i] = i;
  a1 = 14;
  b1 = 34;
  a2 = 2;
  b2 = 21;
  c = 123;
  naggpuRandInitA(NAGGPUTRANDBASE_MRG32K3A, a1, b1, a2, b2,
                  c, seed, &comm, &error);
  checkNagError(&error);

  // Generate N/2 numbers using default tuning parameters
  cout << "Generate with default tuning parameters..." << endl;
  #ifdef SINGLEPRECISION
  naggpuRandGammaA_sp(N/2, NAGGPURANDORDER_CONSISTENT, alpha, beta,
                      d_buff, NULL, 0, &comm, &error);
  #else
  naggpuRandGammaA_sp(N/2, NAGGPURANDORDER_CONSISTENT, alpha, beta,
naggpuRandGammaA(N/2, NAGGPURANDORDER_CONSISTENT, alpha, beta, 
d_buff, NULL, 0, &comm, &error);
#endif
checkNagError(&error);

// Print out the tuning parameters used
cout << "Tuning Parameters used: " << endl;
prientNagTuningParamsUsed(&comm);

// Generate (N- N/2) numbers using specified tuning parameters
cout << "Generate with user supplied tuning parameters..." << endl;
tune.mrgRejConA =
cout << "Tune Parameters supplied: " << endl;
cout << " tune.mrgRejConA = " << (void*)tune.mrgRejConA
<< endl;
#endif SINGLEPRECISION
naggpuRandGammaA_sp(N - N/2, NAGGPURANDORDER_CONSISTENT, alpha, beta,
d_buff + N/2, &tune, 0, &comm, &error);
#else
naggpuRandGammaA(N - N/2, NAGGPURANDORDER_CONSISTENT, alpha, beta,
d_buff + N/2, &tune, 0, &comm, &error);
#endif
checkNagError(&error);

// Print out the tuning parameters used
cout << "Tuning Parameters used: " << endl;
prientNagTuningParamsUsed(&comm);

// Copy back values from the GPU for printing
cuError = cudaMemcpy(h_buff, d_buff, sizeof(FP)*N,
cudaMemcpyDeviceToHost);
checkCudaError(cuError);

// Print random numbers
cout << "The first 50 GPU random numbers:" << endl;
cout.setf(ios::fixed,ios::floatfield);
cout.precision(3);
for(int row = 0; row < 10; row++)
{ 
    for(int col = 0; col < 5; col++)
    { 
        cout << h_buff[row*10 + col] << "\t"
    }
    cout << endl;
}

// Call cleanup for the NAG routine
naggpuRandCleanupA(&comm, &error);
checkNagError(&error);

// Free CPU and GPU memory
delete[] h_buff;
if (d_buff)
{ 
    cuError = cudaFree(d_buff);
    checkCudaError(cuError);
}
return 0;


Mark 0.6

naggpuRandGammaA.7
```c
if (error->code != 0) {
    char *buff;
    buff = new char[error->msgLength];
    naggpuErrorCopyMsg(buff, error);
    cout << buff << endl;
    delete[] buff;
    exit(1);
}
```

```c
void printNagTuningParamsUsed(NagGpuRandComm *comm) {
    cout << " comm.tuneOrigin = ";
    switch (comm->tuneOrigin) {
        case NAGGPUTUNEORIGIN_NA:
            cout << "NAGGPUTUNEORIGIN_NA";
            break;
        case NAGGPUTUNEORIGIN_DEFAULT:
            cout << "NAGGPUTUNEORIGIN_DEFAULT";
            break;
        case NAGGPUTUNEORIGIN_USER:
            cout << "NAGGPUTUNEORIGIN_USER";
            break;
        case NAGGPUTUNEORIGIN_AUTO:
            cout << "NAGGPUTUNEORIGIN_AUTO";
            break;
        default:
            cout << "Unrecognised tuneOrigin";
            break;
    }
    cout << endl;

    cout << " comm.tuneParamsUsed->mrgRejConA = ";
    cout << (void*)comm->tuneParamsUsed->mrgRejConA << endl;
    cout << endl;
}
```

```c
void checkCudaError(cudaError_t cuError) {
    if (cuError != cudaSuccess) {
        cout << cudaGetErrorString(cuError) << endl;
        exit(1);
    }
}
```

### 7.2 Program Data

None.

### 7.3 Program Results

NAG GPU Example Program: naggpuRandGammaA_sp

Initialising generator...

Generate with default tuning parameters...
Tuning Parameters used:
- `comm.tuneOrigin = NAGGPUTUNEORIGIN_DEFAULT`
- `comm.tuneParamsUsed->mrgRejConA = 0x2b755111cab0`

Generate with user supplied tuning parameters...
Tuning Parameters supplied:
- `tune.mrgRejConA = 0x401060`
Tuning Parameters used:
- `comm.tuneOrigin = NAGGPUTUNEORIGIN_USER`
The first 50 GPU random numbers:
1.267 4.824 0.902 1.178 2.641
2.077 1.709 0.679 0.428 4.019
0.426 2.351 3.099 0.903 2.076
2.829 1.440 4.640 3.622 1.027
2.399 0.047 0.103 2.336 0.240
4.441 8.641 5.222 2.708 0.926
2.378 0.843 1.122 2.630 3.052
0.553 2.121 3.696 4.009 1.528
2.402 0.562 0.460 0.044 4.720
2.188 1.400 0.405 2.820 0.029
1 Purpose

naggpuRandNormalA generates \( n \) values \( X_i \) from a Normal distribution with mean \( \mu \) and variance \( \sigma^2 \).

The initialization function naggpuRandInitA must be called prior to the first call to naggpuRandNormalA. Thereafter, this function may be called repeatedly to generate additional sets of random values. Once all desired values have been obtained, the function naggpuRandCleanupA must be called to free allocated system resources.

Note: To obtain the same values from naggpuRandNormalA as from the function \( \text{nag\_gpu\_mrg32k3a\_normal(nb, nt, np, d\_P)} \) in release 0.3 of the library, please see Section 2.1.1.1 in the Random Number Generators Chapter Introduction.

2 Specification

```c
#include <nag_gpu.h>
extern "C"
cudaError_t naggpuRandNormalA_sp(int n, NagGpuRandOrder order, float mu, float sigma, float *d_buff, const NagGpuRandTune *tune, cudaStream_t custream, NagGpuRandComm *comm, NagGpuError *error)
extern "C"
cudaError_t naggpuRandNormalA(int n, NagGpuRandOrder order, double mu, double sigma, double *d_buff, const NagGpuRandTune *tune, cudaStream_t custream, NagGpuRandComm *comm, NagGpuError *error)
```

3 Description

The Normal distribution has probability density function given by

\[
f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(\frac{-(x - \mu)^2}{2\sigma^2}\right)
\]

where \( \sigma > 0 \) and \( \mu \in \mathbb{R} \). This function uses a Box-Muller transform to convert a pair of uniform \((0, 1)\) random numbers into a pair of Normal random variables. Let \( X_0, X_1, X_2, \ldots \) denote the sequence of uniform \((0, 1)\) pseudorandom variates as specified by the base generator algorithm. When \(\text{order} = \text{NAGGPURANDORDER\_CONSISTENT}\), the function will always use successive pairs of uniform variates in the Box-Muller transform to produce successive pairs of Normal variates, i.e. \((X_0, X_1)\rightarrow(Z_0, Z_1), (X_2, X_3)\rightarrow(Z_2, Z_3)\) where \( Z_0, Z_1, Z_2, \ldots \) denotes the output sequence of Normal variates. When \(\text{order} = \text{NAGGPURANDORDER\_OPTIMAL}\):

- the MRG32k3a generator will continue to use successive pairs of uniform variates as described above, but will store the output Normal variates in the permuted order specified in NagGpuRandTune
- the MT19937 generator will select uniform variates and store the output Normal variates in an implementation-dependent manner

3.1 Synchronization

This function is non-blocking. Control will return immediately to the calling program while the computation is executed on the GPU. The user is responsible for synchronization between host and GPU code. Please see the synchronization chapter in the CUDA Programming Guide for further details in this direction. For example, a call to cudaMemcpy in the CUDA runtime library is enough to force the host to wait for the GPU to finish, and then copy the results from the GPU to the host.
3.2 Return Value

Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References

None.

5 Arguments

1: \( n \) – int

*Input*

*On entry:* the number of random values to be generated.

*Constraint:* \( n \geq 1 \).

2: \( \text{order} \) – NagGpuRandOrder

*Input*

*On entry:* the ordering to be observed by the underlying GPU generator:

\( \text{order} = \text{NAGGPURANDORDER\_OPTIMAL} \)

\( \text{order} = \text{NAGGPURANDORDER\_CONSISTENT} \)

See NagGpuRandOrder for further details.

*Constraint:*

\( \text{order} = \text{NAGGPURANDORDER\_OPTIMAL} \) or \( \text{NAGGPURANDORDER\_CONSISTENT} \).

3: \( \mu \) – float

*Input*

4: \( \mu \) – double

*Input*

This parameter has type float or double depending on whether the single or double precision version of this function is called.

*On entry:* the mean, \( \mu \), of the distribution.

5: \( \sigma \) – float

*Input*

6: \( \sigma \) – double

*Input*

This parameter has type float or double depending on whether the single or double precision version of this function is called.

*On entry:* the standard deviation, \( \sigma \), of the distribution.

*Constraint:* \( \sigma > 0 \).

7: \( \text{d\_buff}[n] \) – float *

*Output*

8: \( \text{d\_buff}[n] \) – double *

*Output*

This parameter has type float or double depending on whether the single or double precision version of this function is called.

This buffer must reside in the GPU memory space.

*On exit:* the \( n \) pseudorandom numbers from the specified distribution. The output tuning structure \( \text{comm} \rightarrow \text{tuneParamsUsed} \) will contain the parameters used to launch the kernel. If \( \text{order} = \text{NAGGPURANDORDER\_OPTIMAL} \), these parameters may determine the output ordering (see NagGpuRandTune for details).

9: \( \text{tune} \) – const NagGpuRandTune *

*Input*

This parameter is optional and may be set to NULL.

*On entry:* if specified, points to a NagGpuRandTune structure containing launch parameters for the selected GPU kernel. Upon a successful return from this function, the relevant data will be copied
to the output tuning structure \texttt{comm} \rightarrow \texttt{tuneParamsUsed}. Please see NagGpuRandTune for additional information about performance tuning.

10: \texttt{custream} – cudaStream_t

\textit{Input}

\textit{On entry:} specifies the CUDA stream on which to launch the selected GPU kernel. If no streams are used, set this parameter to 0. Please see the chapter on Streams in the CUDA Programming Guide for further details.

11: \texttt{comm} – NagGpuRandComm *

\textit{Communication Data}

NagGpuRandComm is a structure which holds state and communication information and must not be modified in any way. Once all required points have been obtained, \texttt{comm} must be passed to naggpuRandCleanupA to free allocated system resources.

Upon successful return from this function, the launch configurations applied to the underlying GPU kernels may be observed through \texttt{comm}. This will typically only be of interest to users wanting to fine tune the performance of this function. Please see NagGpuRandTune for details on performance tuning, and consult the NagGpuRandComm documentation for how to observe the launch parameters. Note that these parameters are no longer observable after calling naggpuRandCleanupA.

12: \texttt{error} – NagGpuError *

\textit{Error Reporting}

This parameter contains error information and should not be modified directly. Errors are indicated through the value of \texttt{error} \rightarrow \texttt{code} which should be inspected after each call to this function. If \texttt{error} \rightarrow \texttt{code} = 0 then no error occurred. If \texttt{error} \rightarrow \texttt{code} \neq 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

\section{6 Error Indicators and Warnings}

\texttt{error} \rightarrow \texttt{code} = 1

\textit{On entry:} the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

\texttt{error} \rightarrow \texttt{code} = 2

\textit{During execution:} a CUDA runtime error was detected.

\texttt{error} \rightarrow \texttt{code} = 3

\textit{On entry:} an attempt was made to launch a double precision function on a GPU device that does not support double precision.

\texttt{error} \rightarrow \texttt{code} = 100

\textit{On entry:} the value of \texttt{comm} is NULL.

\texttt{error} \rightarrow \texttt{code} = 101

\textit{On entry:} \texttt{comm} has not been initialized, or the internal state of \texttt{comm} is corrupted.

\texttt{error} \rightarrow \texttt{code} = 110

\textit{On entry:} \texttt{n} \leq 0.

\texttt{error} \rightarrow \texttt{code} = 111

\textit{On entry:} \texttt{order} does not specify a valid ordering. See NagGpuRandOrder for permitted values.

\texttt{error} \rightarrow \texttt{code} = 112

\textit{On entry:} \texttt{d_buff} is NULL.
error → code = 115

On entry: \( \sigma \leq 0 \)

error → code = 200

On entry: the MRG32k3a base generator is selected (see naggpuRandInitA for details),
\( \text{order} = \text{NAGGPUTANDORDER\_CONSISTENT}, \text{tune} \neq \text{NULL} \) and the kernel launch requires too much shared memory. Try reducing the value of 
\( \text{tune} \rightarrow \text{mrgConAThdsPerBlk} \).

error → code = 250

On entry: the MRG32k3a base generator is selected (see naggpuRandInitA for details),
\( \text{order} = \text{NAGGPUTANDORDER\_OPTIMAL}, \text{tune} \neq \text{NULL} \) and 
\( \text{tune} \rightarrow \text{mrgOptAThdsPerBlk} \) is out of bounds. See NagGpuRandTune for further details.

error → code = 251

On entry: the MRG32k3a base generator is selected (see naggpuRandInitA for details),
\( \text{order} = \text{NAGGPUTANDORDER\_OPTIMAL}, \text{tune} \neq \text{NULL} \) and 
\( \text{tune} \rightarrow \text{mrgOptAPtsPerThd} \) is out of bounds. See NagGpuRandTune for further details.

error → code = 252

On entry: the MRG32k3a base generator is selected (see naggpuRandInitA for details),
\( \text{order} = \text{NAGGPUTANDORDER\_CONSISTENT}, \text{tune} \neq \text{NULL} \) and 
\( \text{tune} \rightarrow \text{mrgConAThdsPerBlk} \) is not divisible by \( W \) where \( W = 16 \) on devices of compute 
capability 1.3 or lower and \( W = 32 \) otherwise. See NagGpuRandTune for further details.

error → code = 253

On entry: the MRG32k3a base generator is selected (see naggpuRandInitA for details),
\( \text{order} = \text{NAGGPUTANDORDER\_CONSISTENT}, \text{tune} \neq \text{NULL} \) and 
\( \text{tune} \rightarrow \text{mrgConANumLoops} \) is out of bounds. See NagGpuRandTune for further details.

error → code = 254

On entry: the MRG32k3a base generator is selected (see naggpuRandInitA for details),
\( \text{order} = \text{NAGGPUTANDORDER\_CONSISTENT}, \text{tune} \neq \text{NULL} \) and 
\( \text{tune} \rightarrow \text{mrgConANumLoops} \) is out of bounds. See NagGpuRandTune for further details.

error → code = 300

On entry: the MT19937 base generator is selected (see naggpuRandInitA for details),
\( \text{tune} \neq \text{NULL} \) and \( \text{tune} \rightarrow \text{mtANumBlks} \) is out of bounds. See NagGpuRandTune for further details.

error → code = 301

On entry: the MT19937 base generator is selected (see naggpuRandInitA for details),
\( \text{tune} \neq \text{NULL} \) and \( \text{tune} \rightarrow \text{mtAGen} \) is NULL.

error → code = 302

On entry: the MT19937 base generator is selected (see naggpuRandInitA for details),
\( \text{tune} \neq \text{NULL} \) and \( \text{tune} \rightarrow \text{mtAGen} \) contains negative entries. See NagGpuRandTune for further details.

error → code = 303

On entry: the MT19937 base generator is selected (see naggpuRandInitA for details),
\( \text{tune} \neq \text{NULL} \) and the entries in \( \text{tune} \rightarrow \text{mtAGen} \) do not sum to \( n \). See NagGpuRandTune for further details.

error → code = 304

On entry: the MT19937 base generator is selected (see naggpuRandInitA for details),
\( \text{order} = \text{NAGGPUTANDORDER\_CONSISTENT}, \text{tune} \neq \text{NULL} \) and some of the entries in 
\( \text{tune} \rightarrow \text{mtAGen} \) are odd. See NagGpuRandTune for further details.
7 Example

This example program uses naggpuRandNormalA to print 50 pseudorandom numbers from a uniform distribution using the MT19937 generator. For usage of the MRG32k3a generator, including tuning aspects, please see the example program for naggpuRandUniformA.

7.1 Program Text

```cpp
#include <nag_gpu.h>

// precision defined by Makefile
#ifdef SINGLEPRECISION
#define FP float
#else
#define FP double
#endif

void checkNagError(NagGpuError *error);
void printNagTuningParamsUsed(NagGpuRandComm *comm);
void checkCudaError(cudaError_t cuError);

int main(int argc, char **argv)
{
    // host (CPU) storage for generated GPU random numbers
    FP *h_buff = 0;
    // device (GPU) storage for generated random numbers
    FP *d_buff = 0;

    // total number of points to generate
    int N = 1000001;

    // seed variables
    const int seed_length = 624;
    unsigned int seed[seed_length];

    // skip ahead variables
    int a1, a2, b1, b2;
    long long c;

    // distribution parameters
    FP mu = 0.0;
    FP sigma = 1.0;

    // NAG GPU structures
    NagGpuRandComm comm;
    NagGpuRandTune tune;
    NagGpuError error;

    cudaError_t cuError;

    // Print the title
    cout << "NAG GPU Example Program: ";
    cout << "naggpuRandNormalA";
    if (sizeof(FP)==sizeof(float)) cout << "_sp";
    cout << endl << endl;
```
/ Allocate CPU and GPU memory
h_buff = new FP[N];
cuError = cudaMalloc((void **)d_buff, sizeof(FP)*N);
checkCudaError(cuError);

// Initialise the generator only once
cout << "Initialising generator..." << endl << endl;

// arbitrary seed and skip ahead parameters
for (int i = 0; i < seed_length; i++) seed[i] = i;
a1 = 14;
b1 = 34;
a2 = 2;
b2 = 21;
c = 123;
naggpuRandInitA(NAGGPURANDGEN_MT19937, a1, b1, a2, b2, c, seed, &comm, &error);
checkNagError(&error);

// Generate N/2 numbers using default tuning parameters
cout << "Generate with default tuning parameters..." << endl;
#ifdef SINGLEPRECISION
    naggpuRandNormalA_sp(N/2, NAGGPURANDORDER_CONSISTENT, mu, sigma, d_buff, NULL, 0, &comm, &error);
#else
    naggpuRandNormalA(N/2, NAGGPURANDORDER_CONSISTENT, mu, sigma, d_buff, NULL, 0, &comm, &error);
#endif
checkNagError(&error);

// Print out the tuning parameters used
cout << "Tuning Parameters used: " << endl;
printNagTuningParamsUsed(&comm);

// Generate (N-N/2) numbers using specified tuning parameters
cout << "Generate with user supplied tuning parameters..." << endl;
int N2 = N - N/2;
tune.mtANumBlks = 20;
tune.mtAGen = new int[tune.mtANumBlks];

// check pts_per_block is even
int pts_per_block = N2 / tune.mtANumBlks;
if (pts_per_block%2 == 1) pts_per_block -= 1;
for(int i = 0; i < tune.mtANumBlks - 1; i++)
{
    tune.mtAGen[i] = pts_per_block;
}

// last block does not have to generate an even number of points
int pts_last_block = N2 - (tune.mtANumBlks - 1)*pts_per_block;
tune.mtAGen[tune.mtANumBlks - 1] = pts_last_block;
cout << "Tuning Parameters supplied: " << endl;
cout << " tune.mtANumBlks = ";
cout << tune.mtANumBlks << endl;
for(int i = 0; i < tune.mtANumBlks; i++)
{
    cout << " tune.mtAGen[" << i << "] = ";
    cout << tune.mtAGen[i] << endl;
}
#ifdef SINGLEPRECISION
    naggpuRandNormalA_sp(N2, NAGGPURANDORDER_CONSISTENT, mu, sigma, d_buff + N/2, &tune, 0, &comm, &error);
#endif
# else
    naggpuRandNormalA(N2, NAGGPURANDORDER_CONSISTENT, mu, sigma,
      d_buff + N/2, &tune, 0, &comm, &error);
# endif
checkNagError(&error);
delete[] tune.mtAGen;

// Print out the tuning parameters used
cout << "Tuning Parameters used: " << endl;
printNagTuningParamsUsed(&comm);

// Copy back values from the GPU for printing
cuError = cudaMemcpy(h_buff, d_buff, sizeof(FP)*N,
      cudaMemcpyDeviceToHost);
checkCudaError(cuError);

// Print random numbers
cout << "The first 50 GPU random numbers:" << endl;
cout.setf(ios::fixed, ios::floatfield);
cout.precision(3);
for(int row = 0; row < 10; row++)
  {
    for(int col = 0; col < 5; col++)
      {
        cout << h_buff[row*10 + col] << "\t";
      }
    cout << endl;
  }

// Call cleanup for the NAG routine
naggpuRandCleanupA(&comm, &error);
checkNagError(&error);

// Free CPU and GPU memory
delete[] h_buff;
if (d_buff)
  {
    cuError = cudaFree(d_buff);
    checkCudaError(cuError);
  }
return 0;

void checkNagError(NagGpuError *error)
{
      if (error->code != 0)
      {
        char *buff;
        buff = new char[error->msgLength];
        naggpuErrorCopyMsg(buff, error);
        cout << buff << endl;
        delete[] buff;
        exit(1);
      }
}

void printNagTuningParamsUsed(NagGpuRandComm *comm)
{
      cout << " comm.tuneOrigin = ";
      switch (comm->tuneOrigin)
case NAGGPUTUNEORIGIN NA:
    cout << "NAGGPUTUNEORIGIN NA";
    break;
case NAGGPUTUNEORIGIN_DEFAULT:
    cout << "NAGGPUTUNEORIGIN_DEFAULT";
    break;
case NAGGPUTUNEORIGIN_USER:
    cout << "NAGGPUTUNEORIGIN_USER";
    break;
case NAGGPUTUNEORIGIN_AUTO:
    cout << "NAGGPUTUNEORIGIN_AUTO";
    break;
default:
    cout << "Unrecognised tuneOrigin";
}
cout << endl;
cout << " comm.tuneParamsUsed->mtANumBlks = ";
cout << comm->tuneParamsUsed->mtANumBlks << endl;
for(int i = 0; i < comm->tuneParamsUsed->mtANumBlks; i++)
{
    cout << " comm.tuneParamsUsed->mtAGen[ " << i << " ] = ";
    cout << comm->tuneParamsUsed->mtAGen[i] << endl;
}
cout << endl;

void checkCudaError(cudaError_t cuError)
{
    if (cuError != cudaSuccess)
    {
        cout << cudaGetErrorString(cuError) << endl;
        exit(1);
    }
}

7.2 Program Data
None.

7.3 Program Results
NAG GPU Example Program: naggpuRandNormalA_sp

Initialising generator...
Generate with default tuning parameters...
Tuning Parameters used:
comm.tuneOrigin = NAGGPUTUNEORIGIN_DEFAULT
comm.tuneParamsUsed->mtANumBlks = 18
comm.tuneParamsUsed->mtAGen[0] = 15000
comm.tuneParamsUsed->mtAGen[1] = 28528
comm.tuneParamsUsed->mtAGen[7] = 28528
comm.tuneParamsUsed->mtAGen[8] = 28528
comm.tuneParamsUsed->mtAGen[9] = 28528
comm.tuneParamsUsed->mtAGen[10] = 28528
comm.tuneParamsUsed->mtAGen[12] = 28528
comm.tuneParamsUsed->mtAGen[14] = 28528
comm.tuneParamsUsed->mtAGen[16] = 28528
comm.tuneParamsUsed->mtAGen[17] = 28552
Generate with user supplied tuning parameters...

Tuning Parameters supplied:

tune.mtANumBlks = 20

tune.mtAGen[0] = 25000

tune.mtAGen[1] = 25000

tune.mtAGen[2] = 25000

tune.mtAGen[3] = 25000

tune.mtAGen[4] = 25000

tune.mtAGen[5] = 25000

tune.mtAGen[6] = 25000

tune.mtAGen[7] = 25000

tune.mtAGen[8] = 25000

tune.mtAGen[9] = 25000

tune.mtAGen[10] = 25000


tune.mtAGen[12] = 25000

tune.mtAGen[13] = 25000

tune.mtAGen[14] = 25000

tune.mtAGen[15] = 25000

tune.mtAGen[16] = 25000

tune.mtAGen[17] = 25000

tune.mtAGen[18] = 25000

tune.mtAGen[19] = 25001

Tuning Parameters used:

comm.tuneOrigin = NAGGPUTUNEORIGIN_USER

comm.tuneParamsUsed->mtANumBlks = 20

comm.tuneParamsUsed->mtAGen[0] = 25000

comm.tuneParamsUsed->mtAGen[1] = 25000


comm.tuneParamsUsed->mtAGen[7] = 25000

comm.tuneParamsUsed->mtAGen[8] = 25000

comm.tuneParamsUsed->mtAGen[9] = 25000

comm.tuneParamsUsed->mtAGen[10] = 25000


comm.tuneParamsUsed->mtAGen[12] = 25000


comm.tuneParamsUsed->mtAGen[14] = 25000


comm.tuneParamsUsed->mtAGen[16] = 25000

comm.tuneParamsUsed->mtAGen[17] = 25000

comm.tuneParamsUsed->mtAGen[18] = 25000

comm.tuneParamsUsed->mtAGen[19] = 25001

The first 50 GPU random numbers:

0.300 -0.090 -1.365 0.532 -0.478
1.442 -0.657 -0.683 -0.370 0.298
1.323 0.447 0.226 -1.014 -0.255
0.045 -0.496 -0.693 -0.480 -0.360
0.519 1.637 0.333 0.148 0.207
-3.021 0.040 1.690 -0.718 -0.160
0.645 -0.865 -1.703 1.592 -0.573
0.406 0.276 0.222 1.039 0.295
0.130 1.745 0.592 0.797 0.505
-0.271 -0.619 -2.377 -0.023 0.258
naggpuRandUniformA

1 Purpose

naggpuRandUniformA generates \( n \) values \( X_i \) from a uniform distribution over the interval \([a, b]\) for specified constants \( a \) and \( b \).

The initialization function naggpuRandInitA must be called prior to the first call to naggpuRandUniformA. Thereafter, this function may be called repeatedly to generate additional sets of random values. Once all desired values have been obtained, the function naggpuRandCleanupA must be called to free allocated system resources.

Note: To obtain the same values from naggpuRandUniformA as from the function \texttt{nag_gpu_mrg32-k3a_uniform(nb, nt, np, d_P)} in release 0.3 of the library, please see Section 2.1.1.1 in the Random Number Generators Chapter Introduction.

2 Specification

```c
#include <nag_gpu.h>

extern "C"
 cudaError_t naggpuRandUniformA_sp(int n, NagGpuRandOrder order, float a, float b,
      float *d_buff, const NagGpuRandTune *tune, cudaStream_t custream,
      NagGpuRandComm *comm, NagGpuError *error)

extern "C"
 cudaError_t naggpuRandUniformA(int n, NagGpuRandOrder order, double a, double b,
      double *d_buff, const NagGpuRandTune *tune, cudaStream_t custream,
      NagGpuRandComm *comm, NagGpuError *error)
```

3 Description

If \( a = 0 \) and \( b = 1 \), this function returns the next \( n \) values \( Y_i \) from a uniform \([0, 1]\) generator. For other values of \( a \) and \( b \), the function applies the transformation

\[
X_i = a + \frac{(b - a)}{(b - a)}Y_i
\]

3.1 Synchronization

This function is non-blocking. Control will return immediately to the calling program while the computation is executed on the GPU. The user is responsible for synchronization between host and GPU code. Please see the synchronization chapter in the CUDA Programming Guide for further details in this direction. For example, a call to \texttt{cudaMemcpy} in the CUDA runtime library is enough to force the host to wait for the GPU to finish, and then copy the results from the GPU to the host.

3.2 Return Value

Any CUDA runtime errors that were encountered, or \texttt{cudaSuccess} if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References

None.
5 Arguments

1: \( n \) – int

\emph{Input}

On entry: the number of random values to be generated.

Constraint: \( n \geq 1 \).

2: \( \text{order} \) – NagGpuRandOrder

\emph{Input}

On entry: the ordering to be observed by the underlying GPU generator:

\[ \text{order} = \text{NAGGPURANDORDER\_OPTIMAL} \]

\[ \text{order} = \text{NAGGPURANDORDER\_CONSISTENT} \]

See NagGpuRandOrder for further details.

Constraint:

\[ \text{order} = \text{NAGGPURANDORDER\_OPTIMAL} \text{ or } \text{NAGGPURANDORDER\_CONSISTENT}. \]

3: \( a \) – float

\emph{Input}

4: \( a \) – double

\emph{Input}

This parameter has type float or double depending on whether the single or double precision version of this function is called.

On entry: The lower bound for the uniform random values.

5: \( b \) – float

\emph{Input}

6: \( b \) – double

\emph{Input}

This parameter has type float or double depending on whether the single or double precision version of this function is called.

On entry: The upper bound for the uniform random values.

Constraint: \( b > a \).

7: \( d\_\text{buff}[n] \) – float *

\emph{Output}

8: \( d\_\text{buff}[n] \) – double *

\emph{Output}

This parameter has type float or double depending on whether the single or double precision version of this function is called.

This buffer must reside in the GPU memory space.

On exit: the \( n \) pseudorandom numbers from the specified distribution. The output tuning structure \( \text{comm} \rightarrow \text{tuneParamsUsed} \) will contain the parameters used to launch the kernel. If \( \text{order} = \text{NAGGPURANDORDER\_OPTIMAL} \), these parameters may determine the output ordering (see NagGpuRandTune for details).

9: \( \text{tune} \) – const NagGpuRandTune *

\emph{Input}

This parameter is optional and may be set to NULL.

On entry: if specified, points to a NagGpuRandTune structure containing launch parameters for the selected GPU kernel. Upon a successful return from this function, the relevant data will be copied to the output tuning structure \( \text{comm} \rightarrow \text{tuneParamsUsed} \). Please see NagGpuRandTune for additional information about performance tuning.

10: \( \text{custream} \) – cudaStream_t

\emph{Input}

On entry: specifies the CUDA stream on which to launch the selected GPU kernel. If no streams are used, set this parameter to 0. Please see the chapter on Streams in the CUDA Programming Guide for further details.
11: **comm** – NagGpuRandComm *

NagGpuRandComm is a structure which holds state and communication information and must not be modified in any way. Once all required points have been obtained, **comm** must be passed to naggpuRandCleanupA to free allocated system resources.

Upon successful return from this function, the launch configurations applied to the underlying GPU kernels may be observed through **comm**. This will typically only be of interest to users wanting to fine tune the performance of this function. Please see NagGpuRandTune for details on performance tuning, and consult the NagGpuRandComm documentation for how to observe the launch parameters. Note that these parameters are no longer observable after calling naggpuRandCleanupA.

12: **error** – NagGpuError *

This parameter contains error information and should not be modified directly. Errors are indicated through the value of **error** → **code** which should be inspected after each call to this function. If **error** → **code** = 0 then no error occurred. If **error** → **code** ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

**error** → **code** = 1

*On entry:* the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

**error** → **code** = 2

*During execution:* a CUDA runtime error was detected.

**error** → **code** = 3

*On entry:* an attempt was made to launch a double precision function on a GPU device that does not support double precision.

**error** → **code** = 100

*On entry:* the value of **comm** is NULL.

**error** → **code** = 101

*On entry:* **comm** has not been initialized, or the internal state of **comm** is corrupted.

**error** → **code** = 110

*On entry:* **n** ≤ 0.

**error** → **code** = 111

*On entry:* **order** does not specify a valid ordering. See NagGpuRandOrder for permitted values.

**error** → **code** = 112

*On entry:* **d_buff** is NULL.

**error** → **code** = 113

*On entry:* **b** ≤ **a**

**error** → **code** = 200

*On entry:* the MRG32k3a base generator is selected (see naggpuRandInitA for details), **order** = NAGGPURANDORDER_CONSISTENT, **tune** ≠ NULL and the kernel launch requires too much shared memory. Try reducing the value of **tune** → mrGConAThdsPerBlk.
error → code = 250

On entry: the MRG32k3a base generator is selected (see naggpuRandInitA for details),
order = NAGGPURANDORDER_OPTIMAL, tune ≠ NULL and
tune → mrgOptAThdsPerBlk is out of bounds. See NagGpuRandTune for further details.

error → code = 251

On entry: the MRG32k3a base generator is selected (see naggpuRandInitA for details),
order = NAGGPURANDORDER_OPTIMAL, tune ≠ NULL and
tune → mrgOptAPtsPerThd is out of bounds. See NagGpuRandTune for further details.

error → code = 252

On entry: the MRG32k3a base generator is selected (see naggpuRandInitA for details),
order = NAGGPURANDORDER_CONSISTENT, tune ≠ NULL and
tune → mrgConAThdsPerBlk is out of bounds. See NagGpuRandTune for further details.

error → code = 253

On entry: the MRG32k3a base generator is selected (see naggpuRandInitA for details),
order = NAGGPURANDORDER_CONSISTENT, tune ≠ NULL and
tune → mrgConAThdsPerBlk is not divisible by W where W = 16 on devices of compute
capability 1.3 or lower and W = 32 otherwise. See NagGpuRandTune for further details.

error → code = 254

On entry: the MRG32k3a base generator is selected (see naggpuRandInitA for details),
order = NAGGPURANDORDER_CONSISTENT, tune ≠ NULL and
tune → mrgConANumLoops is out of bounds. See NagGpuRandTune for further details.

error → code = 300

On entry: the MT19937 base generator is selected (see naggpuRandInitA for details),
tune ≠ NULL and tune → mtANumBlks is out of bounds. See NagGpuRandTune for
further details.

error → code = 301

On entry: the MT19937 base generator is selected (see naggpuRandInitA for details),
tune ≠ NULL and tune → mtAGen is NULL.

error → code = 302

On entry: the MT19937 base generator is selected (see naggpuRandInitA for details),
tune ≠ NULL and tune → mtAGen contains negative entries. See NagGpuRandTune for
further details.

error → code = 303

On entry: the MT19937 base generator is selected (see naggpuRandInitA for details),
tune ≠ NULL and the entries in tune → mtAGen do not sum to n. See NagGpuRandTune for
further details.

7 Example

This example program uses naggpuRandUniformA to print 50 pseudorandom numbers from a uniform
distribution using the MRG32k3a generator. For usage of the MT19937 generator, including tuning
aspects, please see the example program for naggpuRandNormalA.
#include <nag_gpu.h>
// precision defined by Makefile
#endif

void checkNagError(NagGpuError *error);
void printNagTuningParamsUsed(NagGpuRandComm *comm);
void checkCudaError(cudaError_t cuError);

int main(int argc, char **argv)
{
    // host (CPU) storage for generated GPU random numbers
    FP *h_buff = 0;
    // device (GPU) storage for generated random numbers
    FP *d_buff = 0;
    // total number of points to generate
    int N = 1000000;
    // seed variables
    const int seed_length = 6;
    unsigned int seed[seed_length];
    // skip ahead variables
    int a1, a2, b1, b2;
    long long c;
    // distribution parameters
    FP a = 0.0;
    FP b = 1.0;
    // NAG GPU structures
    NagGpuRandComm comm;
    NagGpuRandTune tune;
    NagGpuError error;
    cudaMemcpy_t cuError;
    // Print the title
    cout << "NAG GPU Example Program: naggpuRandUniformA";
    cout << "_sp";
    cout << endl << endl;
    // Allocate CPU and GPU memory
    h_buff = new FP[N];
    cuError = cudaMalloc((void **)&d_buff, sizeof(FP)*N);
    checkCudaError(cuError);
    // Initialise the generator only once
cout << "Initialising generator..." << endl << endl;

// arbitrary seed and skip ahead parameters
for (int i = 0; i < seed_length; i++) seed[i] = i;
a1 = 14;
b1 = 34;
a2 = 2;
b2 = 21;
c = 123;

naggpuRandInitA(NAGGPURANDGEN_MRG32K3A, a1, b1, a2, b2, c, seed, &comm, &error);
checkNagError(&error);

// Generate N/2 numbers using default tuning parameters
cout << "Generate with default tuning parameters..." << endl;
#ifdef SINGLEPRECISION
    naggpuRandUniformA_sp(N/2, NAGGPURANDORDER_CONSISTENT, a, b, d_buff, NULL, 0, &comm, &error);
#else
    naggpuRandUniformA(N/2, NAGGPURANDORDER_CONSISTENT, a, b, d_buff, NULL, 0, &comm, &error);
#endif
checkNagError(&error);

// Print out the tuning parameters used
cout << "Tuning Parameters used: " << endl;
printNagTuningParamsUsed(&comm);

// Generate (N- N/2) numbers using specified tuning parameters
cout << "Generate with user supplied tuning parameters..." << endl;
tune.mrgConAThdsPerBlk = 96;
tune.mrgConANumLoops = 30;

#ifdef SINGLEPRECISION
    naggpuRandUniformA_sp(N - N/2, NAGGPURANDORDER_CONSISTENT, a, b, d_buff + N/2, &tune, 0, &comm, &error);
#else
    naggpuRandUniformA(N - N/2, NAGGPURANDORDER_CONSISTENT, a, b, d_buff + N/2, &tune, 0, &comm, &error);
#endif
checkNagError(&error);

// Print out the tuning parameters used
cout << "Tuning Parameters used: " << endl;
printNagTuningParamsUsed(&comm);

// Copy back values from the GPU for printing
cuError = cudaMemcpy(h_buff, d_buff, sizeof(FP)*N, cudaMemcpyDeviceToHost);
checkCudaError(cuError);

// Print random numbers
cout << "The first 50 GPU random numbers:" << endl;
cout.setf(ios::fixed, ios::floatfield);
cout.precision(3);
for(int row = 0; row < 10; row++)
{
    for(int col = 0; col < 5; col++)
    {
        cout << h_buff[row*10 + col] << "\t";
    }
}
cout << endl;

// Call cleanup for the NAG routine
naggpuRandCleanupA(&comm, &error);
checkNagError(&error);

// Free CPU and GPU memory
delete[] h_buff;
if (d_buff)
{
    cuError = cudaFree(d_buff);
    checkCudaError(cuError);
}
return 0;

void checkNagError(NagGpuError *error)
{
    if (error->code != 0)
    {
        char *buff;
        buff = new char[error->msgLength];
naggpuErrorCopyMsg(buff, error);
cout << buff << endl;
delete[] buff;
exit(1);
    }
}

void printNagTuningParamsUsed(NagGpuRandComm *comm)
{
    cout << " comm.tuneOrigin = ";
switch (comm->tuneOrigin)
    {
    case NAGGPUTUNEORIGIN NA:
        cout << "NAGGPUTUNEORIGIN NA";
        break;
    case NAGGPUTUNEORIGIN DEFAULT:
        cout << "NAGGPUTUNEORIGIN DEFAULT";
        break;
    case NAGGPUTUNEORIGIN USER:
        cout << "NAGGPUTUNEORIGIN USER";
        break;
    case NAGGPUTUNEORIGIN AUTO:
        cout << "NAGGPUTUNEORIGIN AUTO";
        break;
    default:
        cout << "Unrecognised tuneOrigin";
    }
cout << endl;

cout << " comm.tuneParamsUsed->mrgConAThdsPerBlk = ";
cout << comm->tuneParamsUsed->mrgConAThdsPerBlk << endl;
cout << " comm.tuneParamsUsed->mrgConANumLoops = ";
cout << comm->tuneParamsUsed->mrgConANumLoops << endl;
cout << endl;
}

void checkCudaError(cudaError_t cuError)
{
    if (cuError != cudaSuccess)
7.2 Program Data

None.

7.3 Program Results

NAG GPU Example Program: naggpuRandUniformA_sp

Initialising generator...

Generate with default tuning parameters...
Tuning Parameters used:
  comm.tuneOrigin = NAGPUTUNEORIGIN_DEFAULT
  comm.tuneParamsUsed->mrgConAThdsPerBlk = 32
  comm.tuneParamsUsed->mrgConANumLoops = 8

Generate with user supplied tuning parameters...
Tuning Parameters supplied:
  tune.mrgConAThdsPerBlk = 96
  tune.mrgConANumLoops = 30
Tuning Parameters used:
  comm.tuneOrigin = NAGPUTUNEORIGIN_USER
  comm.tuneParamsUsed->mrgConAThdsPerBlk = 96
  comm.tuneParamsUsed->mrgConANumLoops = 30

The first 50 GPU random numbers:
0.357 0.958 0.262 0.649 0.703
0.240 0.515 0.307 0.103 0.732
0.982 0.288 0.895 0.511 0.262
0.704 0.398 0.634 0.518 0.559
0.390 0.790 0.670 0.703 0.603
0.206 0.645 0.111 0.595 0.938
0.761 0.896 0.069 0.551 0.405
0.484 0.652 0.148 0.298 0.712
0.860 0.133 0.371 0.706 0.005
0.377 0.227 0.074 0.861 0.195
1 Purpose
naggpuRandCleanupA frees system resources that were allocated by a previous call to naggpuRandInitA.

2 Specification
#include <nag_gpu.h>
extern "C"
cudaError_t naggpuRandCleanupA(NagGpuRandComm *comm, NagGpuError *error)

3 Description
3.1 Synchronization
This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.

3.2 Return Value
Any CUDA runtime errors that were encountered, or cudaMemcpy if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References
None.

5 Arguments
1: comm – NagGpuRandComm * Communication Data
   On entry: the pointer that was passed to a previous call to naggpuRandInitA.

2: error – NagGpuError * Error Reporting
   This parameter contains error information and should not be modified directly. Errors are indicated through the value of error → code which should be inspected after each call to this function. If error → code = 0 then no error occurred. If error → code ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings
error → code = 1
   On entry: the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

error → code = 2
   During execution: a CUDA runtime error was detected.
error → code = 100

On entry: the value of comm is NULL.

error → code = 101

On entry: comm has not been initialized, or the internal state of comm is corrupted.

7 Example

There is no example program specifically for this function. For examples of how this function should be used, please see the example program for naggpuRandUniformA.
naggpuQuasiRandInitA initializes a GPU quasi-random number generator. This function must be called before any call to the GPU generator functions (such as naggpuQuasiRandUniformA) and must ultimately be followed by a call to the cleanup function naggpuQuasiRandCleanupA to release system resources.

2 Specification

#include <nag_gpu.h>
extern "C"
cudaError_t naggpuQuasiRandInitA(NagGpuQuasiGen genid, NagGpuScramTypes stype,
int dim, int offset, NagCPURandComm *pseudoComm, NagGpuQuasiRandComm *comm,
NagGpuError *error)

3 Description

Low discrepancy (quasi-random) sequences are used in numerical integration, simulation and optimization. Like pseudorandom numbers they are uniformly distributed, but they are not statistically independent. Quasi-random sequences are designed to give a more even distribution in multidimensional space (uniformity), and are often more efficient than pseudorandom numbers in multidimensional Monte Carlo methods.

Let \( x_1, x_2, \ldots, x_N \) be a sequence of \( d \)-dimensional points in the unit cube \( I^d = [0,1]^d \). Let \( G \) be a subset of \( I^d \) and define the counting function \( S_N(G) \) as the number of \( d \)-dimensional points \( x \in G \). For each point \( x = (x_1, x_2, \ldots, x_d) \in I^d \), let \( G_x \) be the rectangular \( d \)-dimensional region
\[
G_x = [0, x_1] \times [0, x_2] \times \cdots \times [0, x_d)
\]
with volume \( x_1 \cdot x_2 \cdot \cdots \cdot x_d = \prod_{i=1}^d x_i \). Then one measure of the uniformity of the points \( x_1, x_2, \ldots, x_N \) is the so-called star discrepancy:
\[
D^*_N(x_1, x_2, \ldots, x_N) = \sup_{x \in I^d} \left| S_N(G_x) - \frac{N}{d!} \right|
\]
which satisfies the inequality
\[
D^*_N(x_1, x_2, \ldots, x_N) \leq C_d (\log N)^d + O\left((\log N)^{d-1}\right) \quad \text{for all } \quad N \geq 2.
\]
The principal aim in the construction of low-discrepancy sequences is to find sequences of points in \( I^d \) with a bound of this form where the constant \( C_d \) is as small as possible.

The type of low-discrepancy sequence generated by naggpuQuasiRandInitA depends on the value of \texttt{genid}, and the sequence can optionally be scrambled through the parameter \texttt{stype}. See NagGpuQuasiGen and NagGpuScramTypes respectively for further information.

3.1 Synchronization

This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.

3.2 Return Value

Any CUDA runtime errors that were encountered, or \texttt{cudaSuccess} if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.
4 References

None.

5 Arguments

1: genid – NagGpuQuasiGen

*Input*

On entry: the type of generator to be used:

\( \text{genid} = \text{NAGGPUQUASIGEN\_SOBOL} \)

Constraint: \( \text{genid} = \text{NAGGPUQUASIGEN\_SOBOL} \).

2: stype – NagGpuScramTypes

*Input*

On entry: the type of scrambling to be used:

\( \text{stype} = \text{NAGGPUSCRAMTYPES\_NONE} \)
\( \text{stype} = \text{NAGGPUSCRAMTYPES\_OWEN} \)
\( \text{stype} = \text{NAGGPUSCRAMTYPES\_FAURE\_TEZUKA} \)
\( \text{stype} = \text{NAGGPUSCRAMTYPES\_OWEN\_FAURE\_TEZUKA} \)

Please see NagGpuScramTypes for some of the benefits of scrambling and details about each of available scrambling types.

Constraint: \( \text{stype} = \text{NAGGPUSCRAMTYPES\_NONE} \) or \( \text{NAGGPUSCRAMTYPES\_OWEN} \) or \( \text{NAGGPUSCRAMTYPES\_FAURE\_TEZUKA} \) or \( \text{NAGGPUSCRAMTYPES\_OWEN\_FAURE\_TEZUKA} \).

3: dim – int

*Input*

On entry: the dimension of the quasi-random sequence.

Constraint: \( 1 \leq \text{dim} \leq 50000 \).

4: offset – int

*Input*

On entry: the offset into the sequence at which to start generating.

Constraint: \( \text{offset} \geq 0 \).

5: pseudoComm – NagCPURandComm *

*Input*

On entry: a pointer to a NagCPURandComm structure which has already been initialized by the function nagCPURandInitA.

Constraint: \( \text{pseudoComm} \) must be initialized before being passed to this function.

6: comm – NagGpuQuasiRandComm *

*Communication Data*

NagGpuQuasiRandComm is a structure which holds state and communication information and must not be modified in any way. The structure will be initialized and must be passed to the generator functions (such as naggpuQuasiRandUniformA). Once all required points have been obtained, \( \text{comm} \) must be passed to naggpuQuasiRandCleanupA to free allocated system resources.

7: error – NagGpuError *

*Error Reporting*

This parameter contains error information and should not be modified directly. Errors are indicated through the value of \( \text{error} \rightarrow \text{code} \) which should be inspected after each call to this function. If \( \text{error} \rightarrow \text{code} = 0 \) then no error occurred. If \( \text{error} \rightarrow \text{code} \neq 0 \) then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error.

Please see the Error Handling Chapter Introduction for further details on error handling.
6 Error Indicators and Warnings

error → code = 1

On entry: the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

error → code = 2

During execution: a CUDA runtime error was detected.

error → code = 100

On entry: the value of comm is NULL.

error → code = 110

On entry: genid does not specify a valid quasi-random number generator. See NagGpuQuasiGen for permitted values.

error → code = 111

On entry: stype does not specify a valid scrambling type. See NagGpuScramTypes for permitted values.

error → code = 112

On entry: the value of dim does not satisfy the constraint listed above.

error → code = 113

On entry: the value of offset is negative.

error → code = 114

On entry: the value of pseudoComm is NULL.

error → code = 115

On entry: the pseudorandom generator nagCPURandUniformA returned an error when called by this function: pseudoComm is not initialized, or the internal state of pseudoComm is corrupted.

7 Example

There is no example program specifically for this function. For examples of how this function should be used, please see the example program for naggpuQuasiRandUniformA.
naggpuQuasiRandExpA

1 Purpose

naggpuQuasiRandExpA generates \( n \) points \( x_i \) from a quasi-random exponential distribution with mean \( \lambda \).

The initialization function naggpuQuasiRandInitA must be called prior to the first call to naggpuQuasiRandExpA. Thereafter, this function may be called repeatedly to generate additional sets of quasi-random points. Once all desired points have been obtained, the function naggpuQuasiRandCleanupA must be called to free allocated system resources.

Note: Concerns were raised about the set of Sobol’ direction numbers that were used in release 0.3 of the NAG Numerical Routines for GPUs. These concerns have been addressed by an amended set of direction numbers in Joe and Kuo (2008) which are used in this release. Consequently, the higher dimensions of this Sobol’ generator may not match the higher dimensions of the generator in release 0.3 since the direction numbers are different.

2 Specification

```c
#include <nag_gpu.h>
extern "C"
cudaError_t naggpuQuasiRandExpA_sp(int n, NagGpuQuasiOrient orient,
    float lambda, float *d_buff, const NagGpuQuasiRandTune *tune,
    cudaStream_t custream, NagGpuQuasiRandComm *comm, NagGpuError *error)
```

```c
extern "C"
cudaError_t naggpuQuasiRandExpA(int n, NagGpuQuasiOrient orient, double lambda,
    double *d_buff, const NagGpuQuasiRandTune *tune, cudaStream_t custream,
    NagGpuQuasiRandComm *comm, NagGpuError *error)
```

3 Description

Quasi-random sequences are made up of one or more multidimensional points, with each point composed of several one dimensional values. The dimensionality of the sequence is specified when calling naggpuQuasiRandInitA to initialize the generator. Below we will consider a \( d \)-dimensional quasi-random sequence \( x^0, x^1, \ldots \) so that each point \( x^j = (x^j_1, x^j_2, \ldots, x^j_d) \) is composed of \( d \) one dimensional values.

The exponential distribution has probability density function given by

\[
f(x) = \begin{cases} \frac{1}{\lambda} e^{-x/\lambda} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}
\]

where \( \lambda > 0 \). This function returns the next \( n \) points \( x^j = (x^j_1, x^j_2, \ldots, x^j_d) \) for \( j = 1, 2, \ldots, n \) where

\[
x^j_i = -\lambda \ln(y^j_i + 2^{-32})
\]

for each \( i = 1, 2, \ldots, d \). Here \( y^j = (y^j_1, y^j_2, \ldots, y^j_d) \in [0, 1)^d \) are the next \( n \) points from the quasi-random generator.

3.1 Synchronization

This function is non-blocking. Control will return immediately to the calling program while the computation is executed on the GPU. The user is responsible for synchronization between host and GPU code. Please see the synchronization chapter in the CUDA Programming Guide for further details in this
direction. For example, a call to cudaMemcpy in the CUDA runtime library is enough to force the host to wait for the GPU to finish, and then copy the results from the GPU to the host.

### 3.2 Return Value

Any CUDA runtime errors that were encountered, or cudaMemcpy if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

### 4 References


### 5 Arguments

1: \( n \) – int

*Input*

*On entry:* the number of quasi-random points to be generated.

*Constraint:* \( n \geq 1 \).

2: orient – NagGpuQuasiOrient

*Input*

*On entry:* specifies the orientation with which the generator will store the output points. Currently only NAGGPUQUASIORIENT_DIMVALS_SCATT is supported. See NagGpuQuasiOrient for further details on output orientation.

*Constraint:* \( \text{orient} = \text{NAGGPUQUASIORIENT\_DIMVALS\_SCATT} \).

3: \( \lambda \) – float

*Input*

This parameter has type float or double depending on whether the single or double precision version of this function is called.

*On entry:* The mean, \( \lambda \), of the exponential distribution.

*Constraint:* \( \lambda > 0 \).

4: \( \lambda \) – double

*Input*

This parameter has type float or double depending on whether the single or double precision version of this function is called.

5: \( d \) – float *

*Output*

This parameter has type float or double depending on whether the single or double precision version of this function is called.

This buffer must reside in the GPU memory space.

The value \( d \) is the dimension \( \text{dim} \) of the sequence as specified to the initialization function naggpuQuasiRandInitA.

*On exit:* the \( n \) quasi-random points from the specified distribution. For a given point, the individual dimension values are not stored consecutively in memory. The \( i \)-th dimension \( x_i^j \) of the \( j \)-th quasi-random point will be stored at location \( d \) \( [(i - 1) * n + j] \) for every \( 0 \leq j < n \) and \( 1 \leq i \leq d \). In other words, the first \( n \) values correspond to dimension 1, the second \( n \) to dimension 2, and so on.

6: \( d \) – double *

*Output*

7: tune – const NagGpuQuasiRandTune *

*Input*

This parameter is optional and may be set to NULL.

*On entry:* if specified, points to a NagGpuQuasiRandTune structure containing launch parameters for the selected GPU kernel. Please see NagGpuQuasiRandTune for additional information about performance tuning.
8: **custream** – cudaStream_t

*Input*

On entry: specifies the CUDA stream on which to launch the selected GPU kernel. If no streams are used, set this parameter to 0. Please see the chapter on Streams in the CUDA Programming Guide for further details.

9: **comm** – NagGpuQuasiRandComm *

*Communication Data*

NagGpuQuasiRandComm is a structure which holds state and communication information and must not be modified in any way. Once all required points have been obtained, **comm** must be passed to naggpuQuasiRandCleanupA to free allocated system resources.

Upon successful return from this function, the launch configurations applied to the underlying GPU kernels may be observed through **comm**. This will typically only be of interest to users wanting to fine tune the performance of this function. Please see NagGpuQuasiRandTune for details on performance tuning, and consult the NagGpuQuasiRandComm documentation for how to observe the launch parameters. Note that these parameters are no longer observable after calling naggpuQuasiRandCleanupA.

10: **error** – NagGpuError *

*Error Reporting*

This parameter contains error information and should not be modified directly. Errors are indicated through the value of **error** → **code** which should be inspected after each call to this function. If **error** → **code** = 0 then no error occurred. If **error** → **code** ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 **Error Indicators and Warnings**

**error** → **code** = 1

*On entry*: the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

**error** → **code** = 2

*During execution*: a CUDA runtime error was detected.

**error** → **code** = 3

*On entry*: an attempt was made to launch a double precision function on a GPU device that does not support double precision.

**error** → **code** = 100

*On entry*: the value of **comm** is NULL.

**error** → **code** = 101

*On entry*: **comm** has not been initialized, or the internal state of **comm** is corrupted.

**error** → **code** = 110

*On entry*: n ≤ 0.

**error** → **code** = 111

*On entry*: **orient** does not satisfy the constraint listed above.

**error** → **code** = 112

*On entry*: **d_buff** is NULL.

**error** → **code** = 114

*On entry*: lambda ≤ 0.
error → code = 250
On entry: the Sobol' base generator is selected (see naggpuQuasiRandInitA for details),
tune ≠ NULL and tune → sblAThdsPerBlk is out of bounds. See NagGpuQuasiRandTune
for further details.

error → code = 251
On entry: the Sobol' base generator is selected (see naggpuQuasiRandInitA for details),
tune ≠ NULL and tune → sblAThdsPerBlk is not a power of two.

error → code = 252
On entry: the Sobol' base generator is selected (see naggpuQuasiRandInitA for details),
tune ≠ NULL and tune → sblABlksPerDim is out of bounds. See NagGpuQuasiRandTune
for further details.

error → code = 253
On entry: the Sobol' base generator is selected (see naggpuQuasiRandInitA for details),
tune ≠ NULL and tune → sblABlksPerDim is not a power of two.

7 Example
This example program uses naggpuQuasiRandExpA to print 5 quasi-random numbers of dimension 10
from an exponential distribution. The first point in the sequence is skipped and generation starts at the
second point.

7.1 Program Text
/*
 * Example Program: naggpuQuasiRandExpA
 * Copyright 2009, Numerical Algorithms Group Ltd, Oxford, UK.
 * Version 0.4, 2011.
 */

#include
using namespace std;
#include <nag_gpu.h>
#include <nag_gpu_serial.h>

// precision defined by Makefile
#ifndef SINGLEPRECISION
#define FP float
#else
#define FP double
#endif

void checkNagError(NagGpuError *error);
void printNagTuningParamsUsed(NagGpuQuasiRandComm *comm);
void checkCudaError(cudaError_t cuError);

int main(int argc, char **argv)
{   // host (CPU) storage for generated GPU random numbers
    FP *h_buff = 0;
    // device (GPU) storage for generated random numbers
    FP *d_buff = 0;

    // number of points to generate
    int n = 100000;
    int dim = 100;

    int offset = 1;
unsigned int pseudoSeed[] = {1, 2, 3, 4, 5, 6};

// distribution parameter
FP lambda = 1.0;

// NAG GPU structures
NagGpuQuasiRandComm comm;
NagGpuQuasiRandTune tune;
NagCPURandComm pseudoComm;
NagGpuError error;
NagGpuQuasiOrient orient = NAGGPUQUASIORIENT_DIMVALS_SCATT;

cuError_t cuError;

// Print the title
cout << "NAG GPU Example Program: ";
cout << "naggpuQuasiRandExpA";
if (sizeof(FP)==sizeof(float)) cout << "_sp";
cout << endl << endl;

// Allocate CPU and GPU memory
h_buff = new FP[n*dim];
cuError = cudaMalloc((void **)&d_buff, sizeof(FP)*n*dim);
checkCudaError(cuError);

// Initialise the CPU pseudo-random generator
nagCPURandInitA(NAGGPURANDGEN_MRG32K3A, 0, 0, 0, 0, 0, pseudoSeed,
&pseudoComm, &error);
checkNagError(&error);

// Initialise the generator only once
cout << "Initialising generator..." << endl << endl;
naggpuQuasiRandInitA(NAGGPUQUASIGEN_SOBOL, NAGGPU_SCRAMTYPES_NONE,
    dim, offset, &pseudoComm, &comm, &error);
checkNagError(&error);

// Generate N numbers using default tuning parameters
if (sizeof(FP)==sizeof(float))
    naggpuQuasiRandExpA_sp(n, orient, lambda, d_buff, NULL, 0, &comm, &error);
else
    naggpuQuasiRandExpA(n, orient, lambda, d_buff, NULL, 0, &comm, &error);
checkNagError(&error);

// Print out the tuning parameters used
cout << "Tuning Parameters used: " << endl;
printNagTuningParamsUsed(&comm);

// Copy back values from the GPU for printing
cuError = cudaMemcpy(h_buff, d_buff, sizeof(FP)*n*dim,
    cudaMemcpyDeviceToHost);
checkCudaError(cuError);

// Print random numbers
cout << "The 5 GPU numbers from dimensions 1 to 10:" << endl;
cout.setf(ios::fixed,ios::floatfield);
cout.precision(3);
for(int d = 0; d < 10; d++)
    {cout << "dim" << d+1 << "\t";
        for(int i = 0; i < 5; i++)
            cout << h_buff[d+i] << "\t";
    }


```c++
{
  cout << h_buff[n*d + i] << "\t";
}
cout << endl;

// Generate n numbers using specified tuning parameters
cout << "Generate with user supplied tuning parameters..." << endl;
tune.sblAThdsPerBlk = 32;
tune.sblABlksPerDim = 8;
cout << "Tuning Parameters supplied: " << endl;
cout << "tune.sblAThdsPerBlk = " << tune.sblAThdsPerBlk << endl;
cout << "tune.sblABlksPerDim = " << tune.sblABlksPerDim << endl;
#endif SINGLEPRECISION
naggpuQuasiRandExpA_sp(n, orient, lambda, d_buff, &tune, 0, &comm, &error);
#else
naggpuQuasiRandExpA(n, orient, lambda, d_buff, &tune, 0, &comm, &error);
#endif
checkNagError(&error);

// Print out the tuning parameters used
cout << "Tuning Parameters used: " << endl;
printNagTuningParamsUsed(&comm);

// Call cleanup for the NAG routine
naggpuQuasiRandCleanupA(&comm, &error);
checkNagError(&error);

// Free CPU and GPU memory
delete[] h_buff;
if (d_buff)
{
  cuError = cudaFree(d_buff);
  checkCudaError(cuError);
}
return 0;
}

void checkNagError(NagGpuError *error)
{
  if (error->code != 0)
  {
    char *buff;
    buff = new char[error->msgLength];
    naggpuErrorCopyMsg(buff, error);
    cout << buff << endl;
    delete[] buff;
    exit(1);
  }
}

void printNagTuningParamsUsed(NagGpuQuasiRandComm *comm)
{
  cout << " comm.tuneOrigin = ";
  switch (comm->tuneOrigin)
  {
  case NAGGPUTUNEORIGIN_NA:
    cout << "NAGGPUTUNEORIGIN_NA";
    break;
  case NAGGPUTUNEORIGIN_DEFAULT:
    cout << "NAGGPUTUNEORIGIN_DEFAULT";
    break;
  }
}
```
case NAGGPUTUNEORIGIN_USER:
    cout << "NAGGPUTUNEORIGIN_USER";  
bread;
    case NAGGPUTUNEORIGIN_AUTO:
    cout << "NAGGPUTUNEORIGIN_AUTO";  
bread;
    default:
        cout << "Unrecognised tuneOrigin";
    }
    cout << endl;
    cout << " comm.tuneParamsUsed->sblAThdsPerBlk = ";
    cout << comm->tuneParamsUsed->sblAThdsPerBlk << endl;
    cout << " comm.tuneParamsUsed->sblABlksPerDim = ";
    cout << comm->tuneParamsUsed->sblABlksPerDim << endl;
    cout << endl;
    void checkCudaError(cudaError_t cuError)
    {
        if (cuError != cudaSuccess)
        {
            cout << cudaGetErrorString(cuError) << endl;
            exit(1);
        }
    }

7.2 Program Data
None.

7.3 Program Results
NAG GPU Example Program: naggpuQuasiRandExpA_sp

Initialising generator...

Generate with default tuning parameters...
Tuning Parameters used:
    comm.tuneOrigin = NAGGPUTUNEORIGIN_DEFAULT
    comm.tuneParamsUsed->sblAThdsPerBlk = 64
    comm.tuneParamsUsed->sblABlksPerDim = 16

The 5 GPU numbers from dimensions 1 to 10:
    dim1 0.693 0.288 1.386 0.981 0.134
    dim2 0.693 1.386 0.288 0.981 0.134
    dim3 0.693 1.386 0.288 0.470 2.079
    dim4 0.693 1.386 0.288 0.134 0.981
    dim5 0.693 0.288 1.386 0.981 0.134
    dim6 0.693 0.288 1.386 2.079 0.470
    dim7 0.693 1.386 0.288 0.981 0.134
    dim8 0.693 0.288 1.386 0.134 0.981
    dim9 0.693 0.288 1.386 0.134 0.981
    dim10 0.693 0.288 1.386 0.470 2.079

Generate with user supplied tuning parameters...
Tuning Parameters supplied:
    tune.sblAThdsPerBlk = 32
    tune.sblABlksPerDim = 8
Tuning Parameters used:
    comm.tuneOrigin = NAGGPUTUNEORIGIN_USER
    comm.tuneParamsUsed->sblAThdsPerBlk = 32
    comm.tuneParamsUsed->sblABlksPerDim = 8
1 Purpose

naggpuQuasiRandNormalA generates $N$ points $x_i$ from a quasi-random Normal distribution with mean $\mu$ and variance $\sigma^2$.

The initialization function naggpuQuasiRandInitA must be called prior to the first call to naggpuQuasiRandNormalA. Thereafter, this function may be called repeatedly to generate additional sets of quasi-random points. Once all desired points have been obtained, the function naggpuQuasiRandCleanupA must be called to free allocated system resources.

Note: Concerns were raised about the set of Sobol’ direction numbers that were used in release 0.3 of the NAG Numerical Routines for GPUs. These concerns have been addressed by an amended set of direction numbers in Joe and Kuo (2008) which are used in this release. Consequently, the higher dimensions of this Sobol’ generator may not match the higher dimensions of the generator in release 0.3 since the direction numbers are different.

2 Specification

#include <nag_gpu.h>

extern "C"
cudaError_t naggpuQuasiRandNormalA_sp(int n, NagGpuQuasiOrient orient, float mu,
       float sigma, float *d_buff, const NagGpuQuasiRandTune *tune,
       cudaStream_t custream, NagGpuQuasiRandComm *comm, NagGpuError *error)

extern "C"
cudaError_t naggpuQuasiRandNormalA(int n, NagGpuQuasiOrient orient, double mu,
       double sigma, double *d_buff, const NagGpuQuasiRandTune *tune,
       cudaStream_t custream, NagGpuQuasiRandComm *comm, NagGpuError *error)

3 Description

Quasi-random sequences are made up of one or more multidimensional points, with each point composed of several one dimensional values. The dimensionality of the sequence is specified when calling naggpuQuasiRandInitA to initialize the generator. Below we will consider a $d$-dimensional quasi-random sequence $x^0, x^1, \ldots$ so that each point $x^j = (x^j_1, x^j_2, \ldots, x^j_d)$ is composed of $d$ one dimensional values.

The Normal distribution has probability density function given by

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right)$$

where $\sigma > 0$ and $\mu \in \mathbb{R}$. This function returns the next $n$ points $x^j = (x^j_1, x^j_2, \ldots, x^j_d)$ for $j = 1, 2, \ldots, n$ where

$$x^j_i = \mu + \sigma \sqrt{2} \text{erfinv}(z^j_i)$$

for each $i = 1, 2, \ldots, d$ and erfinv is the inverse error function. Here each $z^j = (z^j_1, z^j_2, \ldots, z^j_d)$ is a low discrepancy point in the interval $(-1, 1)^d$.

3.1 Synchronization

This function is non-blocking. Control will return immediately to the calling program while the computation is executed on the GPU. The user is responsible for synchronization between host and GPU.
code. Please see the synchronization chapter in the CUDA Programming Guide for further details in this direction. For example, a call to `cudaMemcpy` in the CUDA runtime library is enough to force the host to wait for the GPU to finish, and then copy the results from the GPU to the host.

### 3.2 Return Value

Any CUDA runtime errors that were encountered, or `cudaSuccess` if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

### 4 References


### 5 Arguments

1. n – int
   
   **Input**
   
   *On entry:* the number of quasi-random points to be generated.
   
   **Constraint:** \( n \geq 1 \).

2. orient – NagGpuQuasiOrient
   
   **Input**
   
   *On entry:* specifies the orientation with which the generator will store the output points. Currently only `NAGGPUQUASIORIENT_DIMVALS_SCATT` is supported. See `NagGpuQuasiOrient` for further details on output orientation.
   
   **Constraint:** `orient = NAGGPUQUASIORIENT_DIMVALS_SCATT`.

3. mu – float
4. mu – double
   
   **Input**
   
   *On entry:* the mean, \( \mu \), of the distribution.

5. sigma – float
6. sigma – double
   
   **Input**
   
   *On entry:* the standard deviation, \( \sigma \), of the distribution
   
   **Constraint:** \( \sigma > 0 \).

7. d_buffer[n × d] – float *
8. d_buffer[n × d] – double *
   
   **Output**
   
   *On exit:* the \( n \) quasi-random points from the specified distribution. For a given point, the individual dimension values are not stored consecutively in memory. The \( i \)-th dimension \( x_j^i \) of the \( j \)-th quasi-random point will be stored at location `d_buffer[(i - 1) * n + j]` for every \( 0 \leq j < n \) and \( 1 \leq i \leq d \). In other words, the first \( n \) values correspond to dimension 1, the second \( n \) to dimension 2, and so on.
9: **tune** – const NagGpuQuasiRandTune *  
   *Input*

   This parameter is optional and may be set to NULL.

   **On entry:** if specified, points to a NagGpuQuasiRandTune structure containing launch parameters for the selected GPU kernel. Please see NagGpuQuasiRandTune for additional information about performance tuning.

10: **custream** – cudaStream_t  
   *Input*

   **On entry:** specifies the CUDA stream on which to launch the selected GPU kernel. If no streams are used, set this parameter to 0. Please see the chapter on Streams in the CUDA Programming Guide for further details.

11: **comm** – NagGpuQuasiRandComm *  
   *Communication Data*

   NagGpuQuasiRandComm is a structure which holds state and communication information and must not be modified in any way. Once all required points have been obtained, **comm** must be passed to naggpuQuasiRandCleanupA to free allocated system resources.

   Upon successful return from this function, the launch configurations applied to the underlying GPU kernels may be observed through **comm**. This will typically only be of interest to users wanting to fine tune the performance of this function. Please see NagGpuQuasiRandTune for details on performance tuning, and consult the NagGpuQuasiRandComm documentation for how to observe the launch parameters. Note that these parameters are no longer observable after calling naggpuQuasiRandCleanupA.

12: **error** – NagGpuError *  
   *Error Reporting*

   This parameter contains error information and should not be modified directly. Errors are indicated through the value of **error → code** which should be inspected after each call to this function. If **error → code** = 0 then no error occurred. If **error → code** ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 **Error Indicators and Warnings**

   **error → code** = 1

   **On entry:** the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

   **error → code** = 2

   **During execution:** a CUDA runtime error was detected.

   **error → code** = 3

   **On entry:** an attempt was made to launch a double precision function on a GPU device that does not support double precision.

   **error → code** = 100

   **On entry:** the value of **comm** is NULL.

   **error → code** = 101

   **On entry:** **comm** has not been initialized, or the internal state of **comm** is corrupted.

   **error → code** = 110

   **On entry:** n ≤ 0.

   **error → code** = 111

   **On entry:** **orient** does not satisfy the constraint listed above.
Example

This example program uses naggpuQuasiRandNormalA to print 5 quasi-random numbers of dimension 10 from a Normal distribution. The first point in the sequence is skipped and generation starts at the second point.

Program Text

```c
/*
 * Example Program: naggpuQuasiRandNormalA
 * Copyright 2009, Numerical Algorithms Group Ltd, Oxford, UK.
 * Version 0.4, 2011.
 * */

#include
using namespace std;

#include <nag_gpu.h>
#include <nag_gpu_serial.h>

#define FP float

#include

#precision defined by Makefile
#define SINGLEPRECISION
#define FP float
#else
#define FP double
#endif

int main(int argc, char **argv)
{
    // host (CPU) storage for generated GPU random numbers
    FP *h_buff = 0;
```
// device (GPU) storage for generated random numbers
FP *d_buff = 0;

// number of points to generate
int n = 100000;
int dim = 100;
int offset = 1;
unsigned int pseudoSeed[] = {1, 2, 3, 4, 5, 6};

// distribution parameters
FP mu = 0.0;
FP sigma = 1.0;

// NAG GPU structures
NagGpuQuasiRandComm comm;
NagGpuQuasiRandTune tune;
NagCPURandComm pseudoComm;
NagGpuError error;
NagGpuQuasiOrient orient = NAGGPUQUASIORIENT_DIMVALS_SCATT;
cudaError_t cuError;

// Print the title
cout << "NAG GPU Example Program: ";
cout << "naggpuQuasiRandNormalA";
if (sizeof(FP)==sizeof(float)) cout << "_sp";
cout << endl << endl;

// Allocate CPU and GPU memory
h_buff = new FP[n*dim];
if (cuError = cudaMalloc((void **)&d_buff, sizeof(FP)*n*dim)) {checkCudaError(cuError);}

// Initialise the CPU pseudo-random generator
nagCPURandInitA(NAGGPURANDGEN_MRG32K3A, 0, 0, 0, 0, 0, pseudoSeed, &pseudoComm, &error);
checkNagError(&error);

// Initialise the generator only once
cout << "Initialising generator..." << endl << endl;
naggpuQuasiRandInitA(NAGGPUQUASIGEN_SOBOL, NAGGPUSCRAMTYPES_NONE, dim, offset, &pseudoComm, &comm, &error);
checkNagError(&error);

// Generate n numbers using default tuning parameters
out << "Generate with default tuning parameters..." << endl;
#ifdef SINGLEPRECISION
naggpuQuasiRandNormalA_sp(n, orient, mu, sigma, d_buff, NULL, 0, &comm, &error);
#else
naggpuQuasiRandNormalA(n, orient, mu, sigma, d_buff, NULL, 0, &comm, &error);
#endif
checkNagError(&error);

// Print out the tuning parameters used
cout << "Tuning Parameters used: " << endl;
printNagTuningParamsUsed(&comm);

// Copy back values from the GPU for printing
cuError = cudaMemcpy(h_buff, d_buff, sizeof(FP)*n*dim, cudaMemcpyDeviceToHost);
checkCudaError(cuError);

// Print random numbers
for(int d = 0; d < 10; d++)
{
    cout << "dim" << d+1 << "\t";
    for(int i = 0; i < 5; i++)
    {
        cout << h_buff[n*d + i] << "\t";
    }
    cout << endl;
}

// Generate n numbers using specified tuning parameters
naggpuQuasiRandNormalA_sp(n, orient, mu, sigma, d_buff, &tune, 0, &comm, &error);

// Print out the tuning parameters used
printNagTuningParamsUsed(&comm);

// Call cleanup for the NAG routine
naggpuQuasiRandCleanupA(&comm, &error);
checkNagError(&error);

// Free CPU and GPU memory
delete[] h_buff;
if (d_buff)
{
    cuError = cudaFree(d_buff);
    checkCudaError(cuError);
}
return 0;

void checkNagError(NagGpuError *error)
{
    if (error->code != 0)
    {
        char *buff;
        buff = new char[error->msgLength];
        naggpuErrorCopyMsg(buff, error);
        cout << buff << endl;
        delete[] buff;
        exit(1);
    }
}
void printNagTuningParamsUsed(NagGpuQuasiRandComm *comm)
{
    cout << "  comm.tuneOrigin = ";
    switch (comm->tuneOrigin)
    {
        case NAGGPUTUNEORIGIN_NA:
            cout << "NAGGPUTUNEORIGIN_NA"; break;
        case NAGGPUTUNEORIGIN_DEFAULT:
            cout << "NAGGPUTUNEORIGIN_DEFAULT"; break;
        case NAGGPUTUNEORIGIN_USER:
            cout << "NAGGPUTUNEORIGIN_USER"; break;
        case NAGGPUTUNEORIGIN_AUTO:
            cout << "NAGGPUTUNEORIGIN_AUTO"; break;
        default:
            cout << "Unrecognised tuneOrigin"; break;
    }
    cout << endl;
    cout << "  comm.tuneParamsUsed->sblAThdsPerBlk = ";
    cout << comm->tuneParamsUsed->sblAThdsPerBlk << endl;
    cout << "  comm.tuneParamsUsed->sblABlksPerDim = ";
    cout << comm->tuneParamsUsed->sblABlksPerDim << endl;
}

void checkCudaError(cudaError_t cuError)
{
    if (cuError != cudaSuccess)
    {
        cout << cudaGetErrorString(cuError) << endl;
        exit(1);
    }
}

7.2 Program Data
None.

7.3 Program Results

NAG GPU Example Program: naggpuQuasiRandNormalA_sp

Initialising generator...

Generate with default tuning parameters...
Tuning Parameters used:
    comm.tuneOrigin = NAGGPUTUNEORIGIN_DEFAULT
    comm.tuneParamsUsed->sblAThdsPerBlk = 64
    comm.tuneParamsUsed->sblABlksPerDim = 16

The 5 GPU numbers from dimensions 1 to 10:

    dim1  0.000  0.674 -0.674 -0.319  1.150
    dim2  0.000 -0.674  0.674 -0.319  1.150
    dim3  0.000 -0.674  0.674  0.319 -1.150
    dim4  0.000 -0.674  0.674  1.150 -0.319
    dim5  0.000  0.674 -0.674 -0.319  1.150
    dim6  0.000  0.674 -0.674 -1.150  0.319
    dim7  0.000 -0.674  0.674 -0.319  1.150
    dim8  0.000  0.674 -0.674  1.150 -0.319
    dim9  0.000  0.674 -0.674  1.150 -0.319
    dim10 0.000  0.674 -0.674  0.319 -1.150

Generate with user supplied tuning parameters...
Tuning Parameters supplied:
  tune.sblAThdsPerBlk = 32
  tune.sblABlksPerDim = 8
Tuning Parameters used:
  comm.tuneOrigin = NAGGPUTUNEORIGIN_USER
  comm.tuneParamsUsed->sblAThdsPerBlk = 32
  comm.tuneParamsUsed->sblABlksPerDim = 8
Purpose

naggpuQuasiRandUniformA generates \( n \) points \( x_i \) from a quasi-random uniform distribution over the interval \( [a, b] \) for specified constants \( a \) and \( b \).

The initialization function naggpuQuasiRandInitA must be called prior to the first call to naggpuQuasiRandUniformA. Thereafter, this function may be called repeatedly to generate additional sets of quasi-random points. Once all desired points have been obtained, the function naggpuQuasiRandCleanupA must be called to free allocated system resources.

Note: Concerns were raised about the set of Sobol’ direction numbers that were used in release 0.3 of the NAG Numerical Routines for GPUs. These concerns have been addressed by an amended set of direction numbers in Joe and Kuo (2008) which are used in this release. Consequently, the higher dimensions of this Sobol’ generator may not match the higher dimensions of the generator in release 0.3 since the direction numbers are different.

Specification

```c
#include <nag_gpu.h>
int n, NagGpuQuasiOrient orient, float a, float b, float *d_buff, const NagGpuQuasiRandTune *tune, cudaStream_t custream, NagGpuQuasiRandComm *comm, NagGpuError *error)
```

Description

Quasi-random sequences are made up of one or more multidimensional points, with each point composed of several one dimensional values. The dimensionality of the sequence is specified when calling naggpuQuasiRandInitA to initialize the generator. Below we will consider a \( d \)-dimensional quasi-random sequence \( x^0, x^1, \ldots \) so that each point \( x^j = \left(x^j_1, x^j_2, \ldots, x^j_d\right) \) is composed of \( d \) one dimensional values.

If \( a = 0 \) and \( b = 1 \), this function returns the next \( n \) points \( y^j \in [0,1)^d \) from the quasi-random generator. For other values of \( a \) and \( b \), the function applies the transformation

\[
x^j_i = a + (b - a)y^j_i
\]

for each \( i = 1, 2, \ldots, d \) to produce quasi-random points \( x^j \) from the interval \( [a, b]^d \) for each \( j = 1, 2, \ldots, n \).

Synchronization

This function is non-blocking. Control will return immediately to the calling program while the computation is executed on the GPU. The user is responsible for synchronization between host and GPU code. Please see the synchronization chapter in the CUDA Programming Guide for further details in this direction. For example, a call to cudaMemcpy in the CUDA runtime library is enough to force the host to wait for the GPU to finish, and then copy the results from the GPU to the host.
3.2 Return Value

Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References


5 Arguments

1: \(\text{n} \rightarrow \text{int}\)  \hspace{1cm} \text{Input}
   \(\text{On entry:}\) the number of quasi-random points to be generated.
   \(\text{Constraint:}\ \text{n} \geq 1.\)

2: \(\text{orient} \rightarrow \text{NagGpuQuasiOrient}\)  \hspace{1cm} \text{Input}
   \(\text{On entry:}\) specifies the orientation with which the generator will store the output points. Currently only NAGGPUQUASIORIENT_DIMVALS_SCATT is supported. See NagGpuQuasiOrient for further details on output orientation.
   \(\text{Constraint:}\ \text{orient} = \text{NAGGPUQUASIORIENT\_DIMVALS\_SCATT}.\)

3: \(\text{a} \rightarrow \text{float}\)  \hspace{1cm} \text{Input}
4: \(\text{a} \rightarrow \text{double}\)  \hspace{1cm} \text{Input}
   \(\text{This parameter has type float or double depending on whether the single or double precision version of this function is called.}\)
   \(\text{On entry:}\) The lower bound for the uniform random values.

5: \(\text{b} \rightarrow \text{float}\)  \hspace{1cm} \text{Input}
6: \(\text{b} \rightarrow \text{double}\)  \hspace{1cm} \text{Input}
   \(\text{This parameter has type float or double depending on whether the single or double precision version of this function is called.}\)
   \(\text{On entry:}\) The upper bound for the uniform random values.
   \(\text{Constraint:}\ \text{b} > \text{a}.\)

7: \(\text{d_buffer}[\text{n} \times \text{d}] \rightarrow \text{float} *\)  \hspace{1cm} \text{Output}
8: \(\text{d_buffer}[\text{n} \times \text{d}] \rightarrow \text{double} *\)  \hspace{1cm} \text{Output}
   \(\text{This parameter has type float or double depending on whether the single or double precision version of this function is called.}\)
   \(\text{This buffer must reside in the GPU memory space.}\)
   \(\text{The value d is the dimension dim of the sequence as specified to the initialization function naggpuQuasiRandInitA.}\)
   \(\text{On exit:}\) the n quasi-random points from the specified distribution. For a given point, the individual dimension values are not stored consecutively in memory. The i-th dimension \(x_j^i\) of the j-th quasi-random point will be stored at location \(\text{d_buffer}[(i - 1) * \text{n} + j]\) for every \(0 \leq j < \text{n}\) and \(1 \leq i \leq \text{d}.\)
   In other words, the first n values correspond to dimension 1, the second n to dimension 2, and so on.

9: \(\text{tune} \rightarrow \text{const NagGpuQuasiRandTune} *\)  \hspace{1cm} \text{Input}
   \(\text{This parameter is optional and may be set to NULL.}\)
On entry: if specified, points to a NagGpuQuasiRandTune structure containing launch parameters for the selected GPU kernel. Please see NagGpuQuasiRandTune for additional information about performance tuning.

10: custream – cudaStream_t

Input

On entry: specifies the CUDA stream on which to launch the selected GPU kernel. If no streams are used, set this parameter to 0. Please see the chapter on Streams in the CUDA Programming Guide for further details.

11: comm – NagGpuQuasiRandComm *

Communication Data

NagGpuQuasiRandComm is a structure which holds state and communication information and must not be modified in any way. Once all required points have been obtained, comm must be passed to naggpuQuasiRandCleanupA to free allocated system resources.

Upon successful return from this function, the launch configurations applied to the underlying GPU kernels may be observed through comm. This will typically only be of interest to users wanting to fine tune the performance of this function. Please see NagGpuQuasiRandTune for details on performance tuning, and consult the NagGpuQuasiRandComm documentation for how to observe the launch parameters. Note that these parameters are no longer observable after calling naggpuQuasiRandCleanupA.

12: error – NagGpuError *

Error Reporting

This parameter contains error information and should not be modified directly. Errors are indicated through the value of error → code which should be inspected after each call to this function. If error → code = 0 then no error occurred. If error → code ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

error → code = 1

On entry: the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

error → code = 2

During execution: a CUDA runtime error was detected.

error → code = 3

On entry: an attempt was made to launch a double precision function on a GPU device that does not support double precision.

error → code = 100

On entry: the value of comm is NULL.

error → code = 101

On entry: comm has not been initialized, or the internal state of comm is corrupted.

error → code = 110

On entry: n ≤ 0.

error → code = 111

On entry: orient does not satisfy the constraint listed above.

error → code = 112

On entry: d_buff is NULL.
error → code = 113

On entry: \( b \leq a \)

error → code = 250

On entry: the Sobol’ base generator is selected (see naggpuQuasiRandInitA for details),\n\( \text{tune} \neq \text{NULL} \) and \( \text{tune} \rightarrow \text{sblAThdsPerBlk} \) is out of bounds. See NagGpuQuasiRandTune for further details.

error → code = 251

On entry: the Sobol’ base generator is selected (see naggpuQuasiRandInitA for details),\n\( \text{tune} \neq \text{NULL} \) and \( \text{tune} \rightarrow \text{sblAThdsPerBlk} \) is not a power of two.

error → code = 252

On entry: the Sobol’ base generator is selected (see naggpuQuasiRandInitA for details),\n\( \text{tune} \neq \text{NULL} \) and \( \text{tune} \rightarrow \text{sblABlksPerDim} \) is out of bounds. See NagGpuQuasiRandTune for further details.

error → code = 253

On entry: the Sobol’ base generator is selected (see naggpuQuasiRandInitA for details),\n\( \text{tune} \neq \text{NULL} \) and \( \text{tune} \rightarrow \text{sblABlksPerDim} \) is not a power of two.

7 Example

This example program uses naggpuQuasiRandUniformA to print 5 quasi-random numbers of dimension 10 from a uniform distribution. The first point in the sequence is skipped and generation starts at the second point.

7.1 Program Text

/*
 * Example Program: naggpuQuasiRandUniformA
 * Copyright 2009, Numerical Algorithms Group Ltd, Oxford, UK.
 * Version 0.4, 2011.
 */
#include
using namespace std;
#include <nag_gpu.h>
#include <nag_gpu_serial.h>

#define FP float
#ifdef SINGLEPRECISION
#define FP double
#else
#endif

void checkNagError(NagGpuError *error);
void printNagTuningParamsUsed(NagGpuQuasiRandComm *comm);
void checkCudaError(cudaError_t cuError);

int main(int argc, char **argv)
{
    // host (CPU) storage for generated GPU random numbers
    FP *h_buff = 0;
    // device (GPU) storage for generated random numbers
    FP *d_buff = 0;
    // number of points to generate

int n = 100000;
int dim = 100;
int offset = 1;
unsigned int pseudoSeed[] = {1, 2, 3, 4, 5, 6};

// distribution parameters
FP a = 0.0;
FP b = 1.0;

// NAG GPU structures
NagGpuQuasiRandComm comm;
NagGpuQuasiRandTune tune;
NagCPURandComm pseudoComm;
NagGpuError error;
NagGpuQuasiOrient orient = NAGGPUQUASIORIENT_DIMVALS_SCATT;
cudaError_t cuError;

// Print the title
cout << "NAG GPU Example Program: ";
cout << "naggpuQuasiRandUniformA";
if (sizeof(FP)==sizeof(float)) cout << "_sp";
cout << endl << endl;

// Allocate CPU and GPU memory
h_buff = new FP[n*dim];
cuError = cudaMalloc((void **)&d_buff, sizeof(FP)*n*dim);
checkCudaError(cuError);

// Initialise the CPU pseudo-random generator
nagCPURandInitA(NAGGPUURANDGEN_MRG32K3A, 0, 0, 0, 0, 0, pseudoSeed, &pseudoComm, &error);
checkNagError(&error);

// Initialise the generator only once
cout << "Initialising generator..." << endl << endl;
naggpuQuasiRandInitA(NAGGPUQUASIGEN_SOBOL, NAGGPUSCRAMTYPES_NONE, dim, offset, &pseudoComm, &comm, &error);
checkNagError(&error);

// Generate n numbers using default tuning parameters
cout << "Generate with default tuning parameters..." << endl;
#ifdef SINGLEPRECISION
naggpuQuasiRandUniformA_sp(n, orient, a, b, d_buff, NULL, 0, &comm, &error);
#else
naggpuQuasiRandUniformA(n, orient, a, b, d_buff, NULL, 0, &comm, &error);
#endif
checkNagError(&error);

// Print out the tuning parameters used
cout << "Tuning Parameters used: " << endl;
printNagTuningParamsUsed(&comm);

// Copy back values from the GPU for printing
cuError = cudaMemcpy(h_buff, d_buff, sizeof(FP)*n*dim,
cudaMemcpyDeviceToHost);
checkCudaError(cuError);

// Print random numbers
cout << "The 5 GPU numbers from dimensions 1 to 10:" << endl;

cout.setf(ios::fixed, ios::floatfield);

cout.precision(3);

for (int d = 0; d < 10; d++)
{
    cout << "dim" << d+1 << "\t";
    for (int i = 0; i < 5; i++)
    {
        cout << h_buff[n*d + i] << "\t";
    }
    cout << endl;
}

cout << endl;

// Generate n numbers using specified tuning parameters

cout << "Generate with user supplied tuning parameters..." << endl;
tune.sblAThdsPerBlk = 32;
tune.sblABlksPerDim = 8;

cout << "Tuning Parameters supplied: " << endl;
cout << " tune.sblAThdsPerBlk = " << tune.sblAThdsPerBlk << endl;
cout << " tune.sblABlksPerDim = " << tune.sblABlksPerDim << endl;

#ifdef SINGLEPRECISION
naggpuQuasiRandUniformA_sp(n, orient, a, b, d_buff, &tune, 0,
&comm, &error);
#else
naggpuQuasiRandUniformA(n, orient, a, b, d_buff, &tune, 0,
&comm, &error);
#endif

checkNagError(&error);

// Print out the tuning parameters used

cout << "Tuning Parameters used: " << endl;
printNagTuningParamsUsed(&comm);

// Call cleanup for the NAG routine
naggpuQuasiRandCleanupA(&comm, &error);
checkNagError(&error);

// Free CPU and GPU memory
delete[] h_buff;
if (d_buff)
{
    cuError = cudaFree(d_buff);
    checkCudaError(cuError);
}

return 0;

void checkNagError(NagGpuError *error)
{
    if (error->code != 0)
    {
        char *buff;
        buff = new char[error->msgLength];
        naggpuErrorCopyMsg(buff, error);
        cout << buff << endl;
        delete[] buff;
        exit(1);
    }
}

void printNagTuningParamsUsed(NagGpuQuasiRandComm *comm)
{
    cout << " comm.tuneOrigin = ";
}
switch (comm->tuneOrigin)
{
    case NAGGPUTUNEORIGIN_NA:
        cout << "NAGGPUTUNEORIGIN_NA";
        break;
    case NAGGPUTUNEORIGIN_DEFAULT:
        cout << "NAGGPUTUNEORIGIN_DEFAULT";
        break;
    case NAGGPUTUNEORIGIN_USER:
        cout << "NAGGPUTUNEORIGIN_USER";
        break;
    case NAGGPUTUNEORIGIN_AUTO:
        cout << "NAGGPUTUNEORIGIN_AUTO";
        break;
    default:
        cout << "Unrecognised tuneOrigin";
    }
    cout << endl;
    cout << " comm.tuneParamsUsed->sblAThdsPerBlk = ";
    cout << comm->tuneParamsUsed->sblAThdsPerBlk << endl;
    cout << " comm.tuneParamsUsed->sblABlksPerDim = ";
    cout << comm->tuneParamsUsed->sblABlksPerDim << endl;
    cout << endl;
}

void checkCudaError(cudaError_t cuError)
{
    if (cuError != cudaSuccess)
    {
        cout << cudaGetErrorString(cuError) << endl;
        exit(1);
    }
}

7.2 Program Data

None.

7.3 Program Results

NAG GPU Example Program: naggpuQuasiRandUniformA_sp

Initialising generator...

Generate with default tuning parameters...
Tuning Parameters used:
    comm.tuneOrigin = NAGGPUTUNEORIGIN_DEFAULT
    comm.tuneParamsUsed->sblAThdsPerBlk = 64
    comm.tuneParamsUsed->sblABlksPerDim = 16

The 5 GPU numbers from dimensions 1 to 10:

dim1 0.500 0.750 0.250 0.375 0.875
dim2 0.500 0.250 0.750 0.375 0.875
dim3 0.500 0.250 0.750 0.625 0.125
dim4 0.500 0.250 0.750 0.875 0.375
dim5 0.500 0.750 0.250 0.375 0.875
dim6 0.500 0.750 0.250 0.125 0.625
dim7 0.500 0.250 0.750 0.375 0.875
dim8 0.500 0.750 0.250 0.875 0.375
dim9 0.500 0.750 0.250 0.875 0.375
dim10 0.500 0.750 0.250 0.625 0.125

Generate with user supplied tuning parameters...
Tuning Parameters supplied:
    tune.sblAThdsPerBlk = 32
    tune.sblABlksPerDim = 8
Tuning Parameters used:

- comm.tuneOrigin = NAGGPUTUNEORIGIN_USER
- comm.tuneParamsUsed->sblAThdsPerBlk = 32
- comm.tuneParamsUsed->sblABlksPerDim = 8
1 Purpose

naggpuQuasiRandCleanupA frees system resources that were allocated by a previous call to naggpuQuasiRandInitA.

2 Specification

```c
#include <nag_gpu.h>
extern "C"
cudaError_t naggpuQuasiRandCleanupA(NagGpuQuasiRandComm *comm,
                                    NagGpuError *error)
```

3 Description

3.1 Synchronization

This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.

3.2 Return Value

Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References

None.

5 Arguments

1: `comm` – NagGpuQuasiRandComm *
   
   **Communication Data**
   
   *On entry:* the pointer that was passed to a previous call to naggpuQuasiRandInitA.

2: `error` – NagGpuError *
   
   **Error Reporting**
   
   This parameter contains error information and should not be modified directly. Errors are indicated through the value of `error → code` which should be inspected after each call to this function. If `error → code` = 0 then no error occurred. If `error → code` ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

- `error → code` = 1
  
  *On entry:* the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call `cudaGetLastError()` in the CUDA runtime library to clear the runtime error status.

- `error → code` = 2
  
  *During execution:* a CUDA runtime error was detected.
error → code = 100

On entry: the value of comm is NULL.

error → code = 101

On entry: comm has not been initialized, or the internal state of comm is corrupted.

7 Example

There is no example program specifically for this function. For examples of how this function should be used, please see the example program for naggpuQuasiRandUniformA.
1 Purpose

naggpuBBInitA initializes the Brownian bridge generator function naggpuBBA. It must be called before any calls to naggpuBBA and must finally be followed by a call to naggpuBBCleanupA before it can be called a second time.

Note: after the first call to naggpuBBInitA, all subsequent calls (for example, to change time points) must be preceded by a call to naggpuBBCleanupA.

2 Specification

```c
#include <nag_gpu.h>
extern "C"
cudaError_t naggpuBBInitA(double tStart, double tEnd, const double *times, int nTimes, NagGpuBBComm *comm, NagGpuError *error)
```

3 Description

3.1 Brownian Bridge Algorithm

Fix two times \( t_0 < T \) and let \((t_i)_{1 \leq i \leq N}\) be any set of time points satisfying \( t_0 < t_1 < t_2 < \ldots < t_N < T \).

A popular method for constructing a \( d \) dimensional Brownian sample path \((X_t)_{1 \leq i \leq N}\) at these times is via the Brownian bridge algorithm (see Glasserman (2004)). From any two known points \( X_{t_i} \) at time \( t_i \) and \( X_{t_k} \) at time \( t_k \) with \( t_i < t_k \), we can interpolate a new point \( X_{t_j} \) at any time \( t_j \in (t_i, t_k) \) by setting

\[
X_{t_j} = \frac{X_{t_i}(t_k - t_j) + X_{t_k}(t_j - t_i)}{t_k - t_i} + C \sqrt{\frac{(t_k - t_j)(t_j - t_i)}{(t_k - t_i)} Z}
\]

where \( Z \) is a \( d \) dimensional standard normal random variable and \( C \) is any \( d \times d \) matrix such that \( CC^T \) is the desired covariance structure for the Brownian motion \( X \). Clearly this algorithm is iterative in nature. All that is needed to complete the specification is to fix the start point \( X_{t_0} \) and end point \( X_T \), and to specify how successive interpolation times \( t_j \) are chosen. For \( X \) to behave like a Brownian motion we should set \( X_{t_0} \) equal to some value \( x \in \mathbb{R}^d \) and then set \( X_T = x + C \sqrt{T - t_0} Z \) where \( Z \) is any \( d \) dimensional standard normal random variable. However when it comes to deciding how the successive interpolation times \( t_j \) should be chosen, there is virtually no restriction. Any method of choosing which \( t_j \in (t_i, t_k) \) to interpolate next will yield a statistically correct Brownian motion, provided \( t_i \) is the nearest known point to the left of \( t_j \) and \( t_k \) is the nearest known point to the right of \( t_j \). In other words, the interpolation interval \((t_i, t_k)\) must not contain any other known points. If it does, the covariance structure of the process will be incorrect.

The order in which the successive interpolation times \( t_j \) are chosen is called the bridge construction order. Since all construction orders will yield a statistically correct Brownian motion, the question arises whether one construction order should be preferred over another. When the \( Z \) values are drawn from a pseudorandom generator, the answer is typically no. However the bridge algorithm is frequently used with quasi-random numbers, and in this case the bridge construction order can be important.

3.2 Bridge Construction Order and Quasi-Random Sequences

Consider the one dimensional case so that \( d = C = 1 \). The Brownian bridge is frequently combined with low-discrepancy (quasi-random) sequences to perform quasi-Monte Carlo integration. Quasi-random points \( Z^1, Z^2, Z^3, \ldots \) are generated from the standard normal distribution, where each quasi-random point \( Z^i = (Z^i_1, Z^i_2, \ldots, Z^i_D) \) consists of \( D \) one dimensional values. The process \( X \) starts at \( X_{t_0} = x \) which is...
known. There remain \( N + 1 \) time points at which the bridge is to be computed, namely \( (X_t)_{1 \leq i \leq N} \) and \( X_T \). In this case \( D \) is set equal to \( N + 1 \), so that \( N + 1 \) dimensional quasi-random points are generated. A single quasi-random point is used to construct one Brownian sample path.

The question is how to use the dimension values of each \( N + 1 \) dimensional quasi-random point. Often the ‘lower’ dimension values \( (Z_1, Z_2, \ldots) \) display better uniformity properties than the ‘higher’ dimension values \( (Z_{N+1}, Z_N, \ldots) \) so that the ‘lower’ dimension values should be used to construct the most important sections of the Brownian sample path. For example, consider a model which is particularly sensitive to the behaviour of the underlying Brownian motion at time \( 3 \). When constructing the Brownian sample path, one would therefore ensure that time \( 3 \) was one of the interpolation points of the bridge, and that a ‘lower’ dimension value was used in (1) to construct the corresponding bridge point \( X_3 \). Indeed, one would most likely also ensure that time \( X_3 \) was one of the first bridge points that was constructed: ‘lower’ dimension values would be used to construct both the left and right bridge points used in (1) to interpolate \( X_3 \), so that the distribution of \( X_3 \) benefits as much as possible from the uniformity properties of the quasi-random sequence. For further discussions in this regard we refer to Glasserman (2004).

### 3.3 Implementation

The bridge construction order is given by the array \texttt{times}. Suppose we require \( P \) Brownian sample paths each of dimension \( d \). We therefore have corresponding quasi-random points \( Z^1, Z^2, \ldots, Z^p \) where each point \( Z^p = (Z^p_1, Z^p_2, \ldots, Z^p_{d(N+1)}) \) has dimension \( d(N+1) \). The starting value \( X_{t_0} \) of the Brownian motion is assumed to be a known constant.

When \texttt{naggpuBBInitA} is called, the \( p \)-th sample path for \( 1 \leq p \leq P \) is constructed as follows: the end value \( X_T \) is first constructed as

\[
X_T = X_{t_0} + C \sqrt{T - t_0} \begin{bmatrix} Z^p_1 \\ \vdots \\ Z^p_d \end{bmatrix}
\]

where \( C \) is the matrix from Section 3.1. The array \texttt{times} holds the remaining time points \( t_1, t_2, \ldots, t_N \) in the order in which the bridge is to be constructed. For each \( j = 1, \ldots, N \) set \( r = \text{times}[j-1] \) and construct the point \( X_r \) as

\[
X_r = X_q(\frac{s-r}{s-q}) + X_s(\frac{r-q}{s-q}) + C \sqrt{\frac{(s-r)(r-q)}{(s-q)}} \begin{bmatrix} Z^p_{jd+1} \\ \vdots \\ Z^p_{jd+d} \end{bmatrix}
\]

where

\[
q = \max\{t_0, \text{times}[i] : 0 \leq i < j, \text{times}[i] < r\}
\]

and

\[
s = \min\{T, \text{times}[i] : 0 \leq i < j, \text{times}[i] > r\}.\]

Note that in our discussion \( j \) is indexed from 1, and so \( X_r \) is interpolated between the nearest (in time) Brownian points which have already been constructed. The function \texttt{naggpuMakeBridgeOrderA} can be used to initialize the \texttt{times} array for several pre-defined bridge construction orders.

### 3.4 Bridge Construction Order and Working Stack

The efficient implementation of a Brownian bridge algorithm requires the use of a workspace array called the working stack. Since previously computed points will be used to interpolate new points, they should be kept close to the compute units so that the data can be accessed quickly. Ideally the whole stack should be held in hardware cache. Different bridge construction orders may require different amounts of working stack. Indeed, a naive bridge algorithm may require a stack of size \( \frac{N^2}{2} \) or even \( \frac{N}{2} \), which could be very inefficient when \( N \) is large.

For this reason, \texttt{naggpuBBInitA} performs a detailed analysis of the bridge construction order specified by \texttt{times}. Heuristics are used to find an execution strategy which requires a small working stack, while still constructing the bridge in the order required. Due to the GPU implementation of the Brownian bridge algorithm, the maximum size of the working stack is limited. The exact size of the limit is implementation dependent and may change over time (the current limit is 30 for all NVIDIA GPUs). Upon a successful return from \texttt{naggpuBBInitA}, the amount of working stack per bridge dimension required to construct the bridge can be observed through the \texttt{stackSize} member of the NagGpuBBComm structure \texttt{comm}. 
3.5 Synchronization
This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.

3.6 Return Value
Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References

5 Arguments
1: \( t\text{Start} \) – double \( \text{Input} \)
   \( On \ entry: \) the starting value \( t_0 \) of the time interval.

2: \( t\text{End} \) – double \( \text{Input} \)
   \( On \ entry: \) the end value \( T \) of the time interval.
   \( Constraint: \ t\text{End} > t\text{Start}. \)

3: \( \text{times}[\text{nTimes}] \) – const double * \( \text{Input} \)
   \( On \ entry: \) the points in the time interval \( (t\text{Start}, t\text{End}) \) at which the Brownian motion is to be constructed. The order in which points are listed in \( \text{times} \) determines the bridge construction order. The function \text{naggpuMakeBridgeOrderA} can be used to create pre-defined bridge construction orders from a set of input times.
   \( Constraints: \)
   \[ t\text{Start} < \text{times}[i] < t\text{End} \text{ for all } i = 0, 1, \ldots, \text{nTimes} - 1; \]
   \[ \text{times}[i] \neq \text{times}[j] \text{ for all } i, j = 0, 1, \ldots, \text{nTimes} - 1 \text{ and } i \neq j. \]

4: \( \text{nTimes} \) – int \( \text{Input} \)
   \( On \ entry: \) the number of elements in the array \( \text{times} \), denoted by \( N \) in Section 3.1.
   \( Constraint: \ 1 \leq \text{nTimes} \leq 65535. \)

5: \( \text{comm} \) – NagGpuBBComm * \( \text{Communication Data} \)
   \text{NagGpuBBComm} is a structure which holds state and communication information and must not be modified in any way. The structure will be initialized and must be passed to the generator function \text{naggpuBBInitA}. Once all required sample paths have been obtained, \text{comm} must be passed to \text{naggpuBBCleanupA} to free allocated system resources.
   Upon successful return from this function, the size of the working stack per bridge dimension required to construct the bridge may be observed through \text{comm}. Please see the \text{NagGpuBBComm} documentation for further details. Note that this parameter is no longer observable after calling \text{naggpuBBCleanupA}.

6: \( \text{error} \) – NagGpuError * \( \text{Error Reporting} \)
   This parameter contains error information and should not be modified directly. Errors are indicated through the value of \( \text{error} \rightarrow \text{code} \) which should be inspected after each call to this function. If \( \text{error} \rightarrow \text{code} = 0 \) then no error occurred. If \( \text{error} \rightarrow \text{code} \neq 0 \) then an error was detected and a call to \text{naggpuErrorCopyMsg} will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.
6 Error Indicators and Warnings

- **error → code = 1**
  
  *On entry:* the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call `cudaGetLastError()` in the CUDA runtime library to clear the runtime error status.

- **error → code = 2**
  
  *During execution:* a CUDA runtime error was detected.

- **error → code = 100**
  
  *On entry:* the value of `comm` is NULL.

- **error → code = 110**
  
  *On entry:* the value of `times` is NULL.

- **error → code = 111**
  
  *On entry:* the value of `nTimes` does not satisfy the constraint listed above.

- **error → code = 112**
  
  *On entry:* the value of `tEnd` does not satisfy the constraint listed above.

- **error → code = 113**
  
  *On entry:* the values in the `times` array do not satisfy the constraints listed above.

- **error → code = 114**
  
  *On entry:* the bridge construction order specified by the `times` array requires a working stack which is too big. The error message will contain the size of the working stack required and the maximum allowed stack size.

7 Example

There is no example program specifically for this function. For examples of how this function should be used, please see the example program for `naggpuBBA`. 
naggpuBBA

1 Purpose
naggpuBBA uses a Brownian bridge algorithm to construct sample paths for a Brownian motion. It must
be preceded by a call to the initialization function naggpuBBInitA, and must finally be followed by a call
to naggpuBBCleanupA. For further details on the Brownian bridge algorithm and the bridge construction
order, please see the documentation for naggpuBBInitA.

2 Specification

#include <nag_gpu.h>
extern "C"
cudaError_t naggpuBBA_sp(int nPaths, int dim, const float *d_bgStart,
   const float *d_z, const float *d_cholCov, float *d_bgVals,
   const NagGpuBBTune *tune, cudaStream_t custream, NagGpuBBComm *comm,
   NagGpuError *error)

extern "C"
cudaError_t naggpuBBA(int nPaths, int dim, const double *d_bgStart,
   const double *d_z, const double *d_cholCov, double *d_bgVals,
   const NagGpuBBTune *tune, cudaStream_t custream, NagGpuBBComm *comm,
   NagGpuError *error)

3 Description

3.1 Synchronization
This function is non-blocking. Control will return immediately to the calling program while the
computation is executed on the GPU. The user is responsible for synchronization between host and GPU
code. Please see the synchronization chapter in the CUDA Programming Guide for further details in this
direction. For example, a call to cudaMemcpy in the CUDA runtime library is enough to force the host to
wait for the GPU to finish, and then copy the results from the GPU to the host.

3.2 Return Value
Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were
encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References
None.

5 Arguments

1: nPaths – int  
   Input  
   On entry: the number of Brownian sample paths to create.
   Constraint: nPaths ≥ 1.

2: dim – int  
   Input  
   On entry: the dimension of each Brownian sample path.
   Constraint: 1 ≤ dim ≤ 4.
3: \( \text{d\_bgStart}[\text{dim}] \) – const float * \( \text{Input} \)
4: \( \text{d\_bgStart}[\text{dim}] \) – const double * \( \text{Input} \)

This parameter has type float or double depending on whether the single or double precision version of this function is called.

This buffer must reside in the GPU memory space.

On entry: the starting value of the Brownian motion. If \( \text{d\_bgStart} \) is set equal to NULL, the Brownian motion will be taken to start at 0.

5: \( \text{d\_z}[\text{dim} \times (N + 1) \times \text{nPPaths}] \) – const float * \( \text{Input} \)
6: \( \text{d\_z}[\text{dim} \times (N + 1) \times \text{nPPaths}] \) – const double * \( \text{Input} \)

This parameter has type float or double depending on whether the single or double precision version of this function is called.

This buffer must reside in the GPU memory space.

The variable \( N \) denotes the length \( \text{nTimes} \) of the \text{times} array passed to the initialization function \text{naggpuBBInitA}.

On entry: the Normal random numbers used to construct the bridge.

Constraint: \( \text{d\_z} \) must contain \( \text{dim} \times (N + 1) \times \text{nPPaths} \) values. The values should be laid out as a matrix with \( \text{dim} \times (N + 1) \) rows and \( \text{nPPaths} \) columns. If quasi-random numbers are to be used, successive \( \text{dim} \times (N + 1) \) dimensional points should be stored in successive columns of the matrix, i.e. an ordering corresponding to \text{NAGGPUQUASIORIENT\_DIMVALS\_SCATT}.

7: \( \text{d\_cholCov}[\text{dim} \times \text{dim}] \) – const float * \( \text{Input} \)
8: \( \text{d\_cholCov}[\text{dim} \times \text{dim}] \) – const double * \( \text{Input} \)

This parameter has type float or double depending on whether the single or double precision version of this function is called.

This buffer must reside in the GPU memory space.

On entry: the lower triangular Cholesky factorisation \( C \) so that \( CC^T \) gives the covariance matrix of the Brownian motion. The elements must be stored in column major order (see the Linear Equations Chapter Introduction for details on matrix storage schemes), so that \( c_{i,j} \) for \( i, j = 1, 2, \ldots, \text{dim} \) are stored in \( \text{d\_cholCov}[(j - 1) \times \text{dim} + i - 1] \). Elements of the matrix above the diagonal are not referenced.

9: \( \text{d\_bgVals}[\text{dim} \times (N + 1) \times \text{nPPaths}] \) – float * \( \text{Output} \)
10: \( \text{d\_bgVals}[\text{dim} \times (N + 1) \times \text{nPPaths}] \) – double * \( \text{Output} \)

This parameter has type float or double depending on whether the single or double precision version of this function is called.

This buffer must reside in the GPU memory space.

The variable \( N \) denotes the length \( \text{nTimes} \) of the \text{times} array passed to the initialization function \text{naggpuBBInitA}.

On exit: the values of the Brownian bridge. If \( X_{p,i}^d \) denotes the \( d \)-th dimension of the \( i \)-th point of the \( p \)-th sample path where \( 0 \leq d < \text{dim} \), \( 0 \leq i < N + 1 \) and \( 0 \leq p < \text{nPPaths} \), then \( X_{p,i}^d \) will be stored at \( \text{d\_bgVals}[p + \text{nPPaths}(d + i \times \text{dim})] \). The starting value \( \text{d\_bgStart} \) is never stored.

11: \( \text{tune} \) – const NagGpuBBTune * \( \text{Input} \)

This parameter is optional and may be set to NULL.

On entry: if specified, points to a NagGpuBBTune structure containing launch parameters for the Brownian bridge kernel. Upon a successful return from this function, the relevant data will be copied to the output tuning structure \text{comm} \rightarrow \text{tuneParamsUsed}. Please see NagGpuBBTune for additional information about performance tuning.
12: **custream** – cudaStream_t

*Input*

*On entry:* specifies the CUDA stream on which to launch the selected GPU kernel. If no streams are used, set this parameter to 0. Please see the chapter on Streams in the CUDA Programming Guide for further details.

13: **comm** – NagGpuBBComm *

*Communication Data*

NagGpuBBComm is a structure which holds state and communication information and must not be modified in any way. Once all required Brownian sample paths have been obtained, **comm** must be passed to naggpuBBCleanupA to free allocated system resources.

Upon successful return from this function, the launch configurations applied to the underlying GPU kernels may be observed through **comm**. This will typically only be of interest to users wanting to fine tune the performance of this function. Please see NagGpuBBTune for details on performance tuning, and consult the NagGpuBBComm documentation for how to observe the launch parameters. Note that these parameters are no longer observable after calling naggpuBBCleanupA.

14: **error** – NagGpuError *

*Error Reporting*

This parameter contains error information and should not be modified directly. Errors are indicated through the value of **error** → **code** which should be inspected after each call to this function. If **error** → **code** = 0 then no error occurred. If **error** → **code** ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

### 6 Error Indicators and Warnings

**error** → **code** = 1

*On entry:* the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

**error** → **code** = 2

*During execution:* a CUDA runtime error was detected.

**error** → **code** = 3

*On entry:* an attempt was made to launch a double precision function on a GPU device that does not support double precision.

**error** → **code** = 100

*On entry:* the value of **comm** is NULL.

**error** → **code** = 101

*On entry:* **comm** has not been initialized, or the internal state of **comm** is corrupted.

**error** → **code** = 110

*On entry:* the value of **nPaths** does not satisfy the constraint listed above.

**error** → **code** = 111

*On entry:* the value of **dim** does not satisfy the constraint listed above.

**error** → **code** = 112

*On entry:* the value of **d_z** is NULL.

**error** → **code** = 113

*On entry:* the value of **d_cholCov** is NULL.

**error** → **code** = 115

*On entry:* the value of **d_bgVals** is NULL.
error → code = 250

On entry: tune ≠ NULL and tune → bbASHmStackSz is out of bounds. See NagGpuBBTune for further details.

error → code = 251

On entry: tune ≠ NULL and tune → bbATHdsPerBlk is out of bounds. See NagGpuBBTune for further details.

error → code = 252

On entry: tune ≠ NULL and tune → bbANumBlks is out of bounds. See NagGpuBBTune for further details.

7 Example

This example program uses naggpuBBA to print two Brownian bridge sample paths where each path is three dimensional. The inputs are quasi-random Normal numbers from the generator naggpuQuasiRandNormalA.

7.1 Program Text

/*
 * Example Program: naggpu_depthbbA
 *
 * Copyright 2009, Numerical Algorithms Group Ltd, Oxford, UK.
 *
 * Version 0.4, 2011.
 *
 */
#include <stdio.h>
#include using namespace std;
#include <nag_gpu.h>
#include <nag_gpu_serial.h>

void checkNagError(NagGpuError *error);
void checkCudaError(cudaError_t cuError);

int main(int argc, char **argv)
{
    // Number of time steps bridge
    const int nTimes = 30;
    // Dimension of bridge - less than 4
    const int dim = 3;
    // Number of sample paths to generate
    int N = 50;

    // NAG structures
    NagGpuError error;
    NagCPURandComm pcomm;
    NagGpuQuasiRandComm qcomm;
    NagGpuBBCComm bcomm;
    NagGpuQuasiOrient orient = NAGGPUQUASIORIENT_DIMVALS_SCATT;

    cudaError_t cuError;

    // Print the title
    cout << "NAG GPU Example Program: ";
    cout << "naggpuBBA";
    cout << endl << endl;

    return 0;
}
// Initialise the generator only once
cout << "Initialising generators ..." << endl << endl;

// Initialise the CPU pseudorandom generator
unsigned int seed[] = {1, 2, 3, 4, 5, 6};
nagCPURandInitA(NAGGPURANDGEN_MRG32K3A, 0, 0, 0, 0, seed, 
    &pcomm, &error);
checkNagError(&error);
// Initialise the quasi-random generator
naggpuQuasiRandInitA(NAGGPUQUASIGEN_SOBOL, NAGGPUSCRAMTYPES_NONE, 
    dim*(nTimes+1), 501, &pcomm, &qcomm, &error);
checkNagError(&error);
// Cleanup the pseudorandom generator
nagCPURandCleanupA(&pcomm, &error);
checkNagError(&error);

// Generate the Sobol numbers
double *d_z = NULL;
cuError = cudaMalloc((void **)&d_z, sizeof(double)*(nTimes+1)*dim*N);
checkCudaError(cuError);
naggpuQuasiRandNormalA(N, orient, 0, 1, d_z, NULL, 0, 
    &qcomm, &error);
checkNagError(&error);

// Create the bridge time points
double times[nTimes];
double t[nTimes+1];
for(int i=0; i<nTimes+1; i++) t[i] = (i+1)*0.477;
double tEnd;
NagGpuBridgeOrder order = NAGGPUBRIDGEORDER_BISECT_ASCEND_ROUNDDOWN;
naggpuMakeBridgeOrderA(order, 0, t, NULL, 0, times, nTimes, 
    &tEnd, &error);
checkNagError(&error);

// Initialise the bridge generator
naggpuBBInitA(0, tEnd, times, nTimes, &bbcomm, &error);
checkNagError(&error);

// Create covariance structure and copy to GPU
double cov[dim*dim];
for(int i=0; i<dim*dim; i++) cov[i] = 0.1;
for(int i=0; i<dim; i++) cov[i*(dim+1)] = 0.31;
double *d_cov = NULL;
cuError = cudaMalloc((void **)&d_cov, sizeof(double)*dim*dim);
checkCudaError(cuError);
cuError = cudaMemcpy(d_cov, cov, sizeof(double)*dim*dim, 
    cudaMemcpyHostToDevice);
checkCudaError(cuError);

// Compute Cholesky factorisation
NagGpuLinAlgComm lacommm;
naggpuLinAlgInitA(&lacommm, &error);
checkNagError(&error);
naggpuDpotrfA(NAGGPDUMATRIXUPLOW_LOWER, dim, d_cov, dim, &lacommm, &error);
checkNagError(&error);
naggpuLinAlgCleanupA(&lacommm, &error);
checkNagError(&error);

cout << "Creating the bridge ..." << endl << endl;

// Generate bridge and copy back to host
double *d_buff = NULL;
cuError = cudaMalloc((void **)&d_buff, sizeof(double)*dim*(nTimes+1)*N);
checkCudaError(cuError);
double *h_buff = new double[dim*(nTimes+1)*N];
naggpuBBA(N, dim, NULL, d_z, d_cov, d_buff, NULL, 0, &bbcomm, &error);
checkNagError(&error);
cuError = cudaMemcpy(h_buff, d_buff, sizeof(double)*dim*(nTimes+1)*N, cudaMemcpyDeviceToHost);
checkCudaError(cuError);

// Cleanup quasi-generator and bridge
naggpuQuasiRandCleanupA(&qcomm, &error);
checkNagError(&error);
naggpuBBBCleanupA(&bbcomm, &error);
checkNagError(&error);
cuError = cudaFree(d_z);
checkCudaError(cuError);
cuError = cudaFree(d_cov);
checkCudaError(cuError);
cuError = cudaFree(d_buff);
checkCudaError(cuError);

del;}

// Transpose the data in order to display it
const int nPrint = 2;
double *print = new double[dim*(nTimes+1)*nPrint];
for(int p=0; p<nPrint; p++)
{
    for(int i=0; i<nTimes+1; i++)
    {
        for(int d=0; d<dim; d++)
        {
            print[d+p*dim+i*dim*nPrint] = h_buff[p+N*(d+i*dim)];
        }
    }
}
delete[] h_buff;

// Print Brownian Bridge Sample Paths
cout << "The first " << nPrint << " bridge paths of dimension " << dim << ":" << endl;
cout << " \	";
for(int p = 0; p < nPrint; p++)
{
    int nspaces = (8*dim - 9)/2;
    for(int s=0; s<nspaces; s++) cout << "-"; 
    cout << " Path" << p+1 << " ";
    for(int s=0; s<nspaces; s++) cout << "-"; 
    cout << " \t ";
}
cout << endl;
cout << " t_i\t";
for(int p=0; p<nPrint; p++)
{
    for(int d=1; d<dim; d++) cout << " dim " << d << " ";
    cout << "\t";
}
for(int i=0; i<nTimes+1; i++)
{
    cout << "n " << i+1 << "\t";
    for(int p=0; p<nPrint; p++)
    {
        for(int d=0; d<dim; d++)
        {
            double val = print[i*dim*nPrint + p*dim + d];
            if (val < 10 && val > -10) printf("% .3f ", val);
            else printf("% .2f ", val);
        }
    cout << "\t";
}
cout << endl << endl;
delete[] print;
return 0;
}

void checkNagError(NagGpuError *error)
{
    if (error->code != 0)
    {
        char *buff;
        buff = new char[error->msgLength];
        naggpuErrorCopyMsg(buff, error);
        cout << buff << endl;
        delete[] buff;
        exit(1);
    }
}

void checkCudaError(cudaError_t cuError)
{
    if (cuError != cudaSuccess)
    {
        cout << cudaGetErrorString(cuError) << endl;
        exit(1);
    }
}

7.2 Program Data
None.

7.3 Program Results
NAG GPU Example Program: naggpuBBA
Initialising generators ...
Creating the bridge ...
The first 2 bridge paths of dimension 3:
--------- Path1 ------- ------- Path2 -------
t   dim1  dim2  dim3  dim1  dim2  dim3
  1  0.539  0.178 -0.028 -0.093 -0.239 -0.379
  2  0.938  0.140 -0.795  0.451 -0.303 -0.312
  3  0.988 -0.494 -0.984 -0.275 -0.877 -1.104
  4  1.307 -0.779 -0.809 -0.488 -1.706 -1.369
  5  1.544 -0.216 -0.286 -0.432 -2.172 -1.637
  6  1.672 -0.721 -0.309 -0.570 -2.222 -2.315
  7  1.699 -0.677 -0.665 -0.637 -2.060 -2.032
  8  1.195 -0.793 -0.502 -1.091 -2.274 -2.760
  9  1.793 -0.476  0.437 -1.252 -1.851 -3.181
 10  1.837 -0.044  0.999 -0.665 -1.213 -2.831
 11  1.524 -0.732  0.621 -0.071 -0.879 -2.715
 12  2.347 -0.333  0.942  0.105 -0.671 -2.546
 13  2.327 -0.570  1.333  0.012 -0.563 -2.396
 14  2.639 -0.919  1.715 -0.509 -1.052 -2.767
 15  3.029 -0.625  1.405 -0.560 -1.212 -2.539
 16  3.134 -1.214  1.531 -0.083 -0.303 -2.326
 17  3.220 -1.010  1.206  0.002  0.028 -2.543
 18  3.503 -1.133  1.050  0.406 -0.069 -2.822
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naggpuBBCleanupA

1 Purpose

naggpuBBCleanupA frees system resources which were allocated by a previous call to naggpuBBInitA.

2 Specification

#include <nag_gpu.h>

extern "C"
cudaError_t naggpuBBCleanupA(NagGpuBBComm *comm, NagGpuError *error)

3 Description

3.1 Synchronization

This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.

3.2 Return Value

Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References

None.

5 Arguments

1: comm – NagGpuBBComm *

The structure which was initialized by a previous call to naggpuBBInitA.

2: error – NagGpuError *

This parameter contains error information and should not be modified directly. Errors are indicated through the value of error → code which should be inspected after each call to this function. If error → code = 0 then no error occurred. If error → code ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

error → code = 1

On entry: the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

epsilon → code = 2

During execution: a CUDA runtime error was detected.
error → code = 100 

On entry: the value of comm is NULL.

error → code = 101 

On entry: comm has not been initialized, or the internal state of comm is corrupted.

7 Example

There is no example program specifically for this function. For examples of how this function should be used, please see the example program for naggpuBBA.
1 Purpose

naggpuBBIncInitA initializes the Brownian bridge increments generator function naggpuBBIncA. It must be called before any calls to naggpuBBIncA and must finally be followed by a call to naggpuBBIncCleanupA before it can be called a second time.

Note: after the first call to naggpuBBIncInitA, all subsequent calls (for example, to change time points) must be preceded by a call to naggpuBBIncCleanupA.

2 Specification

```c
#include <nag_gpu.h>
extern "C"
cudaError_t naggpuBBIncInitA(double tStart, double tEnd, const double *times, int nTimes, NagGpuBBIncComm *comm, NagGpuError *error)
```

3 Description

3.1 Brownian Bridge Algorithm

Fix two times \( t_0 < T \) and let \( (t_i)_{1 \leq i \leq N} \) be any set of time points satisfying \( t_0 < t_1 < t_2 < \ldots < t_N < T \). A popular method for constructing a \( d \) dimensional Brownian sample path \( (X_{t_i})_{1 \leq i \leq N} \) at these times is via the Brownian bridge algorithm (see Glasserman (2004)). From any two known points \( X_{t_i} \) at time \( t_i \) and \( X_{t_k} \) at time \( t_k \) with \( t_i < t_k \), we can interpolate a new point \( X_{t_j} \) at any time \( t_j \in (t_i, t_k) \) by setting

\[
X_{t_j} = \frac{X_{t_i}(t_k - t_j) + X_{t_k}(t_j - t_i)}{t_k - t_i} + CZ\sqrt{\frac{(t_k - t_j)(t_j - t_i)}{(t_k - t_i)}},
\]

where \( Z \) is a \( d \) dimensional standard Normal random variable and \( C \) is any \( d \times d \) matrix such that \( CC^T \) is the desired covariance structure for the Brownian motion \( X \). The Brownian bridge increments generator uses the Brownian bridge algorithm to construct the Brownian sample path \( X \), and then uses this to compute the scaled Brownian increments

\[
\frac{X_{t_i} - X_{t_0}}{t_1 - t_0}, \frac{X_{t_2} - X_{t_1}}{t_2 - t_1}, \ldots, \frac{X_{t_N} - X_{t_{N-1}}}{t_N - t_{N-1}}, \frac{X_T - X_{t_N}}{T - t_N}
\]

Such increments can be useful in computing numerical solutions to stochastic differential equations driven by Brownian motion.

For further details about the Brownian bridge algorithm, the working stack, bridge construction orders and the connection with quasi-random sequences, please see the naggpuBBInitA documentation.

3.2 Synchronization

This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.

3.3 Return Value

Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.
4 References


5 Arguments

1: \( t_{\text{Start}} \) – double \( \text{Input} \)

\( On \text{ entry:} \) the starting value \( t_0 \) of the time interval.

2: \( t_{\text{End}} \) – double \( \text{Input} \)

\( On \text{ entry:} \) the end value \( T \) of the time interval.

\( Constraint: \ t_{\text{End}} > t_{\text{Start}}. \)

3: \( \text{times}[\text{nTimes}] \) – const double * \( \text{Input} \)

\( On \text{ entry:} \) the points in the time interval \((t_{\text{Start}}, t_{\text{End}})\) at which the Brownian motion is to be constructed. The order in which points are listed in \( \text{times} \) determines the bridge construction order. The function \( \text{naggpuMakeBridgeOrderA} \) can be used to create pre-defined bridge construction orders from a set of input times.

\( Constraints: \)

\( t_{\text{Start}} < \text{times}[i] < t_{\text{End}} \) for all \( i = 0, 1, \ldots, \text{nTimes} - 1; \)

\( \text{times}[i] \neq \text{times}[j] \) for all \( i, j = 0, 1, \ldots, \text{nTimes} - 1 \) and \( i \neq j. \)

4: \( \text{nTimes} \) – int \( \text{Input} \)

\( On \text{ entry:} \) the number of elements in the array \( \text{times} \), denoted by \( N \) in Section 3.1.

\( Constraint: \ 1 \leq \text{nTimes} \leq 65535. \)

5: \( \text{comm} \) – NagGpuBBIncComm * \( \text{Communication Data} \)

NagGpuBBIncComm is a structure which holds state and communication information and must not be modified in any way. The structure will be initialized and must be passed to the generator function \( \text{naggpuBBIncA} \). Once all required sample paths have been obtained, \( \text{comm} \) must be passed to \( \text{naggpuBBIncCleanupA} \) to free allocated system resources.

Upon successful return from this function, the size of the working stack per bridge dimension required to construct the bridge may be observed through \( \text{comm} \). Please see the NagGpuBBIncComm documentation for further details. Note that this parameter is no longer observable after calling \( \text{naggpuBBIncCleanupA} \).

6: \( \text{error} \) – NagGpuError * \( \text{Error Reporting} \)

This parameter contains error information and should not be modified directly. Errors are indicated through the value of \( \text{error} \rightarrow \text{code} \) which should be inspected after each call to this function. If \( \text{error} \rightarrow \text{code} = 0 \) then no error occurred. If \( \text{error} \rightarrow \text{code} \neq 0 \) then an error was detected and a call to \( \text{naggpuErrorCopyMsg} \) will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

\( \text{error} \rightarrow \text{code} = 1 \)

\( On \text{ entry:} \) the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call \( \text{cudaGetLastError}() \) in the CUDA runtime library to clear the runtime error status.

\( \text{error} \rightarrow \text{code} = 2 \)

\( During \text{ execution:} \) a CUDA runtime error was detected.
error → code = 100

On entry: the value of comm is NULL.

error → code = 110

On entry: the value of times is NULL.

error → code = 111

On entry: the value of nTimes does not satisfy the constraint listed above.

error → code = 112

On entry: the value of tEnd does not satisfy the constraint listed above.

error → code = 113

On entry: the values in the times array do not satisfy the constraints listed above.

error → code = 114

On entry: the bridge construction order specified by the times array requires a working stack which is too big. The error message will contain the size of the working stack required and the maximum allowed stack size.

7 Example

There is no example program specifically for this function. For examples of how this function should be used, please see the example program for naggpuBBIncA.
1 Purpose
naggpuBBIncA computes scaled increments of Brownian sample paths which are constructed using a
Brownian bridge algorithm. It must be preceded by a call to the initialization function naggpuBBIncInitA,
and must finally be followed by a call to naggpuBBIncCleanupA. For further details on the Brownian
bridge algorithm and the bridge construction order, please see the documentation for naggpuBBInitA and
naggpuBBIncInitA.

2 Specification
#include <nag_gpu.h>
extern "C"
cudaError_t naggpuBBIncA_sp(int nPaths, int dim, const float *d_z,
    const float *d_cholCov, float *d_bgIncs, const NagGpuBBTune *tune,
    cudaStream_t custream, NagGpuBBIncComm *comm, NagGpuError *error)
extern "C"
cudaError_t naggpuBBIncA(int nPaths, int dim, const double *d_z,
    const double *d_cholCov, double *d_bgIncs, const NagGpuBBTune *tune,
    cudaStream_t custream, NagGpuBBIncComm *comm, NagGpuError *error)

3 Description
3.1 Synchronization
This function is non-blocking. Control will return immediately to the calling program while the
computation is executed on the GPU. The user is responsible for synchronization between host and GPU
code. Please see the synchronization chapter in the CUDA Programming Guide for further details in this
direction. For example, a call to cudaMemcpy in the CUDA runtime library is enough to force the host to
wait for the GPU to finish, and then copy the results from the GPU to the host.

3.2 Return Value
Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were
encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References
None.

5 Arguments
1: nPaths – int
   On entry: the number of Brownian sample paths to create.
   Constraint: nPaths ≥ 1.
2: dim – int
   On entry: the dimension of each Brownian sample path.
   Constraint: 1 ≤ dim ≤ 4.
This parameter has type float or double depending on whether the single or double precision version of this function is called.

This buffer must reside in the GPU memory space.

The variable \( N \) denotes the length \( nTimes \) of the \( \text{times} \) array passed to the initialization function \( \text{naggpuBBIncInitA} \).

**On entry:** the Normal random numbers used to construct the bridge.

**Constraint:** \( \text{d}_z \) must contain \( \text{dim} \times (N + 1) \times \text{nPaths} \) values. The values should be laid out as a matrix with \( \text{dim} \times (N + 1) \) rows and \( \text{nPaths} \) columns. If quasi-random numbers are to be used, successive \( \text{dim} \times (N + 1) \) dimensional points should be stored in successive columns of the matrix, i.e. an ordering corresponding to \( \text{NAGGPUQUASIORIENT\_DIMVALS\_SCATT} \).

This parameter has type float or double depending on whether the single or double precision version of this function is called.

This buffer must reside in the GPU memory space.

**On entry:** the lower triangular Cholesky factorisation \( C \) so that \( CC^T \) gives the covariance matrix of the Brownian motion. The elements must be stored in column major order (see the Linear Equations Chapter Introduction for details on matrix storage schemes), so that \( c_{i,j} \) for \( i, j = 1, 2, \ldots, \text{dim} \) are stored in \( \text{d}_\text{cholCov}[(j - 1) \times \text{dim} + i - 1] \). Elements of the matrix above the diagonal are not referenced.

This parameter has type float or double depending on whether the single or double precision version of this function is called.

This buffer must reside in the GPU memory space.

The variable \( N \) denotes the length \( nTimes \) of the \( \text{times} \) array passed to the initialization function \( \text{naggpuBBIncInitA} \).

**On exit:** the scaled increments of the Brownian motion. If \( X_{p,i}^d \) denotes the \( d \)-th dimension of the \( i \)-th point of the \( p \)-th sample path where \( 0 \leq d < \text{dim} \), \( 0 \leq i < N + 1 \) and \( 0 \leq p < \text{nPaths} \), then the scaled increment \( (X_{p,i+1}^d - X_{p,i}^d)/(t_{i+1} - t_i) \) will be stored at \( \text{d}_\text{bgIncs}[p + \text{nPaths}(d + i \times \text{dim})] \).

This parameter is optional and may be set to NULL.

**On entry:** if specified, points to a NagGpuBBTune structure containing launch parameters for the Brownian bridge kernel. Upon a successful return from this function, the relevant data will be copied to the output tuning structure \( \text{comm} \rightarrow \text{tuneParamsUsed} \). Please see NagGpuBBTune for additional information about performance tuning.

On **entry:** specifies the CUDA stream on which to launch the selected GPU kernel. If no streams are used, set this parameter to 0. Please see the chapter on Streams in the CUDA Programming Guide for further details.
comm – NagGpuBBIncComm *

NagGpuBBIncComm is a structure which holds state and communication information and must not be modified in any way. Once all required Brownian sample paths have been obtained, comm must be passed to naggpuBBIncCleanupA to free allocated system resources.

Upon successful return from this function, the launch configurations applied to the underlying GPU kernels may be observed through comm. This will typically only be of interest to users wanting to fine tune the performance of this function. Please see NagGpuBBTune for details on performance tuning, and consult the NagGpuBBIncComm documentation for how to observe the launch parameters. Note that these parameters are no longer observable after calling naggpuBBIncCleanupA.

error – NagGpuError *

This parameter contains error information and should not be modified directly. Errors are indicated through the value of error → code which should be inspected after each call to this function. If error → code = 0 then no error occurred. If error → code ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

error → code = 1

On entry: the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

error → code = 2

During execution: a CUDA runtime error was detected.

error → code = 3

On entry: an attempt was made to launch a double precision function on a GPU device that does not support double precision.

error → code = 100

On entry: the value of comm is NULL.

error → code = 101

On entry: comm has not been initialized, or the internal state of comm is corrupted.

error → code = 110

On entry: the value of nPaths does not satisfy the constraint listed above.

error → code = 111

On entry: the value of dim does not satisfy the constraint listed above.

error → code = 112

On entry: the value of d_z is NULL.

error → code = 113

On entry: the value of d_cholCov is NULL.

error → code = 114

On entry: the value of d_bgIncs is NULL.

error → code = 250

On entry: tune ≠ NULL and tune → bbAShmStackSz is out of bounds. See NagGpuBBTune for further details.
On entry: \( t_{\text{tune}} \neq \text{NULL} \) and \( t_{\text{tune}} \rightarrow \text{bbAThdsPerBlk} \) is out of bounds. See NagGpuBBTune for further details.

On entry: \( t_{\text{tune}} \neq \text{NULL} \) and \( t_{\text{tune}} \rightarrow \text{bbANumBlks} \) is out of bounds. See NagGpuBBTune for further details.

This example program uses naggpuBBIncA and naggpuBBA to print a Brownian sample path and the corresponding scaled path increments side by side. Each sample path is three dimensional. To aid comparison, the distance between time points \( \Delta t_i := t_{i+1} - t_i = 1 \) so that the scaled increments \( (X_{t_{i+1}} - X_{t_i})/(t_{i+1} - t_i) \) are in fact the true increments \( X_{t_{i+1}} - X_{t_i} \).

7.1 Program Text

```c
#include <stdio.h>
#include <nag_gpu.h>
#include <nag_gpu_serial.h>

void checkNagError(NagGpuError *error);
void checkCudaError(cudaError_t cuError);

int main(int argc, char **argv)
{
    // Number of time steps bridge
    const int nTimes = 30;
    // Dimension of bridge - less than 4
    const int dim = 3;
    // Number of sample paths to generate
    int N = 2;

    // NAG structures
    NagGpuError error;
    NagCPURandComm pcomm;
    NagGpuQuasiRandComm qcomm;
    NagGpuBComm bcomm;
    NagGpuBBIncComm bbincComm;
    NagGpuQuasiComm qcomm = NAGGPUQUASIORT_DIMVALS_SCATT;

    // Print the title
    cout << "NAG GPU Example Program: ":
    cout << "naggpuBBIncA"
    cout << endl;
    // Initialise the CPU pseudorandom generator
    unsigned int seed[] = {1,2,3,4,5,6};
```

nagCPURandInitA(NAGGPURANDGEN_MRG32K3A, 0, 0, 0, 0, seed, &pcomm, &error);
checkNagError(&error);

// Initialise the quasi-random generator
naggpuQuasiRandInitA(NAGGPUQUASIGEN_SOBOLO, NAGGPUSCRAMTYPES_NONE,
                        dim*(nTimes+1), 10, &pcomm, &qcomm, &error);
checkNagError(&error);

// Cleanup the pseudorandom generator
nagCPURandCleanupA(&pcomm, &error);
checkNagError(&error);

// Generate the Sobol numbers and clean up
double *d_z = NULL;
cuError = cudaMalloc((void **)&d_z, sizeof(double)*(nTimes+1)*dim*N);
checkCudaError(cuError);

naggpuQuasiRandNormalA(N, orient, 0.0f, 1.0f, d_z, NULL, 0,
                        &qcomm, &error);
checkNagError(&error);

naggpuQuasiRandCleanupA(&qcomm, &error);
checkNagError(&error);

// Create the bridge time points
double t[nTimes+1];
for(int i=0; i<nTimes+1; i++) t[i] = (i+1)*1.0;
double times[nTimes];
double tEnd;
NagGpuBridgeOrder order =
    NAGGPUBRIDGEORDER_BISECT_ASCEND_ROUNDDOWN;

naggpuMakeBridgeOrderA(order, 0, t, NULL, 0, times, nTimes,
                        &tEnd, &error);
checkNagError(&error);

// Create covariance structure and copy to GPU
double cov[dim*dim];
for(int i=0; i<dim*dim; i++) cov[i] = 0.1;
for(int i=0; i<dim; i++) cov[i*(dim+1)] = 0.31;
double *d_cov = NULL;
cuError = cudaMalloc((void **)&d_cov, sizeof(double)*dim*dim);
checkCudaError(cuError);
cuError = cudaMemcpy(d_cov, cov, sizeof(double)*dim*dim,
                        cudaMemcpyHostToDevice);
checkCudaError(cuError);

// Compute Cholesky factorisation
NagGpuLinAlgComm lacomm;
naggpuLinAlgInitA(&lacomm, &error);
checkNagError(&error);
naggpuDpotrfA(NAGGPUMATRIXUPLOW_LOWER, dim, d_cov, dim, &lacomm, &error);
checkNagError(&error);
naggpuLinAlgCleanupA(&lacomm, &error);
checkNagError(&error);

// Create storage for bridge numbers on host and device
double *d_buff = NULL;
cuError = cudaMalloc((void **)&d_buff, sizeof(double)*dim*(nTimes+1)*N);
checkCudaError(cuError);
double *h_bb = new double[dim*(nTimes+1)*N];
double *h_bbinc = new double[dim*(nTimes+1)*N];

// Initialise the bridge generator, generate and clean up
naggpuBBInitA(0, tEnd, times, nTimes, &bbcomm, &error);
checkNagError(&error);
naggpuBA(N, dim, NULL, d_z, d_cov, d_buff, NULL, 0, &bbcomm, &error);
checkNagError(&error);
cuError = cudaMemcpy(h_bb, d_buff, sizeof(double)*dim*(nTimes+1)*N,
                        cudaMemcpyDeviceToHost);
checkCudaError(cuError);
naggpuBBCleanupA(&bbcomm, &error);
checkNagError(&error);

// Initialise the bridge increments generator, generate and clean up
naggpuBBIincInitA(0, tEnd, times, nTimes, &bbinccomm, &error);
checkNagError(&error);
naggpuBBIincA(N, dim, d_z, d_cov, d_buff,
NULL, 0, &bbinccomm, &error);
checkNagError(&error);
cuError = cudaMemcpy(h_bbinc, d_buff,
sizeof(double)*dim*(nTimes+1)*N, cudaMemcpyDeviceToHost);
checkCudaError(cuError);
naggpuBBIincCleanupA(&bbinccomm, &error);
checkNagError(&error);

// Free GPU memory
cuError = cudaFree(d_z);
checkCudaError(cuError);
cuError = cudaFree(d_buff);
checkCudaError(cuError);
cuError = cudaFree(d_cov);
checkCudaError(cuError);

// Print Brownian Bridge Sample Paths and increments
printf("\nThe %d Brownian Bridge paths of dimension %d and "
"corresponding increments:\n", N, dim);
printf("Bridge starts at 0, and t_{i+1}-t_i = dt = %g\n", t[0]);

// Top level grouping
for(int p=0; p<N; p++)
{
    int nspaces = (9*dim - 9)/2;
    printf("  t ");
    for(int s = 0; s < nspaces; s++) printf("-");
    printf(" Path%d ", p+1);
    for(int s = 0; s < nspaces; s++) printf("-");
    printf("-----");

    for(int s = 0; s < nspaces; s++) printf("-");
    printf(" Incr%d ", p+1);
    for(int s = 0; s < nspaces; s++) printf("-");

    // Time and Dimension column headings
    printf("\n  t_i\t");
    for(int d=1; d<=dim; d++) printf(" dim%d ", d);
    printf(" ");
    for(int d=1; d<=dim; d++) printf(" dim%d ", d);
    // Values
    for(int i = -1; i < nTimes+1; i++)
    {
        printf("\n  %d\t", i+1);
        // Print path
        for(int d=0; d<dim; d++)
        {
            double val = 0;
            if (i < 0) val = 0;
            else val = h_bb[p+N*(d+i*dim)];
            if (val < 10 && val > -10) printf("% .4f ", val);
            else printf("% .3f ", val);
        }
        printf(" ");
        // Print increments
        for(int d=0; d<dim; d++)
        {
            // Code continues...
```c
if (i < 0) printf(" ");
else {
    double val = h_bbinc[p+N*(d+i*dim)];
    if (val < 10 && val > -10) printf("% .4f ", val);
    else printf("% .3f ", val);
}
}
printf("\n\n\n");
}
delete[] h_bb;
delete[] h_bbinc;

return 0;
}

void checkNagError(NagGpuError *error)
{
    if (error->code != 0) {
        char *buff;
        buff = new char[error->msgLength];
        naggpuErrorCopyMsg(buff, error);
        cout << buff << endl;
        delete[] buff;
        exit(1);
    }
}

void checkCudaError(cudaError_t cuError)
{
    if (cuError != cudaSuccess) {
        cout << cudaGetErrorString(cuError) << endl;
        exit(1);
    }
}

7.2 Program Data
None.

7.3 Program Results
NAG GPU Example Program: naggpuBBIncA

The 2 Brownian Bridge paths of dimension 3 and corresponding increments:
Bridge starts at 0, and \( t_{(i+1)} - t_i = dt = 1 \)

<table>
<thead>
<tr>
<th>t_i</th>
<th>dim1</th>
<th>dim2</th>
<th>dim3</th>
<th>dim1</th>
<th>dim2</th>
<th>dim3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-0.3591</td>
<td>0.0563</td>
<td>-0.1112</td>
<td>-0.3591</td>
<td>0.0563</td>
<td>-0.1112</td>
</tr>
<tr>
<td>2</td>
<td>0.0792</td>
<td>0.1278</td>
<td>-0.2589</td>
<td>0.4383</td>
<td>0.0715</td>
<td>-0.1477</td>
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<tr>
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<td>0.0534</td>
<td>-0.7139</td>
<td>-1.0744</td>
</tr>
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<td>0.5419</td>
<td>0.2525</td>
</tr>
<tr>
<td>5</td>
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<td>-0.4203</td>
<td>-1.2352</td>
<td>0.6127</td>
<td>-0.3762</td>
<td>-0.1544</td>
</tr>
<tr>
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<td>-1.4617</td>
<td>-0.5288</td>
<td>-0.7990</td>
<td>-0.2265</td>
</tr>
</tbody>
</table>
```
<table>
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<tr>
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**nagpuBBIncA.8 (last)**

Mark 0.6

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**nagpuBBIncA**

NAG Numerical Routines for GPUs Manual
1 Purpose
naggpuBBIncCleanupA frees system resources which were allocated by a previous call to naggpuBBIncInitA.

2 Specification
#include <nag_gpu.h>
extern "C"
cudaError_t naggpuBBIncCleanupA(NagGpuBBIncComm *comm, NagGpuError *error)

3 Description
3.1 Synchronization
This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.

3.2 Return Value
Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References
None.

5 Arguments
1: comm – NagGpuBBIncComm * Communication Data
   The structure which was initialized by a previous call to naggpuBBIncInitA.
2: error – NagGpuError * Error Reporting
   This parameter contains error information and should not be modified directly. Errors are indicated through the value of error → code which should be inspected after each call to this function. If error → code = 0 then no error occurred. If error → code ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings
   error → code = 1
   *On entry:* the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

   error → code = 2
   *During execution:* a CUDA runtime error was detected.
error → code = 100
   On entry: the value of comm is NULL.
error → code = 101
   On entry: comm has not been initialized, or the internal state of comm is corrupted.

7 Example

There is no example program specifically for this function. For examples of how this function should be used, please see the example program for naggpuBBIncA.
1 Purpose

naggpuMakeBridgeOrderA takes a set of input times and permutes them to specify one of several pre-defined Brownian bridge construction orders. The permuted times can be passed to naggpuBBInitA or naggpuBBIncInitA to initialize the Brownian bridge generators with the chosen bridge construction order.

2 Specification

```c
#include <nag_gpu.h>
extern "C"
void naggpuMakeBridgeOrderA(NagGpuBridgeOrder order, double tStart, const double *inTimes, const int *specials, int nSpecials, double *times, int nTimes, double *tEnd, NagGpuError *error)
```

3 Description

The Brownian bridge algorithm (see Glasserman (2004)) is a popular method for constructing a Brownian motion $X$ on an interval $[t_0, T]$ at a set of discrete times $t_0 < t_1 < t_2, \ldots, t_N < T$ for some $N \geq 1$. Inherent in the algorithm is the notion of a bridge construction order which specifies the order in which points $X_{t_i}$ for $i = 1, 2, \ldots, N$ are generated. The value of $X_{t_0}$ is always assumed known, and the first Brownian point to be generated is always the final time $X_T$. Thereafter, successive points are generated iteratively by an interpolation formula, using points which were computed at previous iterations. In many cases the bridge construction order is not important, since any construction order will yield a statistically accurate Brownian motion. However, in certain cases, for example when using quasi-random variates to construct the Brownian paths, the bridge construction order can be important. For further details, please see the documentation for naggpuBBInitA.

naggpuMakeBridgeOrderA accepts as input an array `inTimes` of time points corresponding to the values $t_1, t_2, \ldots, t_N$ above at which the Brownian motion is to be sampled. These time points are then permuted to reflect the bridge construction order specified by `order`, and the result is stored in `times`. In addition, the selected construction order can be modified. An array `specials` can be provided which contains a set of indices into the input array `inTimes`. To see the effect of this array, suppose that an array $A$ holds the output of naggpuMakeBridgeOrderA when $nSpecials = 0$ (i.e. the unmodified bridge construction order as specified by `order`). Let $B = \{ \text{inTimes[specials[i]]} : 0 \leq i \leq nSpecials - 1 \}$ be the array of all times identified by `specials`, and let $C$ be the array $A$ with all the elements in $B$ removed, i.e.

$C = \{ A[i] : 0 \leq i \leq nTimes - 1 \text{ and } A[i] \notin B \}$

Then the output of naggpuMakeBridgeOrderA when $nSpecials > 0$ is given by

$[ B[0] \ B[1] \ldots \ B[nSpecials - 1] \ C[0] \ C[1] \ldots \ C[nTimes - nSpecials - 1] ]$

When the Brownian bridge is used with quasi-random variates, this functionality could be used to allow certain sections of the bridge to be constructed using the lowest dimensions of the quasi-random points.

3.1 Synchronization

As this function only uses the CPU there is no requirement for synchronization.
5 Arguments

1: order – NagGpuBridgeOrder
   
   On entry: the bridge construction order to use
   
   order = NAGGPUBRIDGEORDER_BISECT_ASCEND_ROUNDDOWN
   order = NAGGPUBRIDGEORDER_BISECT_ASCEND_ROUNDUP
   order = NAGGPUBRIDGEORDER_BISECT_DESCEND_ROUNDDOWN
   order = NAGGPUBRIDGEORDER_BISECT_DESCEND_ROUNDUP

   Please see NagGpuBridgeOrder for descriptions of the various construction orders.

   Constraint:
   
   order = NAGGPUBRIDGEORDER_BISECT_ASCEND_ROUNDDOWN or
   NAGGPUBRIDGEORDER_BISECT_ASCEND_ROUNDUP or
   NAGGPUBRIDGEORDER_BISECT_DESCEND_ROUNDDOWN or
   NAGGPUBRIDGEORDER_BISECT_DESCEND_ROUNDUP

2: tStart – double
   
   On entry: the start value $t_0$ of the time interval $[t_0, T]$ on which the Brownian motion is constructed.

3: inTimes[nTimes + 1] – const double *
   
   On entry: the time points $t_1, t_2, \ldots, t_N, T$ at which the Brownian motion is to be constructed. Note
   that the final time $T$ is included in this array.

   Constraint: the values in inTimes must be unique, must be in increasing order, and each must be
   greater than tStart.

4: specials[nSpecials] – const int *
   
   On entry: the indices of the entries in inTimes which should be moved to the front of the times
   array. When nSpecials = 0 then specials is not referenced.

   Constraint: the values in specials must all be unique and must lie in the interval $[0, nTimes - 1] .

5: nSpecials – int
   
   On entry: the number of elements in the array specials.

   Constraint: $0 \leq nSpecials \leq nTimes .

6: times[nTimes] – double *
   
   On exit: the output bridge construction order. This should be passed to naggpuBBInitA or
   naggpuBBIncInitA.

7: nTimes – int
   
   On entry: the size of the array times. This corresponds to $N$ in Section 3 above.

   Constraint: $nTimes \geq 1$.

8: tEnd – double *
   
   On exit: the value of inTimes[nTimes], which corresponds to $T$ in Section 3 above. This should be
   passed unchanged to naggpuBBInitA or naggpuBBIncInitA.
9: \textbf{error} – NagGpuError *

This parameter contains error information and should not be modified directly. Errors are indicated through the value of \textbf{error} \rightarrow \textbf{code} which should be inspected after each call to this function. If \textbf{error} \rightarrow \textbf{code} = 0 then no error occurred. If \textbf{error} \rightarrow \textbf{code} \neq 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 \textbf{Error Indicators and Warnings}

\textbf{error} \rightarrow \textbf{code} = 110

\emph{On entry:} \textbf{order} does not specify a valid bridge construction order. See NagGpuBridgeOrder for permitted values.

\textbf{error} \rightarrow \textbf{code} = 111

\emph{On entry:} the value of \textbf{inTimes} is NULL.

\textbf{error} \rightarrow \textbf{code} = 112

\emph{On entry:} the values in the \textbf{inTimes} array do not satisfy the constraints listed above.

\textbf{error} \rightarrow \textbf{code} = 113

\emph{On entry:} the value of \textbf{nTimes} does not satisfy the constraint listed above.

\textbf{error} \rightarrow \textbf{code} = 114

\emph{On entry:} the values in the \textbf{specials} array do not satisfy the constraints listed above.

\textbf{error} \rightarrow \textbf{code} = 115

\emph{On entry:} the value of \textbf{nSpecials} does not satisfy the constraint listed above.

\textbf{error} \rightarrow \textbf{code} = 116

\emph{On entry:} the value of \textbf{times} is NULL.

\textbf{error} \rightarrow \textbf{code} = 117

\emph{On entry:} the value of \textbf{tEnd} is NULL.

7 \textbf{Example}

There is no example program specifically for this function. For examples of how this function should be used, please see the example program for naggpuBBA.
NAG Numerical Routines for GPUs Function Document

**naggpuDepthBBInitA**

1 **Purpose**

naggpuDepthBBInitA initializes the depth-order Brownian bridge generator naggpuDepthBBA. It must be called before any calls to naggpuDepthBBA and must finally be followed by a call to naggpuDepthBBCleanupA.

**Note:** after the first call to naggpuDepthBBInitA, all subsequent calls (for example, to change the time points) must be preceded by a call to naggpuDepthBBCleanupA.

2 **Specification**

```c
#include <nag_gpu.h>

extern "C"

cudaError_t naggpuDepthBBInitA(float tStart, const float *times, int nTimes,
    bool isBridgeFree, NagGpuDepthBBComm *comm, NagGpuError *error)
```

3 **Description**

3.1 **Synchronization**

This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.

3.2 **Return Value**

Any CUDA runtime errors that were encountered, or `cudaSuccess` if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 **References**

None.

5 **Arguments**

1: `tStart` – float
   
   *Input*
   
   *On entry:* the starting value of the time interval.

2: `times[nTimes]` – const float *
   
   *Input*
   
   *On entry:* the vector of times at which to compute the Brownian bridge.
   
   *Constraint:* the values in `times` must be in increasing order, and each must be greater than `tStart`.

3: `nTimes` – int
   
   *Input*
   
   *On entry:* the length of the vector `times`.
   
   *Constraint:* $1 \leq nTimes \leq 4095$.

4: `isBridgeFree` – bool
   
   *Input*
   
   *On entry:* specifies whether a free or ‘pinned’ Brownian bridge is to be constructed. See naggpuDepthBBA for more details.
If \( \text{isBridgeFree} = \text{true} \), \text{naggpuDepthBBA} will construct a free Brownian motion via a depth-order Brownian bridge algorithm.

If \( \text{isBridgeFree} = \text{false} \), \text{naggpuDepthBBA} will construct a non-free or ‘pinned’ Brownian motion.

5: \text{comm} – \text{NagGpuDepthBBComm} * 

\text{NagGpuDepthBBComm} is a structure which holds state and communication information and must not be modified in any way. The structure will be initialized and must be passed to the generator function \text{naggpuDepthBBA}. Once all required bridge sample paths have been obtained, \text{comm} must be passed to \text{naggpuDepthBBCleanupA} to free allocated system resources.

6: \text{error} – \text{NagGpuError} * 

This parameter contains error information and should not be modified directly. Errors are indicated through the value of \text{error} → \text{code} which should be inspected after each call to this function. If \text{error} → \text{code} = 0 then no error occurred. If \text{error} → \text{code} \neq 0 then an error was detected and a call to \text{naggpuErrorCopyMsg} will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

\text{error} → \text{code} = 1

\textit{On entry:} the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call \text{cudaGetLastError}() in the CUDA runtime library to clear the runtime error status.

\text{error} → \text{code} = 2

\textit{During execution:} a CUDA runtime error was detected.

\text{error} → \text{code} = 100

\textit{On entry:} the value of \text{comm} is NULL.

\text{error} → \text{code} = 110

\textit{On entry:} the value of \text{times} is NULL.

\text{error} → \text{code} = 111

\textit{On entry:} the value of \text{nTimes} does not satisfy the constraint listed above.

\text{error} → \text{code} = 112

\textit{On entry:} the values in the \text{times} array do not satisfy the constraints listed above.

7 Example

There is no example program specifically for this function. For examples of how this function should be used, please see the example program for \text{naggpuDepthBBA}. 

naggpuDepthBBInitA
1 Purpose

naggpuDepthBBA constructs sample paths for a Brownian bridge or for a free Brownian motion using a depth-order bridge interpolation algorithm. It must be preceded by a call to the initialization function naggpuDepthBBInitA, and must finally be followed by a call to naggpuDepthBBCleanupA.

2 Specification

```c
#include <nag_gpu.h>
extern "C"
cudaError_t naggpuDepthBBA(int nPaths, int dim, float bgStart, float bgEnd,
const float *d_z, const float *d_cholCov, float *d_bgVals,
cudaStream_t custream, NagGpuDepthBBComm *comm, NagGpuError *error)
```

3 Description

3.1 Background

Fix $T > 0$ and let $W = (W_t)_{0 \leq t \leq T}$ be a standard $d$-dimensional Wiener process. A standard $d$-dimensional Brownian bridge $B = (B_t)_{0 \leq t \leq T}$ is defined (see Revuz and Yor (1999)) as

$$B_t = W_t - \frac{t}{T} W_T$$

for all $t \in [0, T]$. This process is continuous, starts at zero at time 0 and ends at zero at time $T$. It is Gaussian, has zero mean and has a covariance structure given by

$$\mathbb{E}(B_s B_t') = s \left(1 - \frac{t}{T}\right) I_d = \frac{s(T-t)}{T} I_d$$

for any $s \leq t$ in $[0, T]$ where $I_d$ is the $d$-dimensional identity matrix. The Brownian bridge is often called a non-free or 'pinned' Brownian motion, since it is forced to be equal to 0 at time $T$ but is otherwise very similar to a standard Brownian motion.

We can generalize this construction as follows. Fix points $x, w \in \mathbb{R}^d$, let $\Sigma$ be a $d \times d$ covariance matrix and choose any $d \times d$ matrix $C$ such that $CC' = \Sigma$. We will define the generalized $d$-dimensional Brownian bridge $X = (X_t)_{0 \leq t \leq T}$ by setting

$$X_t = tw + (T - t)x + CB_t = \frac{tw + (T - t)x}{T} + CW_t - \frac{t}{T} CW_T$$

for all $t \in [0, T]$. The process $X$ is therefore continuous, starts at $x$ at time zero and ends at $w$ at time $T$. It has time-dependent mean $(tw + (T - t)x)/T$ and has the covariance structure

$$\mathbb{E}(X_s - \mathbb{E}X_t)(X_t - \mathbb{E}X_t)' = \mathbb{E}(CB_s B_t'C') = \frac{s(T-t)}{T} CC' = \frac{s(T-t)}{T} \Sigma$$

for all $s \leq t$ in $[0, T]$. This is a non-free bridge since it is forced to be equal to $w$ at time $T$. However if we set $w = x + CW_T$, then $X$ simplifies to

$$X_t = x + CW_t$$

for all $t \in [0, T]$ which is a free $d$-dimensional Brownian motion with covariance given by $\Sigma$.

3.2 Implementation

The bridge is generated in a modified depth-first order. Suppose there are $N$ time points $t_1, \ldots, t_N$ at which the bridge is to be computed. The algorithm starts by taking the known values $X_{t_0} = x$ and...
$X_t = w$ and then generating

$$X_{t[N/2]}, X_{t[N/4]}, X_{t[N/8]}, \ldots, X_t$$

according to the standard Brownian bridge interpolation formula (see Glasserman (2004)). Once $X_t$ is reached, the algorithm moves upwards from $t_1$ searching for an interval $[t_i, t_k]$ such that both $X_{t_i}$ and $X_{t_k}$ are already known, but all $X_{t_j}$ for $i < j < k$ are not. This interval is then treated in the same way as the interval $[t_0, t_N]$, and the process repeats until all points are computed.

The main input to the bridge algorithm is an array of standard Normal random numbers. If these come from a quasi-random generator (e.g., Sobol numbers), then the order in which these numbers are used becomes important. Suppose that the bridge is one-dimensional and that we have an $N$-dimensional quasi-random point. Roughly speaking, the algorithm uses the dimensions in this point in breadth-first order: the first dimension is used to compute $X_{t[N/2]}$, the second dimension is used to compute $X_{t[N/4]}$, the third to compute $X_{t[N/8]}$, the fourth to compute $X_{t[N/16]}$, and so on. For a $d$-dimensional bridge, and corresponding $N \times d$ dimensional quasi-random point, the first $d$ dimensions are used to compute $X_{t[N/2]}$, the second $d$ to compute $X_{t[N/4]}$, the third $d$ to compute $X_{t[N/8]}$, and so on. If the bridge is free, in other words $X_t = x + CW_t$, then the first $d$ dimensions are used to compute $X_{t_1}$, the second $d$ to compute $X_{t_2}$, the third $d$ to compute $X_{t_3}$, and so on.

The boolean parameter `isBridgeFree` in the initialization function `naggpuDepthBBInitA` whether a free or non-free Brownian sample path is created. Note that the final value $w$ of the bridge is always stored, whereas the starting value $x$ is never stored. The algorithm therefore only produces the values $X_{t_1}, X_{t_2}, X_{t_3}, \ldots, X_{t_N}$.

### 3.3 Synchronization

This function is non-blocking. Control will return immediately to the calling program while the computation is executed on the GPU. The user is responsible for synchronization between host and GPU code. Please see the synchronization chapter in the CUDA Programming Guide for further details in this direction. For example, a call to `cudaMemcpy` in the CUDA runtime library is enough to force the host to wait for the GPU to finish, and then copy the results from the GPU to the host.

### 3.4 Return Value

Any CUDA runtime errors that were encountered, or `cudaSuccess` if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

### 4 References


Revuz D and Yor M (1999) *Continuous Martingales and Brownian Motion* Springer

### 5 Arguments

1: `nPaths` – `int` 
   
   *Input*

   *On entry:* the number of Brownian bridge sample paths to create.

   *Constraint:* $nPaths \geq 1$.

2: `dim` – `int` 

   *Input*

   *On entry:* the dimension of each Brownian bridge sample path.

   *Constraint:* $1 \leq dim \leq 8$.

3: `bgStart` – `float` 

   *Input*

   *On entry:* the starting value $x$ of the bridge.
4: \texttt{bgEnd} – float

\textit{Input}

On entry: the final value $w$ of the bridge. If \texttt{naggpuDepthBBInitA} was called with \texttt{isBridgeFree = true}, this value is ignored and $w$ is set equal to $x + C W_T$.

5: \texttt{d_z} [\texttt{dim} \times \texttt{N} \times \texttt{nPaths}] – const float *

\textit{Input}

This buffer must reside in the GPU memory space.

The variable $N$ denotes the length \texttt{nTimes} of the \texttt{times} array passed to the initialization function \texttt{naggpuDepthBBInitA}.

On entry: the Normal random numbers used to construct the bridge.

\textit{Constraints:}

If \texttt{naggpuDepthBBInitA} was called with \texttt{isBridgeFree = true}, then \texttt{d_z} must contain $N \times \texttt{dim} \times \texttt{nPaths}$ values. The values should be laid out as a matrix with $\texttt{dim} \times \texttt{N}$ rows and \texttt{nPaths} columns. If quasi-random numbers are to be used, successive $\texttt{dim} \times \texttt{N}$-dimensional points should be stored in successive columns of the matrix, i.e. an ordering corresponding to \texttt{NAGGPUQUASIORIENT_DIMVALS_SCATT};

If \texttt{naggpuDepthBBInitA} was called with \texttt{isBridgeFree = false}, then \texttt{d_z} must contain $(N-1) \times \texttt{dim} \times \texttt{nPaths}$ values. The values should be laid out as a matrix with $\texttt{dim} \times (N-1)$ rows and \texttt{nPaths} columns. If quasi-random numbers are to be used, successive $\texttt{dim} \times \texttt{N}$-dimensional points should be stored in successive columns of the matrix, i.e. an ordering corresponding to \texttt{NAGGPUQUASIORIENT_DIMVALS_SCATT}.

6: \texttt{d_cholCov} [\texttt{dim} \times \texttt{dim}] – const float *

\textit{Input}

This buffer must reside in the GPU memory space.

On entry: the matrix $C$ which specifies the correlation structure of the Brownian bridge. $C$ should be chosen such that $C C^T = \Sigma$ where $\text{Cov}(X_s, X_t) = s(T - t) / T \Sigma$ for all $s \leq t$ in $[0, T]$.

7: \texttt{d_bgVals} [\texttt{dim} \times \texttt{N} \times \texttt{nPaths}] – float *

\textit{Output}

This buffer must reside in the GPU memory space.

The variable $N$ denotes the length \texttt{nTimes} of the \texttt{times} array passed to the initialization function \texttt{naggpuDepthBBInitA}.

On exit: the values of the Brownian bridge. If $x_{p,i}^d$ denotes the $d$-th dimension of the $i$-th point of the $p$-th sample path where $0 \leq d < \texttt{dim}$, $0 \leq i < \texttt{N}$ and $0 < p < \texttt{nPaths}$, then $x_{p,i}^d$ will be stored at \texttt{d_bgVals}[p + \texttt{nPaths} \times (d + i \times \texttt{dim})].$ The starting value $\texttt{bgStart}$ is never stored, while the terminal value $\texttt{bgEnd}$ is always stored.

8: \texttt{custream} – cudaStream_t

\textit{Input}

On entry: specifies the CUDA stream on which to launch the selected GPU kernel. If no streams are used, set this parameter to 0. Please see the chapter on Streams in the CUDA Programming Guide for further details.

9: \texttt{comm} – NagGpuDepthBBComm *

\textit{Communication Data}

NagGpuDepthBBComm is a structure which holds state and communication information and must not be modified in any way. Once all required bridge sample paths have been obtained, \texttt{comm} must be passed to \texttt{naggpuDepthBBCleanupA} to free allocated system resources.

10: \texttt{error} – NagGpuError *

\textit{Error Reporting}

This parameter contains error information and should not be modified directly. Errors are indicated through the value of \texttt{error -> code} which should be inspected after each call to this function. If \texttt{error -> code} = 0 then no error occurred. If \texttt{error -> code} $\neq 0$ then an error was detected and a call to \texttt{naggpuErrorCopyMsg} will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.
6 Error Indicators and Warnings

**error → code = 1**

*On entry:* the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call `cudaGetLastError()` in the CUDA runtime library to clear the runtime error status.

**error → code = 2**

*During execution:* a CUDA runtime error was detected.

**error → code = 100**

*On entry:* the value of `comm` is NULL.

**error → code = 101**

*On entry:* `comm` has not been initialized, or the internal state of `comm` is corrupted.

**error → code = 110**

*On entry:* the value of `nPaths` does not satisfy the constraint listed above.

**error → code = 111**

*On entry:* the value of `dim` does not satisfy the constraint listed above.

**error → code = 112**

*On entry:* the value of `d_z` is NULL.

**error → code = 113**

*On entry:* the value of `d_cholCov` is NULL.

**error → code = 115**

*On entry:* the value of `d_bgVals` is NULL.

7 Example

This example program uses `naggpuDepthBBA` to print two Brownian bridge sample paths where each path is three dimensional. The bridge is pinned to end at a fixed value, and the inputs are quasi-random Normal numbers from the generator `naggpuQuasiRandNormalA`.

7.1 Program Text

```c
#include <stdio.h>
#include using namespace std;
#include <nag_gpu.h>
#include <nag_gpu_serial.h>

void checkNagError(NagGpuError *error);
void checkCudaError(cudaError_t cuError);

int main(int argc, char **argv)
{
```
// Number of time steps bridge - less than 4096
const int nTimes = 30;
// Dimension of bridge - less than 8
const int dim = 3;
// Number of sample paths to generate
int N = 50;

// NAG structures
NagGpuError error;
NagCPURandComm pcomm;
NagGpuQuasiRandComm qcomm;
NagGpuDepthBBComm bbcomm;
NagGpuQuasiOrient orient = NAGGPUQUASIORIENT_DIMVALS_SCATT;

// Print the title
cout << "NAG GPU Example Program: ";
cout << "naggpuDepthBBA";
cout << endl << endl;

// Initialise the generator only once
cout << "Initialising generators ..." << endl << endl;

// Initialise the CPU pseudorandom generator
unsigned int seed[] = {1, 2, 3, 4, 5, 6};
nagCPURandInitA(NAGGPURANDGEN_MRG32K3A, 0, 0, 0, 0, 0, seed,
                         &pcomm, &error);
checkNagError(&error);

// Initialise the quasi-random generator
naggpuQuasiRandInitA(NAGGPUQUASIGEN_SOBOL, NAGGPUSCRAMTYPES_NONE,
                         dim*nTimes, 0, &pcomm, &qcomm, &error);
checkNagError(&error);

// Cleanup the pseudorandom generator
nagCPURandCleanupA(&pcomm, &error);
checkNagError(&error);

// Generate the Sobol numbers
float *d_z = NULL;
cuError = cudaMalloc((void **)&d_z, sizeof(float)*nTimes*dim*N);
checkCudaError(cuError);
naggpuQuasiRandNormalA_sp(N, orient, 0.0f, 1.0f, d_z, NULL, 0,
                                &qcomm, &error);
checkNagError(&error);

// Create the bridge time points
float times[nTimes];
for(int i=0; i<nTimes; i++) times[i] = (i+1)*0.477f;

// Specify bridge setup
float bridgeStart = -0.5;
float bridgeEnd = 1.5;
bool isBridgeFree = false;

// Initialise the bridge generator
naggpuDepthBBInitA(0.0f, times, nTimes, isBridgeFree,
                           &bbcomm, &error);
checkNagError(&error);

// Create covariance structure and copy to GPU
float cov[dim*dim];
for(int i=0; i<dim*dim; i++) cov[i] = 0.1f;
for(int i=0; i<dim; i++)
   for(int j=(i+1)*dim) cov[i*dim+j] = 0.31f;
float *d_cov = NULL;
cuError = cudaMalloc((void **)&d_cov, sizeof(float)*dim*dim);
checkNagError(&error);
checkCudaError(cuError);
cuError = cudaMemcpy(d_cov, cov, sizeof(float)*dim*dim,
    cudaMemcpyHostToDevice);
checkCudaError(cuError);

cout << "Creating the bridge ..." << endl << endl;

// Generate bridge and copy back to host
float *d_buff = NULL;
cuError = cudaMalloc((void **)&d_buff, sizeof(float)*dim*nTimes*N);
checkCudaError(cuError);
float *h_buff = new float[dim*nTimes*N];
naggpuDepthBBA(N, dim, bridgeStart, bridgeEnd, d_z, d_cov, d_buff,
    0, &bbcomm, &error);
checkNagError(&error);
cuError = cudaMemcpy(h_buff, d_buff, sizeof(float)*dim*nTimes*N,
    cudaMemcpyDeviceToHost);
checkCudaError(cuError);

// Cleanup quasi-generator and bridge
naggpuQuasiRandCleanupA(&qcomm, &error);
checkNagError(&error);
naggpuDepthBBCleanupA(&bbcomm, &error);
checkNagError(&error);
cuError = cudaFree(d_z);
checkCudaError(cuError);
cuError = cudaFree(d_cov);
checkCudaError(cuError);
cuError = cudaFree(d_buff);
checkCudaError(cuError);

// Transpose the data in order to display it
const int nPrint = 2;
float *print = new float[dim*nTimes*nPrint];
for(int p=0; p<nPrint; p++)
{    for(int i=0; i<nTimes; i++)
        {            for(int d=0; d<dim; d++)
                {                    print[d+p*dim+i*dim*nPrint] = h_buff[p+N*(d+i*dim)];                }
            }
    delete[] h_buff;
}

// Print Brownian Bridge Sample Paths
cout << "The first " << nPrint << " bridge paths of dimension "
    << dim << ":" << endl;
cout << " t_i 	";
for(int p = 0; p < nPrint; p++)
{    for(int s=0; s<nSpaces; s++) cout << "-";
        cout << " Path " << p+1 << " ";
    for(int s=0; s<nSpaces; s++) cout << "-";
        cout << " t_i\t";
    for(int d=1; d<dim; d++) cout << " dim " << d << " ";
}

cout << "\t";
for(int i=0; i<nTimes; i++)
{
    cout << "\n " << i+1 << "\t";
    for(int p=0; p<nPrint; p++)
    {
        for(int d=0; d<dim; d++)
        {
            float val = print[i*dim*nPrint + p*dim + d];
            if (val < 10 && val > -10) printf("% .3f ", val);
            else printf("% .2f ", val);
        }
        cout << "\t";
    }
    cout << endl;
}
return 0;

void checkNagError(NagGpuError *error)
{
    if (error->code != 0)
    {
        char *buff;
        buff = new char[error->msgLength];
        naggpuErrorCopyMsg(buff, error);
        cout << buff << endl;
        delete[] buff;
        exit(1);
    }
}

void checkCudaError(cudaError_t cuError)
{
    if (cuError != cudaSuccess)
    {
        cout << cudaGetErrorString(cuError) << endl;
        exit(1);
    }
}

7.2 Program Data

None.

7.3 Program Results

NAG GPU Example Program: naggpuDepthBBA

Initialising generators ...

Creating the bridge ...

The first 2 bridge paths of dimension 3:

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<th>dim3</th>
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1 Purpose
naggpuDepthBBIncInitA initializes the depth-order Brownian bridge increments generator naggpuDepthBBIncA. This function must be called before any calls to naggpuDepthBBIncA and must finally be followed by a call to naggpuDepthBBCleanupA.

Note: after the first call to naggpuDepthBBIncInitA, all subsequent calls (for example, to change the time points) must be preceded by a call to naggpuDepthBBCleanupA.

2 Specification

```c
#include <nag_gpu.h>
extern "C"

cudaError_t naggpuDepthBBIncInitA(float tStart, const float *times, int nTimes,
    bool isBridgeFree, NagGpuDepthBBComm *comm, NagGpuError *error)
```

3 Description
3.1 Synchronization
This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.

3.2 Return Value
Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References
None.

5 Arguments
1: tStart – float  
   Input  
   On entry: the starting value of the time interval.
2: times[nTimes] – const float *  
   Input  
   On entry: the vector of times at which to compute the Brownian bridge.  
   Constraint: the values in times must be in increasing order, and each must be greater than tStart.
3: nTimes – int  
   Input  
   On entry: the length of the vector times.  
   Constraint: 1 ≤ nTimes ≤ 4095.
4: isBridgeFree – bool  
   Input  
   On entry: specifies whether scaled increments for a free or ‘pinned’ Brownian bridge is to be constructed. See naggpuDepthBBIncA for more details.
If `isBridgeFree = true`, `naggpuDepthBBIncA` will construct scaled increments of a free Brownian motion via a depth-order Brownian bridge algorithm.

If `isBridgeFree = false`, `naggpuDepthBBIncA` will construct scaled increments of a non-free or 'pinned' Brownian motion.

5: `comm` – NagGpuDepthBBComm *

Communication Data

NagGpuDepthBBComm is a structure which holds state and communication information and must not be modified in any way. The structure will be initialized and must be passed to the generator function `naggpuDepthBBIncA`. Once all required bridge increments have been obtained, `comm` must be passed to `naggpuDepthBBCleanupA` to free allocated system resources.

6: `error` – NagGpuError *

Error Reporting

This parameter contains error information and should not be modified directly. Errors are indicated through the value of `error -> code` which should be inspected after each call to this function. If `error -> code = 0` then no error occurred. If `error -> code ≠ 0` then an error was detected and a call to `naggpuErrorCopyMsg` will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

`error -> code = 1`

*On entry:* the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call `cudaGetLastError()` in the CUDA runtime library to clear the runtime error status.

`error -> code = 2`

*During execution:* a CUDA runtime error was detected.

`error -> code = 100`

*On entry:* the value of `comm` is NULL.

`error -> code = 110`

*On entry:* the value of `times` is NULL.

`error -> code = 111`

*On entry:* the value of `nTimes` does not satisfy the constraint listed above.

`error -> code = 112`

*On entry:* the values in the `times` array do not satisfy the constraints listed above.

7 Example

There is no example program specifically for this function. For examples of how this function should be used, please see the example program for `naggpuDepthBBIncA`.

__________________________________________________________________________________________________________________________ ... ______________________________________________________________________
### NAG Numerical Routines for GPUs Function Document

#### naggpuDepthBBIncA

1 **Purpose**

naggpuDepthBBIncA computes scaled increments of a depth-order Brownian bridge or free Brownian motion. It must be preceded by a call to the initialization function naggpuDepthBBIncInitA, and must finally be followed by a call to naggpuDepthBBCleanupA.

2 **Specification**

```c
#include <nag_gpu.h>
extern "C"
cudaError_t naggpuDepthBBIncA(int nPaths, int dim, float startEndDiff,
                              const float *d_z, const float *d_cholCov, float *d_bgIncs,
                              cudaStream_t custream, NagGpuDepthBBComm *comm, NagGpuError *error)
```

3 **Description**

Fix $T > 0$ and suppose that $0 = t_0 < t_1 < \cdots < t_N = T$. Conceptually, this algorithm first constructs a depth-order Brownian bridge $X = (X_t)_{0 \leq t \leq T}$ in the same way as naggpuDepthBBA and then computes

$$
\frac{X_{t_0} - X_{t_1}}{t_1 - t_0}, \frac{X_{t_1} - X_{t_2}}{t_2 - t_1}, \ldots, \frac{X_{t_N} - X_{t_{N-1}}}{t_N - t_{N-1}}.
$$

Such increments can be useful when computing numerical solutions to stochastic differential equations driven by either a Brownian bridge or a free Brownian motion. For more details on the Brownian bridge and how it is constructed, see the discussion in naggpuDepthBBA.

We briefly recall some notation: for further details see naggpuDepthBBA. We let $W = (W_t)_{0 \leq t \leq T}$ be a standard $d$-dimensional Wiener process, we let $\Sigma$ be a $d \times d$ covariance matrix, we choose $C$ to be a $d \times d$ matrix such that $CC' = \Sigma$, and we fix two points $x$ and $w$ in $\mathbb{R}^d$. The generalized Brownian bridge $X = (X_t)_{0 \leq t \leq T}$ is defined as

$$
X_t = \frac{tw + (T-t)x}{T} + CW_t - \frac{t}{T}CW_T
$$

for all $t \in [0, T]$ so that $X_0 = x$, $X_T = w$ and $\text{Cov}(X_s, X_t) = s(T-t)/T\Sigma$ for all $s \leq t$ in $[0, T]$. This process is a non-free or ‘pinned’ Brownian motion since $X_T = w$. However if we set $w = x + CW_T$ then $X_t = x + CW_t$ becomes a standard, correlated $d$-dimensional Brownian motion. The boolean parameter `isBridgeFree` in the initialization routine naggpuDepthBBIncInitA controls whether a free or non-free Brownian sample path is created.

3.1 **Synchronization**

This function is non-blocking. Control will return immediately to the calling program while the computation is executed on the GPU. The user is responsible for synchronization between host and GPU code. Please see the synchronization chapter in the CUDA Programming Guide for further details in this direction. For example, a call to `cudaMemcpy` in the CUDA runtime library is enough to force the host to wait for the GPU to finish, and then copy the results from the GPU to the host.

3.2 **Return Value**

Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.
4 References
None.

5 Arguments

1: \(nPaths\) – int

\(On\ entry: \) the number of Brownian bridge sample paths that are created.

\(Constraint:\ nPaths \geq 1.\)

2: \(dim\) – int

\(On\ entry: \) the dimension of each Brownian bridge sample path.

\(Constraint: \ 1 \leq dim \leq 8.\)

3: \(startEndDiff\) – float

\(On\ entry: \) the difference between \(X_{t_N}\) and \(X_{t_0}.\) If \(naggpuDepthBBIncInitA\) was called with \(isBridgeFree = \text{true},\) this value is ignored and \(X_{t_N}\) is set equal to \(x + CW_T.\)

4: \(d_z[\text{dim} \times N \times nPaths]\) – const float *

This buffer must reside in the GPU memory space.

The variable \(N\) denotes the length \(nTimes\) of the \(\text{times}\) array passed to the initialization function \(naggpuDepthBBIncInitA.\)

\(On\ entry: \) the Normal random numbers used to construct the bridge.

\(Constraints:\)

If \(naggpuDepthBBIncInitA\) was called with \(isBridgeFree = \text{true},\) then \(d_z\) must contain \(N \times \text{dim} \times \text{nPaths}\) values. The values should be laid out as a matrix with \(\text{dim} \times N\)-dimensional points should be stored in successive columns of the matrix, i.e. an ordering corresponding to \(NAGGPUQUASIORIENT_DIMVALS_SCATT;\)

If \(naggpuDepthBBIncInitA\) was called with \(isBridgeFree = \text{false},\) then \(d_z\) must contain \((N-1) \times \text{dim} \times \text{nPaths}\) values. The values should be laid out as a matrix with \(\text{dim} \times (N-1)\) rows and \(\text{nPaths}\) columns. If quasi-random numbers are to be used, successive \(\text{dim} \times N\)-dimensional points should be stored in successive columns of the matrix, i.e. an ordering corresponding to \(NAGGPUQUASIORIENT_DIMVALS_SCATT.\)

5: \(d_{\text{cholCov}}[\text{dim} \times \text{dim}]\) – const float *

This buffer must reside in the GPU memory space.

\(On\ entry: \) the matrix \(C\) which specifies the correlation structure of the Brownian bridge. \(C\) should be chosen such that \(CC' = \Sigma\) where \(\text{Cov}(X_s, X_t) = s(T-t)/T\Sigma\) for all \(s \leq t\) in \([0, T].\)

6: \(d_{\text{bgIncs}}[\text{dim} \times N \times \text{nPaths}]\) – float *

This buffer must reside in the GPU memory space.

The variable \(N\) denotes the length \(nTimes\) of the \(\text{times}\) array passed to the initialization function \(naggpuDepthBBIncInitA.\)

\(On\ exit: \) the scaled increments of the Brownian bridge. If \(x_{p,i}^{d}\) denotes the \(d\)-th dimension of the \(i\)-th point of the \(p\)-th sample path where \(0 \leq d < \text{dim}, 0 \leq i < N\) and \(0 \leq p < \text{nPaths},\) then the scaled increment \(((x_{p,i+1}^{d} - x_{p,i}^{d})/(t_{i+1} - t_i)\) will be stored at \(d_{\text{bgIncs}}[p + \text{nPaths}(d + i \times \text{dim})].\)
7: **custream** – cudaStream_t 

*Input*

On entry: specifies the CUDA stream on which to launch the selected GPU kernel. If no streams are used, set this parameter to 0. Please see the chapter on Streams in the CUDA Programming Guide for further details.

8: **comm** – NagGpuDepthBBComm *  

*Communication Data*

NagGpuDepthBBComm is a structure which holds state and communication information and must not be modified in any way. Once all required bridge sample paths have been obtained, **comm** must be passed to naggpuDepthBBCleanupA to free allocated system resources.

9: **error** – NagGpuError *  

*Error Reporting*

This parameter contains error information and should not be modified directly. Errors are indicated through the value of **error** → **code** which should be inspected after each call to this function. If **error** → **code** = 0 then no error occurred. If **error** → **code** ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

**error** → **code** = 1

*On entry:* the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

**error** → **code** = 2

*During execution:* a CUDA runtime error was detected.

**error** → **code** = 100

*On entry:* the value of **comm** is NULL.

**error** → **code** = 101

*On entry:* **comm** has not been initialized, or the internal state of **comm** is corrupted.

**error** → **code** = 110

*On entry:* the value of **nPaths** does not satisfy the constraint listed above.

**error** → **code** = 111

*On entry:* the value of **dim** does not satisfy the constraint listed above.

**error** → **code** = 112

*On entry:* the value of **d_z** is NULL.

**error** → **code** = 113

*On entry:* the value of **d_cholCov** is NULL.

**error** → **code** = 114

*On entry:* the value of **d_bgIncs** is NULL.

7 Example

This example program uses naggpuDepthBBIncA and naggpuDepthBBA to print a Brownian bridge sample path and the corresponding scaled path increments side by side. Each sample path is three dimensional and the bridge is pinned to end at a fixed value. To aid comparison, the distance between time points \( \Delta t \) := \( t_{i+1} - t_i = 1 \) so that the scaled increments \( (X_{t_{i+1}} - X_{t_i})/(t_{i+1} - t_i) \) are in fact the true bridge increments \( X_{t_{i+1}} - X_{t_i} \).
# Program Text

```c
/*
 * Example Program: naggpu_depthbbA
 * Copyright 2009, Numerical Algorithms Group Ltd, Oxford, UK.
 * Version 0.4, 2011.
 */

#include <stdio.h>
#include <nag_gpu.h>
#include <nag_gpu_serial.h>

void checkNagError(NagGpuError *error);
void checkCudaError(cudaError_t cuError);

int main(int argc, char **argv)
{
    // Number of time steps bridge - less than 4096
    const int nTimes = 30;
    // Dimension of bridge - less than 8
    const int dim = 3;
    // Number of sample paths to generate
    int N = 2;

    // NAG structures
    NagGpuError error;
    NagCPURandComm pcomm;
    NagGpuQuasiRandComm qcomm;
    NagGpuDepthBBComm bbcomm;
    NagGpuQuasiOrient orient = NAGGPUQUASIORIENT_DIMVALS_SCATT;

    cudaError_t cuError;
    // Print the title
    cout << "NAG GPU Example Program: ";
    cout << "naggpuDepthBBIncA";
    cout << endl << endl;

    // Initialise the CPU pseudorandom generator
    unsigned int seed[] = {1,2,3,4,5,6};
    nagCPURandInitA(NAGGPURANDGEN_MRG32K3A, 0, 0, 0, 0, 0, seed,
                    &pcomm, &error);
    checkNagError(&error);
    // Initialise the quasi-random generator
    naggpuQuasiRandInitA(NAGGPUQUASIGEN_SOBOL, NAGGPUQUASICTYPES_NONE,
                          dim*nTimes, 0, &pcomm, &error);
    checkNagError(&error);
    // Cleanup the pseudorandom generator
    nagCPURandCleanupA(&pcomm, &error);
    checkNagError(&error);

    // Generate the Sobol numbers and clean up
    float *d_z = NULL;
    cuError = cudaMalloc((void **)&d_z, sizeof(float)*nTimes*dim*N);
    checkCudaError(cuError);
    naggpuQuasiRandNormalA_sp(N, orient, 0.0f, 1.0f, d_z, NULL, 0,
                               &qcomm, &error);
    checkNagError(&error);
    naggpuQuasiRandCleanupA(&qcomm, &error);
    checkNagError(&error);
}
```
// Create the bridge time points
float times[nTimes];
for(int i=0; i<nTimes; i++) times[i] = (i+1)*1.0f;

// Specify bridge setup
float bridgeStart = -0.5;
float bridgeEnd = 1.5;
bool isBridgeFree = false;

// Create covariance structure and copy to GPU
float cov[dim*dim];
for(int i=0; i<dim*dim; i++) cov[i] = 0.1f;
for(int i=0; i<dim; i++) cov[i*(dim+1)] = 0.31f;
float *d_cov = NULL;
cuError = cudaMalloc((void **)&d_cov, sizeof(float)*dim*dim);
checkCudaError(cuError);

cuError = cudaMemcpy(d_cov, cov, sizeof(float)*dim*dim, cudaMemcpyHostToDevice);
checkCudaError(cuError);

// Create storage for bridge numbers on host and device
float *d_buff = NULL;
cuError = cudaMalloc((void**)&d_buff, sizeof(float)*dim*nTimes*N);
checkCudaError(cuError);
float *h_bb = new float[dim*nTimes*N];
float *h_bbinc = new float[dim*nTimes*N];

// Initialise the bridge generator, generate and clean up
naggpuDepthBBInitA(0.0f, times, nTimes, isBridgeFree, 
                    &bbcomm, &error);
checkNagError(&error);
naggpuDepthBBRA(N, dim, bridgeStart, bridgeEnd, d_z, d_cov, d_buff, 0,
                    &bbcomm, &error);
checkNagError(&error);
cuError = cudaMemcpy(h_bb, d_buff, sizeof(float)*dim*nTimes*N, 
                        cudaMemcpyDeviceToHost);
checkCudaError(cuError);
naggpuDepthBBCleanupA(&bbcomm, &error);
checkNagError(&error);

// Initialise the bridge increments generator, generate and clean up
naggpuDepthBBIncInitA(0.0f, times, nTimes, isBridgeFree, 
                      &bbcomm, &error);
checkNagError(&error);
naggpuDepthBBIncA(N, dim, bridgeEnd-bridgeStart, d_z, d_cov, 
                   d_buff, 0, &bbcomm, &error);
checkNagError(&error);
cuError = cudaMemcpy(h_bbinc, d_buff, sizeof(float)*dim*nTimes*N, 
                      cudaMemcpyDeviceToHost);
checkCudaError(cuError);
naggpuDepthBBCleanupA(&bbcomm, &error);
checkNagError(&error);

// Free GPU memory
cuError = cudaFree(d_z);
checkCudaError(cuError);
cuError = cudaFree(d_buff);
checkCudaError(cuError);
cuError = cudaFree(d_cov);
checkCudaError(cuError);

// Print Brownian Bridge Sample Paths and increments
The %d Brownian Bridge paths of dimension %d and corresponding increments:
Bridge starts at \( x = %g \), and \( t_{i+1} - t_i = \Delta t = %g \n\):

// Top level grouping
for(int p=0; p<N; p++)
{
    int nspaces = (9*dim - 9)/2;
    printf("\
\t \t\t \
\t\t\t--
\t\t\t--
\t\t\t--
Path%d ", p+1);
    printf("-----");
    for(int s = 0; s < nspaces; s++) printf("--");
    printf(" Incr%d ", p+1);
    for(int s = 0; s < nspaces; s++) printf("--");

    // Time and Dimension column headings
    printf(" t_i\t" );
    for(int d=1; d<=dim; d++) printf(" dim%d ", d);
    printf(" ");
    for(int d=1; d<=dim; d++) printf(" dim%d ", d);

    // Values
    for(int i = -1; i < nTimes; i++)
    {
        float val = 0;
        if (i < 0) val = bridgeStart;
        else val = h_bb[p+N*(d+i*dim)];
        if (val < 10 && val > -10) printf("%.4f ", val);
        else printf("%.3f ", val);
    }
    printf(" ");
    // Print increments
    for(int d=0; d<dim; d++)
    {
        if (i < 0) printf(" ");
        else
        {
            float val = h_bbinc[p+N*(d+i*dim)];
            if (val < 10 && val > -10) printf("%.4f ", val);
            else printf("%.3f ", val);
        }
    }
    printf("\n\n\n\n");
}

delete[] h_bb;
delete[] h_bbinc;

return 0;
}

void checkNagError(NagGpuError *error)
{
    if (error->code != 0)
7.2 Program Data

None.

7.3 Program Results

NAG GPU Example Program: naggpuDepthBBIncA

The 2 Brownian Bridge paths of dimension 3 and corresponding increments:
Bridge starts at $x = -0.5$, and $t_{i+1} - t_i = dt = 1$

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<th>dim2</th>
<th>dim3</th>
<th>dim1</th>
<th>dim2</th>
<th>dim3</th>
</tr>
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<td>Path1</td>
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<td></td>
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NAG Numerical Routines for GPUs Function Document

naggpuDepthBBCleanupA

1 Purpose
naggpuDepthBBCleanupA frees system resources which were allocated by a previous call to naggpuDepthBBInitA or naggpuDepthBBIncInitA.

2 Specification
#include <nag_gpu.h>
extern "C"
cudaError_t naggpuDepthBBCleanupA(NagGpuDepthBBComm *comm, NagGpuError *error)

3 Description
3.1 Synchronization
This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.

3.2 Return Value
Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References
None.

5 Arguments
1: comm – NagGpuDepthBBComm * Communication Data
The structure which was initialized by a previous call to naggpuDepthBBInitA or naggpuDepthBBIncInitA.

2: error – NagGpuError * Error Reporting
This parameter contains error information and should not be modified directly. Errors are indicated through the value of error → code which should be inspected after each call to this function. If error → code = 0 then no error occurred. If error → code ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings
error → code = 1

On entry: the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastErrror() in the CUDA runtime library to clear the runtime error status.

error → code = 2

During execution: a CUDA runtime error was detected.
error → code = 100

On entry: the value of comm is NULL.

error → code = 101

On entry: comm has not been initialized, or the internal state of comm is corrupted.

7 Example

There is no example program specifically for this function. For examples of how this function should be used, please see the example program for naggpuDepthBBA.
naggpuMrg32k3aDeviceInitA

1 Purpose
naggpuMrg32k3aDeviceInitA creates initialization data on the GPU for the MRG32k3a device function generators. It must be called prior to a call to naggpudevMrg32k3aInitA and must ultimately be followed by a call to naggpuMrg32k3aDeviceCleanupA to free allocated system resources.

2 Specification
#include <nag_gpu.h>
extern "C"
cudaError_t naggpuMrg32k3aDeviceInitA(int a1, int b1, int a2, int b2, long long c,
             unsigned int *seed, NagGpuMrg32k3aDeviceComm **devComm,
             NagGpuRandComm *comm, NagGpuError *error)

3 Description
To use the MRG32k3a device function generators, the memory address of a NagGpuMrg32k3aDeviceComm structure must be obtained from the device generator initialization function naggpuMrg32k3aDeviceInitA. This structure will reside in the GPU memory space and will contain communication data for use by the device function generator. This GPU memory address must then be passed to the MRG32k3a device generator initialization function naggpudevMrg32k3aInitA. Once all values have been obtained from the device function generators, the same NagGpuMrg32k3aDeviceComm structure memory address must be passed to naggpuMrg32k3aDeviceCleanupA to free allocated system resources.

This function will create the NagGpuMrg32k3aDeviceComm communication structure in the GPU memory space and will assign its address to devComm. It also requires the seed for the generator to be passed in for error checking and verification. The seed can optionally be skipped ahead (see below) by a user-specified amount. Note that this function does NOT copy the seed to the device: the supplied seed is verified and skipped ahead (if required), but is left on the host. The user must transfer the seed to the device and ultimately pass it to naggpudevMrg32k3aInitA. This is for performance reasons: users will typically wish to exploit the GPU’s memory hierarchy and place the seed in a specific location, be it registers, shared memory, or elsewhere.

3.1 Parallelization
For different values of seed, a given generator will yield different sequences of random numbers. Alternatively, the same sequence of random numbers will be generated if the same value of seed is used. In general there is no guarantee of statistical properties between sequences, only within sequences. This is important when generators are used in parallel. This function can ‘skip ahead’ or advance the seed by an amount
\[ s = a_1 2^{b_1} + a_2 2^{b_2} + c \]  
so that the generator will produce the sequence of random numbers \( X_0, X_1, X_2, \ldots \) instead of the original sequence \( X_0, X_1, X_2, \ldots \). This technique is useful to produce independent generators, often also called independent streams and substreams. Please see the Random Number Generators Chapter Introduction for further information.

The skip ahead functionality provided by naggpuMrg32k3aDeviceInitA will typically only be important to applications which use multiple GPUs simultaneously. Users would write their own kernel, embedding the inline MRG32k3a device function generator, and then launch the kernel on multiple GPUs simultaneously to distribute the computation. In this case, each user kernel would require its own NagGpuRandComm and NagGpuMrg32k3aDeviceComm structures to pass to the MRG32k3a device functions. Each
NagGpuRandComm and NagGpuMrg32k3aDeviceComm pair must be initialized by a call to naggpuMrg32k3aDeviceInitA. Note that when initializing a given pair, the call to naggpuMrg32k3aDeviceInitA must access the same GPU device context which will be used to execute the instance of the user’s kernel associated with that pair. Please consult the CUDA documentation for details on how to achieve this and how to access multiple GPUs simultaneously from a single application.

Note that the initialization function nagpudevMrg32k3aInitA provides another opportunity to advance the seed before generating values. The skip ahead performed by naggpuMrg32k3aDeviceInitA therefore corresponds to the GPU upon which the user’s kernel will run (e.g. determined by an OpenMP thread number and/or a GPU device index), while the skip ahead performed by nagpudevMrg32k3aInitA will correspond to the thread number of the CUDA thread in the user’s GPU kernel.

If the user’s kernel is not to be distributed across multiple GPUs, the skip ahead s above can be set to zero and the comments about arrays of communication structures can be ignored.

3.2 Synchronization

This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.

3.3 Return Value

Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References


5 Arguments

1: \( a_1 \) – int 
   
   *Input*
   
   *On entry:* the value of \( a_1 \) in the skip ahead equation (1) above.
   
   *Constraint:* \( a_1 \geq 0 \).

2: \( b_1 \) – int 
   
   *Input*
   
   *On entry:* the value of \( b_1 \) in the skip ahead equation (1) above.
   
   *Constraint:* \( 0 \leq b_1 \leq 191 \).

3: \( a_2 \) – int 
   
   *Input*
   
   *On entry:* the value of \( a_2 \) in the skip ahead equation (1) above.
   
   *Constraint:* \( a_2 \geq 0 \).

4: \( b_2 \) – int 
   
   *Input*
   
   *On entry:* the value of \( b_2 \) in the skip ahead equation (1) above.
   
   *Constraint:* \( 0 \leq b_2 \leq 191 \).

5: \( c \) – long long 
   
   *Input*
   
   *On entry:* the value of \( c \) in the skip ahead equation (1) above.
   
   *Constraint:* \( c \geq 0 \).
6:  **seed[6]** – unsigned int *  
*Input/Output*

The device function generator seed. Note that this function does NOT copy the seed to the device. This is left to the user, who may wish to control where the seed is placed in the GPU memory hierarchy.

*On entry:* the seed which is to be used.

*On exit:* the seed skipped ahead by 
$$s = a_12^{b_1} + a_22^{b_2} + c$$ steps.

*Constraints:*

- for $$i = 0, 1, 2$$, 
  $$seed[i] < 2^{32} - 209 \text{ and } seed[i] \neq 0$$ for at least one $$i$$;
- for $$i = 3, 4, 5$$, 
  $$seed[i] < 2^{32} - 22853 \text{ and } seed[i] \neq 0$$ for at least one $$i$$.

7:  **devComm** – NagGpuMrg32k3aDeviceComm **  
*Communication Data*

NagGpuMrg32k3aDeviceComm is a structure which holds state and communication information and must not be modified in any way. A structure will be allocated in the GPU memory space and its address will be assigned to the location pointed to by **devComm**. This address must be passed to the MRG32k3a device generator initialization function naggpuDevMrg32k3aInitA. Once all required values have been obtained, this address must be passed to naggpuMrg32k3aDeviceCleanupA to free allocated system resources.

8:  **comm** – NagGpuRandComm *  
*Communication Data*

NagGpuRandComm is a structure which holds state and communication information and must not be modified in any way. Once all required values have been obtained from the MRG32k3a device function generators, **comm** must be passed to naggpuMrg32k3aDeviceCleanupA to free allocated system resources.

9:  **error** – NagGpuError *  
*Error Reporting*

This parameter contains error information and should not be modified directly. Errors are indicated through the value of **error → code** which should be inspected after each call to this function. If **error → code** = 0 then no error occurred. If **error → code** ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6  Error Indicators and Warnings

**error → code** = 1

*On entry:* the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

**error → code** = 2

*During execution:* a CUDA runtime error was detected.

**error → code** = 100

*On entry:* the value of **comm** is NULL.

**error → code** = 111

*On entry:* the value of **a1** is negative.

**error → code** = 112

*On entry:* the value of **b1** does not satisfy the constraints listed above.

**error → code** = 113

*On entry:* the value of **a2** is negative.
error \rightarrow \text{code} = 114

\textit{On entry}: the value of \texttt{b2} does not satisfy the constraints listed above.

error \rightarrow \text{code} = 115

\textit{On entry}: the value of \texttt{c} is negative.

error \rightarrow \text{code} = 116

\textit{On entry}: the value of \texttt{seed} is NULL.

error \rightarrow \text{code} = 117

\textit{On entry}: the values in the \texttt{seed} array do not satisfy the constraints listed above.

error \rightarrow \text{code} = 118

\textit{On entry}: the value of \texttt{devComm} is NULL.

7 Example

There is no example program specifically for this function. For examples of how this function should be used, please see the example program for \texttt{naggpudevMrg32k3aUniformA}.
NAG Numerical Routines for GPUs Function Document

naggpudevMrg32k3aInitA

1 Purpose

naggpudevMrg32k3aInitA initializes the MRG32k3a device function generators. It must be preceded by a call to naggpuMrg32k3aDeviceInitA and must ultimately be followed by a call to naggpuMrg32k3aDeviceCleanupA to free allocated system resources. naggpudevMrg32k3aInitA must be called prior to calling any of the device generator functions such as naggpudevMrg32k3aUniformA.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_mrg32k3aDevFuncs.h>
__device__ void naggpudevMrg32k3aInitA(unsigned int a1, unsigned int b1,
                               unsigned int a2, unsigned int b2, unsigned long long c, unsigned int *seed,
                               const NagGpuMrg32k3aDeviceComm *devComm)
```

3 Description

The MRG32k3a generator is a ‘light weight’ generator with a small state and very good statistical properties. Due to its small state, it is well suited for use on a GPU and can be embedded in a user’s GPU kernel. Typically each CUDA thread will have its own copy of state and will run as an independent generator. The NVIDIA compiler will often store the state in registers, provided the array is handled with some care (see the CUDA Programming Guide section on Local Memory). There is no need for threads to communicate in order to generate values, and ideally threads should consume the values as they come off the generator: this avoids any storage overhead and maximizes speed.

3.1 Parallelization

For different values of seed, a given generator will yield different sequences of random numbers. Alternatively, the same sequence of random numbers will be generated if the same value of seed is used. In general there is no guarantee of statistical properties between sequences, only within sequences. This is important when generators are used in parallel. This function can ‘skip ahead’ or advance the seed by an amount

\[ s = a_12^{b_1} + a_22^{b_2} + c \]  

so that the generator will produce the sequence of random numbers \( X_s, X_{s+1}, X_{s+2}, \ldots \) instead of the original sequence \( X_0, X_1, X_2, \ldots \). This technique is useful to produce independent generators, often also called independent streams and substreams. Please see the Random Number Generators Chapter Introduction for further information.

Each CUDA thread can skip the seed ahead according to the formula above. Which skip aheads are chosen depends on the application and on how the CUDA threads are to cooperate. Applications where the number of random variates required is known in advance will probably use a block splitting scheme as described in the Random Number Generators Chapter Introduction. Others may wish to use independent streams.

3.2 Error Handling

This is a GPU device function, and no error handling is performed.

4 References

5 Arguments

1: \texttt{a1} – unsigned int \hspace{1cm} \textit{Input}
\textit{On entry:} the value of \(a_1\) in the skip ahead equation (1) above.

2: \texttt{b1} – unsigned int \hspace{1cm} \textit{Input}
\textit{On entry:} the value of \(b_1\) in the skip ahead equation (1) above.
\textit{Constraint:} \(b_1 \leq 191\).

3: \texttt{a2} – unsigned int \hspace{1cm} \textit{Input}
\textit{On entry:} the value of \(a_2\) in the skip ahead equation (1) above.

4: \texttt{b2} – unsigned int \hspace{1cm} \textit{Input}
\textit{On entry:} the value of \(b_2\) in the skip ahead equation (1) above.
\textit{Constraint:} \(b_2 \leq 191\).

5: \texttt{c} – unsigned long long \hspace{1cm} \textit{Input}
\textit{On entry:} the value of \(c\) in the skip ahead equation (1) above.

6: \texttt{seed[6]} – unsigned int * \hspace{1cm} \textit{Input/Output}
\textit{On entry:} the seed which is to be used.
\textit{On exit:} the state of the generator. This is the seed skipped ahead by \(s = a_12^{b_1} + a_22^{b_2} + c\) steps.
\textit{Constraints:}
\begin{align*}
&\text{for } i = 0, 1, 2, \texttt{seed}[i] < 2^{32} - 209 \text{ and } \texttt{seed}[i] \neq 0 \text{ for at least one } i; \\
&\text{for } i = 3, 4, 5, \texttt{seed}[i] < 2^{32} - 22853 \text{ and } \texttt{seed}[i] \neq 0 \text{ for at least one } i.
\end{align*}

7: \texttt{devComm} – const NagGpuMrg32k3aDeviceComm * \hspace{1cm} \textit{Communication Data}
\text{The NagGpuMrg32k3aDeviceComm memory address obtained from the host initialization function \texttt{naggpuMrg32k3aDeviceInitA.}}

6 Error Indicators and Warnings
None.

7 Example
There is no example program specifically for this function. For examples of how this function should be used, please see the example program for \texttt{naggpudevMrg32k3aUniformA.}
NAG Numerical Routines for GPUs Function Document

naggpudevMrg32k3aExpA

1 Purpose

naggpudevMrg32k3aExpA generates the next value from an exponential distribution with mean $\lambda$. The initialization function naggpudevMrg32k3aInitA must be called prior to the first call to naggpudevMrg32k3aExpA. Thereafter, this function may be called repeatedly to generate the next value in the sequence. Care should be taken to ensure that sequences from successive CUDA threads do not overlap.

2 Specification

```cpp
#include <nag_gpu.h>
#include <nag_gpu_mrg32k3aDevFuncs.h>

template <typename FP>
__device__ void naggpudevMrg32k3aExpA(FP &x, FP lambda, unsigned int *state)
```

3 Description

The exponential distribution has probability density function given by

$$f(x) = \begin{cases} \frac{1}{\lambda} e^{-x/\lambda} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\lambda > 0$. This function returns

$$X = -\lambda \ln Y$$

where $Y$ is the next value generated by the MRG32k3a uniform $(0, 1]$ generator.

3.1 Error Handling

This is a GPU device function, and no error handling is performed.

4 References

None.

5 Arguments

1: $x$ – FP & \hspace{1cm} Output

The template argument parameter FP can take type double or float.

On exit: the next pseudorandom value from the sequence.

2: lambda $\sim$ FP \hspace{1cm} Input

The template argument parameter FP can take type double or float.

On entry: the mean, $\lambda$, of the exponential distribution.

Constraint: lambda $> 0$. 
3: \textbf{state} – unsigned int *  

\textit{Input/Output}

\textit{On entry}: the output state from the previous call to any of the MRG32k3a device function generators, or if this is the first call to any of the MRG32k3a device generators, the value of \textit{seed} obtained from the initialization call \texttt{naggpudevMrg32k3aInitA}

\textit{On exit}: the output state of the generator

6 \textbf{Error Indicators and Warnings}

None.

7 \textbf{Example}

This example program uses \texttt{naggpudevMrg32k3aExpA} to print 50 pseudorandom numbers from an exponential distribution using the MRG32k3a device function generator.

7.1 \textbf{Program Text}

```c
#include <nag_gpu.h>
#include <nag_gpu_mrg32k3aDevFuncs.h>

// precision defined by Makefile
#ifdef SINGLEPRECISION
#define FP float
#else
#define FP double
#endif

void checkNagError(NagGpuError *error);
void checkCudaError(cudaError_t cuError);

__global__ void mrg32k3a_device_functions_exponential_test(
    int npts, // Number of points per thread
    int N, // Total number of points to create
    FP *d_buff, // memory address to store points
    unsigned int *d_seed,
    NagGpuMrg32k3aDeviceComm *devComm)
{
    const FP lambda = 1.0;

    unsigned int state[6];
    for (int i = 0; i < 6; i++) state[i] = d_seed[i];
```

naggpudevMrg32k3aExpA.2 Mark 0.6
// Generation offset
int offset = ((blockIdx.x * blockDim.x) + threadIdx.x)*npts;

// Storage offset
int store = threadIdx.x + npts*blockIdx.x*blockDim.x;

FP x = 0;

// Initialise state for this thread
naggpudevMrg32k3aInitA(0, 0, 0, 0, offset, state, devComm);

// Compute only as many points as requested
for (int i = 0; i < npts; i++) {
    if (store < N) {
        naggpudevMrg32k3aExpA(x, lambda, state);
        d_buff[store] = x;
        store += blockDim.x;
    } else {
        break;
    }
}

int main(int argc, char **argv) {
    const int N = 2621440;
    FP *h_buff = 0, *d_buff = 0;
    unsigned int seed[] = {1, 2, 3, 1, 2, 3};
    unsigned int *d_seed;
    unsigned long long offset = 1;
    NagGpuMrg32k3aDeviceComm *devComm;
    NagGpuRandComm comm;
    NagGpuError error;
    cudaError_t cuError;

    // Print the title
    cout << "NAG GPU Example Program: ";
    cout << "naggpudevMrg32k3aExpA, ";
    if (sizeof(FP)==sizeof(float))
        cout << "single precision";
    else
        cout << "double precision";
    cout << endl << endl;

    // Allocate CPU and GPU memory
    h_buff = new FP[N];
    cuError = cudaMalloc((void **)d_buff, sizeof(FP)*N);
    checkCudaError(cuError);
    cuError = cudaMemcpy(d_seed, seed, sizeof(unsigned int)*6, cudaMemcpyHostToDevice);

    // Host initialisation call
    naggpudevMrg32k3aDeviceInitA(0, 0, 0, 0, offset, seed,
    &devComm, &comm, &error);
    checkNagError(&error);

    // Copy seed to device
    cuError = cudaMemcpy(d_seed, seed, sizeof(unsigned int)*6, cudaMemcpyHostToDevice);
checkCudaError(cuError);

// Launch GPU computation asynchronously. Feel free to experiment
// with nblks=#thread blocks, nthds=#threads/block & ppt=#points/thread
const int ppt = 1024;
const int nthds = 64;
const int nblks = N/(ppt*nthds) + 1;
mrg32k3a_device_functions_exponential_test<<<nblks, nthds>>>(ppt, N, d_buff, d_seed, devComm);

/*
 * One can now launch other kernels to operate on the random numbers,
 * or copy the numbers to the host and operate on them there.
 * Here we simply copy them to the host in order to print them
 */
cuError = cudaMemcpy(h_buff, d_buff, sizeof(FP)*N, cudaMemcpyDeviceToHost);
checkCudaError(cuError);

// Print random numbers
cout << "The first 50 GPU random numbers:" << endl;
cout.setf(ios::fixed,ios::floatfield);
cout.precision(4);
for(int row = 0; row < 10; row++)
{
    for(int col = 0; col < 5; col++)
    {
        cout << h_buff[row*10 + col] << "\t";
    }
    cout << endl;
}

// Call cleanup for the NAG routine
naggpuMrg32k3aDeviceCleanupA(devComm, &comm, &error);
checkNagError(&error);

// Free CPU and GPU memory
delete[] h_buff;
if (d_buff)
{
    cuError = cudaFree(d_buff);
    checkCudaError(cuError);
}
if (d_seed)
{
    cuError = cudaFree(d_seed);
    checkCudaError(cuError);
}
return 0;
}
void checkNagError(NagGpuError *error)
{
    if (error->code != 0)
    {
        char *buff;
        buff = new char[error->msgLength];
        naggpuErrorCopyMsg(buff, error);
        cout << buff << endl;
        delete[] buff;
        exit(1);
    }
}
void checkCudaError(cudaError_t cuError)
{
    if (cuError != cudaSuccess)
    {
        cout << cudaGetErrorString(cuError) << endl;
        exit(1);
    }
}

7.2 Program Data
None.

7.3 Program Results
NAG GPU Example Program: naggpudevMrg32k3aExpA, single precision

The first 50 GPU random numbers:
0.0000 0.7015 0.3593 1.0333 0.0692
1.9998 1.5521 2.3616 0.6293 2.1551
0.9912 0.9287 1.0029 3.8324 0.4377
0.2346 0.2850 1.8797 0.8697 0.0256
0.1589 0.9164 0.2278 0.6564 0.1516
0.1700 0.7241 1.1626 0.2227 0.3614
0.2547 0.5586 0.0025 0.5932 7.7862
1.4530 1.1058 1.4885 0.4088 0.2544
0.8717 0.5517 0.0337 0.9418 0.5588
2.1155 1.3872 3.2928 1.2697 1.4017
NAG Numerical Routines for GPUs Function Document
naggpudevMrg32k3aGammaA

1 Purpose
naggpudevMrg32k3aGammaA generates the next value from a gamma distribution with shape parameter $\alpha$ and scale parameter $\beta$. The function naggpudevMrg32k3aGammaSetParamsA must be called first to set the parameters of the distribution and to compute certain constants which are used by this function. Each time the parameters of the distribution change, naggpudevMrg32k3aGammaSetParamsA must be called to recompute these constants, which must then be passed to naggpudevMrg32k3aGammaA.

The initialization function naggpudevMrg32k3aInitA must be called prior to the first call to naggpudevMrg32k3aGammaA. Thereafter, this function may be called repeatedly to generate the next value in the sequence. Care should be taken to ensure that sequences from successive CUDA threads do not overlap.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_mrg32k3aDevFuncs.h>

template <typename FP>
__device__ void naggpudevMrg32k3aGammaA(FP &x, unsigned int *state,
                                              FP gammaComm1, FP gammaComm2, FP gammaComm3, FP gammaComm4, FP &normalComm)
```

3 Description

The gamma distribution has probability density function given by

$$f(x) = \begin{cases} \frac{1}{\beta^\alpha x^{\alpha-1} e^{-x/\beta}} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\alpha, \beta > 0$. The rejection algorithm described in Marsaglia and Tsang (2000) is used to generate the gamma pseudorandom variates when $\alpha \geq 1$. When $0 < \alpha < 1$, the scaling

$$\gamma_\alpha = \gamma_{1+\alpha} U^\alpha$$

is used where $U$ denotes a uniform random variable in the interval $[0, 1]$ and $\gamma_\alpha$ denotes a gamma random variable with shape parameter $\alpha$ and scale parameter $\beta = 1$.

This function uses a rejection algorithm to generate variates from the required distribution. A key feature of rejection algorithms is that a random number of uniform variates is required to generate a single variate from the target distribution. Creating independent generators for use in a parallel setting is therefore not simple: for each of these generators one does not know how far to skip ahead the state to ensure that the generators do not overlap. Care must also be taken to ensure the independent generators preserve the good statistical properties of the underlying uniform generator. The recommended way of using naggpudevMrg32k3aGammaA is to place each CUDA thread on a different independent stream or substream. Please see the Random Number Generators Chapter Introduction for recommendations on choices of streams and substreams, and naggpuMrg32k3aDeviceInitA and naggpudevMrg32k3aInitA for details on how to create such streams and substreams.

Using naggpudevMrg32k3aGammaA will lead to a certain amount of warp divergence (please see NVIDIA CUDA (2011) for a thorough discussion of this concept and its performance implications). The probability of warp divergence is closely linked to the acceptance probability of the rejection algorithm, which in the case of Marsaglia and Tsang (2000) is approximately 0.951, 0.981 and 0.992 when $\alpha = 1, 2$ and 4 respectively. In the worst case when $\alpha = 1$, the probability of warp divergence is slightly less than $1 - 0.951^{32} = 0.80$. However the separate code branches executed in divergent warps are relatively small and not computationally demanding, so that the impact of the divergence should not be great.
3.1 Error Handling
This is a GPU device function, and no error handling is performed.

4 References

5 Arguments
1: \(x\) – FP &  
   The template argument parameter FP can take type double or float.  
   *On exit*: the next pseudorandom value from the sequence.

2: state – unsigned int *  
   *On entry*: the output state from the previous call to any of the MRG32k3a device function generators, or if this is the first call to any of the MRG32k3a device generators, the value of seed obtained from the initialization call naggpudevMrg32k3aInitA  
   *On exit*: the output state of the generator

3: gammaComm1 – FP  
   The template argument parameter FP can take type double or float.  
   This parameter is for internal use and must not be modified in any way.  
   *On entry*: the value of gammaComm1 obtained from naggpudevMrg32k3aGammaSetParamsA

4: gammaComm2 – FP  
   The template argument parameter FP can take type double or float.  
   This parameter is for internal use and must not be modified in any way.  
   *On entry*: the value of gammaComm2 obtained from naggpudevMrg32k3aGammaSetParamsA

5: gammaComm3 – FP  
   The template argument parameter FP can take type double or float.  
   This parameter is for internal use and must not be modified in any way.  
   *On entry*: the value of gammaComm3 obtained from naggpudevMrg32k3aGammaSetParamsA

6: gammaComm4 – FP  
   The template argument parameter FP can take type double or float.  
   This parameter is for internal use and must not be modified in any way.  
   *On entry*: the value of gammaComm4 obtained from naggpudevMrg32k3aGammaSetParamsA

7: normalComm – FP &  
   The template argument parameter FP can take type double or float.  
   This parameter is for internal use and must not be modified in any way.  
   The argument normalComm contains information required by both naggpudevMrg32k3aNormalA and naggpudevMrg32k3aGammaA. If both these functions are referenced in a GPU kernel, a single instance of normalComm must be created and must be used when calling either function. On the very first call to either naggpudevMrg32k3aNormalA or naggpudevMrg32k3aGammaA,
normalComm must be set to (FP)nanf(""). Thereafter the variable must be passed unchanged to every subsequent call to these functions.

6 Error Indicators and Warnings

None.

7 Example

This example program uses naggpudevMrg32k3aGammaA to print 50 pseudorandom numbers from a gamma distribution using the MRG32k3a device function generator.

7.1 Program Text

```c
#include
#include <nag_gpu.h>
#include <nag_gpu_mrg32k3aDevFuncs.h>

// precision defined by Makefile
#ifdef SINGLEPRECISION
#define FP float
#else
#define FP double
#endif

void checkNagError(NagGpuError *error);
void checkCudaError(cudaError_t cuError);

__global__ void mrg32k3a_device_functions_gamma_test(
    int ppt,    // Number of points per thread
    int N,      // Total number of points
    FP *d_buff, // memory address to store points
    unsigned int *d_seed,
    NagGpuMrg32k3aDeviceComm *devComm)
{
    /*
     * Assumptions:
     * Grid is one dimensional
     * Thread block is one dimensional
     */
    const FP alpha = FP(1.3);
    const FP beta = FP(3.7);

    FP gammaComm1, gammaComm2, gammaComm3, gammaComm4;
    FP normComm = (FP)nanf("");

    unsigned int state[6];
    for (int i = 0; i < 6; i++) state[i] = d_seed[i];

    // Initialise state for this thread
```
// Place each CUDA thread on a separate substream
int tid = threadIdx.x + blockDim.x*blockIdx.x;
naggpudevMrg32k3aInitA(tid, 76, 0, 0, 0, state, devComm);

// Storage offset
int store = threadIdx.x + ppt*blockIdx.x*blockDim.x;

// Set parameters of gamma distribution
naggpudevMrg32k3aGammaSetParamsA(alpha, beta,
    gammaComm1, gammaComm2, gammaComm3, gammaComm4);

FP x = 0;
// Compute only as many points as requested
for (int i = 0; i < ppt; i++)
{
    if (store < N)
    {
        naggpudevMrg32k3aGammaA(x, state,
            gammaComm1, gammaComm2, gammaComm3, gammaComm4, normComm);
        d_buff[store] = x;
        store += blockDim.x;
    }
    else
    {
        break;
    }
}

int main(int argc, char **argv)
{
    const int N = 2621440;
    FP *h_buff = 0, *d_buff = 0;
    unsigned int seed[] = {1, 2, 4, 1, 2, 3};
    unsigned int *d_seed;
    NagGpuMrg32k3aDeviceComm *devComm;
    NagGpuRandComm comm;
    NagGpuError error;
    cudaError_t cuError;

    // Print the title
    cout << "NAG GPU Example Program: ";
    cout << "naggpudevMrg32k3aGammaA, ";
    if (sizeof(FP)==sizeof(float))
        cout << "single precision";
    else
        cout << "double precision";
    cout << endl << endl;

    // Allocate CPU and GPU memory
    h_buff = new FP[N];
    cuError = cudaMalloc((void **)&d_buff, sizeof(FP)*N);
    checkCudaError(cuError);
    cuError = cudaMalloc((void**)&d_seed, sizeof(unsigned int) * 6);
    checkCudaError(cuError);

    // Host initialisation call
    naggpug32k3aDeviceInitA(0, 0, 0, 0, seed,
        &devComm, &comm, &error);
    checkNagError(&error);

    // Copy seed to device
    cuError = cudaMemcpy(d_seed, seed, sizeof(unsigned int) * 6,
cudaMemcpyHostToDevice);
checkCudaError(cuError);

// Launch GPU computation asynchronously. Feel free to experiment
// with nblks=#thread blocks, nthds=#threads/block & ppt=#points/thread
const int ppt = 1024;
const int nthds = 64;
const int nblks = N/(ppt*nthds) + 1;
mrg32k3a_device_functions_gamma_test<<<nblks, nthds>>>(ppt, N, d_buff, d_seed, devComm);

/*
 * One can now launch other kernels to operate on the random numbers,
 * or copy the numbers to the host and operate on them there.
 * Here we simply copy them to the host in order to print them
 */
cuError = cudaMemcpy(h_buff, d_buff, sizeof(FP)*N,
cudaMemcpyDeviceToHost);
checkCudaError(cuError);

// Print random numbers
cout << "The first 50 GPU random numbers:" << endl;
cout.setf(ios::fixed,ios::floatfield);
cout.precision(4);
for(int row = 0; row < 10; row++)
{
    for(int col = 0; col < 5; col++)
    {
        cout << h_buff[row*10 + col] << "\t";
    }
    cout << endl;
}

// Call cleanup for the NAG routine
naggpuMrg32k3aDeviceCleanupA(devComm, &comm, &error);
checkNagError(&error);

// Free CPU and GPU memory
delete[] h_buff;
if (d_buff)
{
    cuError = cudaFree(d_buff);
    checkCudaError(cuError);
}
if (d_seed)
{
    cuError = cudaFree(d_seed);
    checkCudaError(cuError);
}
return 0;
}

void checkNagError(NagGpuError *error)
{
    if (error->code != 0)
    {
        char *buff;
        buff = new char[error->msgLength];
        naggpuErrorCopyMsg(buff, error);
        cout << buff << endl;
        delete[] buff;
        exit(1);
    }
}
void checkCudaError(cudaError_t cuError)
{
    if (cuError != cudaSuccess)
    {
        cout << cudaGetErrorString(cuError) << endl;
        exit(1);
    }
}

### 7.2 Program Data

None.

### 7.3 Program Results

NAG GPU Example Program: naggpudevMrg32k3aGammaA, single precision

The first 50 GPU random numbers:

3.6403  0.9104  6.8153  4.3084  4.1830
0.7710  10.8153 16.9506  1.5821  1.8948
2.9317  4.6541  0.2354  1.6106  3.0009
4.0389  3.4582 11.5590  1.3871  3.2133
1.2653  2.6843  4.3136  5.7665  2.3449
5.5406  4.3038  5.9099  1.9624  0.8370
0.4379  7.3254  4.8952 11.3394  2.7389
0.6569  5.6433  0.7745  0.0692  0.1111
2.4630  2.6579  0.5987  4.1963  4.0539
6.5393 11.2430  5.1479  0.3441  4.6715
1 Purpose

NagppudevMrg32k3aGammaSetParamsA accepts the shape parameter $\alpha$ and scale parameter $\beta$ of a gamma distribution and computes constants which are used by the generator function NagppudevMrg32k3aGammaA.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_mrg32k3aDevFuncs.h>

template <typename FP>
__device__ void nagppudevMrg32k3aGammaSetParamsA(FP alpha, FP beta,
    FP &gammaComm1, FP &gammaComm2, FP &gammaComm3, FP &gammaComm4)
```

3 Description

The function nagppudevMrg32k3aGammaA uses certain constants which only change when the parameters of the gamma distribution change. For performance reasons, these constants should only be recomputed when necessary and not each time nagppudevMrg32k3aGammaA is called. This function should therefore be called before the first call to nagppudevMrg32k3aGammaA and each time the gamma distribution’s parameters change, and the output must be passed to nagppudevMrg32k3aGammaA.

3.1 Error Handling

This is a GPU device function, and no error handling is performed.

4 References

None.

5 Arguments

1. **alpha** – FP
   
   The template argument parameter FP can take type double or float.
   
   *On entry:* the shape parameter, $\alpha$, of the distribution.
   
   *Constraint:* $\alpha > 0$.

2. **beta** – FP
   
   The template argument parameter FP can take type double or float.
   
   *On entry:* the scale parameter, $\beta$, of the distribution.
   
   *Constraint:* $\beta > 0$.

3. **gammaComm1** – FP &
   
   The template argument parameter FP can take type double or float.
   
   *On exit:* a constant which must be passed to nagppudevMrg32k3aGammaA.
4: **gammaComm2** – FP &
   The template argument parameter FP can take type double or float.
   *On exit:* a constant which must be passed to naggpudevMrg32k3aGammaA.

5: **gammaComm3** – FP &
   The template argument parameter FP can take type double or float.
   *On exit:* a constant which must be passed to naggpudevMrg32k3aGammaA.

6: **gammaComm4** – FP &
   The template argument parameter FP can take type double or float.
   *On exit:* a constant which must be passed to naggpudevMrg32k3aGammaA.

### 6 Error Indicators and Warnings

None.

### 7 Example

There is no example program specifically for this function. For an example of how this function should be used, please see the example program for naggpudevMrg32k3aGammaA.
1 Purpose

naggpudevMrg32k3aNormalA generates the next value from a Normal distribution with mean $\mu$ and variance $\sigma^2$.

The initialization function naggpudevMrg32k3aInitA must be called prior to the first call to naggpudevMrg32k3aNormalA. Thereafter, this function may be called repeatedly to generate the next value in the sequence. Care should be taken to ensure that sequences from successive CUDA threads do not overlap.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_mrg32k3aDevFuncs.h>

template <typename FP>
__device__ void naggpudevMrg32k3aNormalA(FP &x, FP mu, FP sigma,
                                        unsigned int *state, FP &normalComm)
```

3 Description

The Normal distribution has probability density function given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right)$$

where $\sigma > 0$ and $\mu \in \mathbb{R}$. This function uses a Box-Muller transform to convert a pair of uniform $(0, 1)$ random numbers into a pair of Normal random numbers. Let $X_0, X_1, X_2, \ldots$ denote the sequence of uniform $(0, 1)$ pseudorandom variates as specified by the MRG32k3a algorithm. This function uses successive pairs of uniform variates in the Box-Muller transform to produce successive pairs of Normal variates, i.e. $(X_0, X_1) \rightarrow (Z_0, Z_1), (X_2, X_3) \rightarrow (Z_2, Z_3)$ where $Z_0, Z_1, Z_2, \ldots$ denotes the output sequence of Normal variates.

3.1 Error Handling

This is a GPU device function, and no error handling is performed.

4 References

None.

5 Arguments

1: $x$ – FP &

   The template argument parameter FP can take type double or float.

   On exit: the next pseudorandom value from the sequence.

2: $\mu$ – FP

   The template argument parameter FP can take type double or float.

   On entry: the mean, $\mu$, of the distribution.
3: sigma – FP  
   The template argument parameter FP can take type double or float.
   
   On entry: the standard deviation, $\sigma$, of the distribution
   
   Constraint: $\sigma > 0$.

4: state – unsigned int *  
   
   On entry: the output state from the previous call to any of the MRG32k3a device function
   generators, or if this is the first call to any of the MRG32k3a device generators, the value of seed
   obtained from the initialization call naggpuDevMrg32k3aInitA
   
   On exit: the output state of the generator

5: normalComm – FP &  
   
   The template argument parameter FP can take type double or float.
   
   This parameter is for internal use and must not be modified in any way.
   
   The argument normalComm contains information required by both naggpuDevMrg32k3aNormalA and
   naggpuDevMrg32k3aGammaA. If both these functions are referenced in a GPU kernel, a single
   instance of normalComm must be created and must be used when calling either function. On the
   very first call to either naggpuDevMrg32k3aNormalA or naggpuDevMrg32k3aGammaA, normalComm
   must be set to (FP)nanf(""). Thereafter the variable must be passed unchanged
   to every subsequent call to these functions.

6 Error Indicators and Warnings

None.

7 Example

This example program uses naggpuDevMrg32k3aNormalA to print 50 pseudorandom numbers from a
Normal distribution using the MRG32k3a device function generator.

7.1 Program Text

```c++
#include <nag_gpu.h>
#include <nag_gpu_mrg32k3aDevFuncs.h>

void checkNagError(NagGpuError *error);
void checkCudaError(cudaError_t cuError);
```

* Test kernel to try out device level functions
* Note: this will NOT output numbers in the same order
* as the CPU routine
*/
__global__ void mrg32k3a_device_functions_normal_test(
    int npts, // Number of points per thread
    int N, // Total number of points to create
    FP *d_buff, // memory address to store points
    unsigned int *d_seed,
    NagGpuMrg32k3aDeviceComm *devComm)
{
    
    const FP mu = 0.0, sigma = 1.0;
    FP normComm = (FP)nanf("");
    unsigned int state[6];
    for (int i = 0; i < 6; i++) state[i] = d_seed[i];

    // Generation offset
    int offset = ((blockIdx.x * blockDim.x) + threadIdx.x)*npts;

    // Storage offset
    int store = threadIdx.x + npts*blockIdx.x*blockDim.x;
    FP x = 0;

    // Initialise state for this thread
    naggpudevMrg32k3aInitA(0, 0, 0, 0, offset, state, devComm);

    // Compute only as many points as requested
    for (int i = 0; i < npts; i++)
    {
        if (store < N)
        {
            naggpudevMrg32k3aNormalA(x, mu, sigma, state, normComm);
            d_buff[store] = x;
            store += blockDim.x;
        }
        else
        {
            break;
        }
    }
}

int main(int argc, char **argv)
{
    
    const int N = 2621440;
    FP *h_buff = 0, *d_buff = 0;
    unsigned int seed[] = {1, 2, 3, 1, 2, 3};
    unsigned int *d_seed;
    unsigned long long offset = 1;
    NagGpuMrg32k3aDeviceComm *devComm;
    NagGpuRandComm comm;
    NagGpuError error;
    cudaError_t cuError;

    // Print the title
    cout << "NAG GPU Example Program: ";
    cout << "naggpudevMrg32k3aNormalA, ";

    ...
if (sizeof(FP)==sizeof(float))
    cout << "single precision";
else
    cout << "double precision";
    cout << endl << endl;

// Allocate CPU and GPU memory
    h_buff = new FP[N];
    cuError = cudaMalloc((void**)&d_buff, sizeof(FP)*N);
    checkCudaError(cuError);
    cuError = cudaMalloc((void**)&d_seed, sizeof(unsigned int) * 6);
    checkCudaError(cuError);

// Host initialisation call
    naggpuMrg32k3aDeviceInitA(0, 0, 0, 0, offset, seed,
                               &devComm, &comm, &error);
    checkNagError(&error);

// Copy seed to device
    cuError = cudaMemcpy(d_seed, seed, sizeof(unsigned int)*6,
                          cudaMemcpyHostToDevice);
    checkCudaError(cuError);

// Launch GPU computation asynchronously. Feel free to experiment
    const int ppt = 1024;
    const int nthds = 64;
    const int nblks = N/(ppt*nthds) + 1;
    mrg32k3a_device_functions_normal_test<<<nblks, nthds>>>(ppt, N, d_buff, d_seed, devComm);

    /*
    * One can now launch other kernels to operate on the random numbers,
    * or copy the numbers to the host and operate on them there.
    * Here we simply copy them to the host in order to print them
    */
    cuError = cudaMemcpy(h_buff, d_buff, sizeof(FP)*N,
                          cudaMemcpyDeviceToHost);
    checkCudaError(cuError);

    // Print random numbers
    cout << "The first 50 GPU random numbers:

    int row = 0; row < 10; row++)
    {
        for(int col = 0; col < 5; col++)
            cout << h_buff[row*10 + col] << " \t";
        cout << endl;
    }

    // Call cleanup for the NAG routine
    naggpuMrg32k3aDeviceCleanupA(devComm, &comm, &error);
    checkNagError(&error);

    // Free CPU and GPU memory
    delete[] h_buff;
    if (d_buff)
        {cuError = cudaFree(d_buff);
         checkCudaError(cuError);
        }
    if (d_seed)
{ 
    cuError = cudaFree(d_seed);
    checkCudaError(cuError);
}

return 0;
}

void checkNagError(NagGpuError *error)
{
    if (error->code != 0)
    {
        char *buff;
        buff = new char[error->msgLength];
        naggpuErrorCopyMsg(buff, error);
        cout << buff << endl;
        delete[] buff;
        exit(1);
    }
}

void checkCudaError(cudaError_t cuError)
{
    if (cuError != cudaSuccess)
    {
        cout << cudaGetErrorString(cuError) << endl;
        exit(1);
    }
}

7.2 Program Data

None.

7.3 Program Results

NAG GPU Example Program: naggpudevMrg32k3aNormalA, single precision

The first 50 GPU random numbers:
0.0000 0.3201 -0.7592 -0.4116 -0.0888
-1.9745 -0.3208 -2.0949 -0.1084 1.4794
-0.6145 0.9015 0.2775 -1.4955 0.7982
0.6848 -0.5189 -0.2508 -0.4190 0.0253
0.5189 0.8332 -0.5497 -1.0040 0.2467
-0.0231 -1.1487 -1.3612 0.4385 0.1324
-0.6566 -0.9416 -0.0496 0.8405 0.0003
0.1454 -0.4827 0.1190 -0.5212 0.3178
-1.5864 -0.5816 0.7089 -0.5835 -1.2668
2.0041 0.0033 0.9920 -0.1001 0.0164

1 Purpose

naggpudevMrg32k3aUniformA generates the next value from a uniform distribution over the interval \((a, b)\) for specified constants \(a\) and \(b\).

The initialization function naggpudevMrg32k3aInitA must be called prior to the first call to naggpudevMrg32k3aUniformA. Thereafter, this function may be called repeatedly to generate the next value in the sequence. Care should be taken to ensure that sequences from successive CUDA threads do not overlap.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_mrg32k3aDevFuncs.h>
template <typename FP>
__device__ void naggpudevMrg32k3aUniformA(FP &x, FP a, FP b, unsigned int *state)
```

3 Description

If \(a = 0\) and \(b = 1\), this function returns the next value \(Y\) from the MRG32k3a uniform \((0, 1)\) sequence. For other values of \(a\) and \(b\), the function applies the transformation

\[
X = a + \frac{b - a}{a} Y
\]

to produce random numbers from the interval \((a, b)\).

3.1 Error Handling

This is a GPU device function, and no error handling is performed.

4 References

None.

5 Arguments

1: \(x\) – FP &

   The template argument parameter FP can take type double or float.

   On exit: the next pseudorandom value from the sequence.

2: \(a\) – FP

   The template argument parameter FP can take type double or float.

   On entry: The lower bound for the uniform random values.

3: \(b\) – FP

   The template argument parameter FP can take type double or float.

   On entry: The upper bound for the uniform random values.

   Constraint: \(b > a\).
4: state – unsigned int *  

*Input/Output*

On entry: the output state from the previous call to any of the MRG32k3a device function generators, or if this is the first call to any of the MRG32k3a device generators, the value of seed obtained from the initialization call naggpudevMrg32k3aInitA

On exit: the output state of the generator

6 Error Indicators and Warnings

None.

7 Example

This example program uses naggpudevMrg32k3aUniformA to print 50 pseudorandom numbers from a uniform distribution using the MRG32k3a device function generator.

7.1 Program Text

```c
#include
using namespace std;

#include <nag_gpu.h>
#include <nag_gpu_mrg32k3aDevFuncs.h>

// precision defined by Makefile
#ifdef SINGLEPRECISION
#define FP float
#else
#define FP double
#endif

void checkNagError(NagGpuError *error);
void checkCudaError(cudaError_t cuError);

__global__ void mrg32k3a_device_functions_uniform_test(
    int npts, // Number of points per thread
    int N, // Total number of points to create
    FP *d_buff, // memory address to store points
    unsigned int *d_seed,
    NagGpuMrg32k3aDeviceComm *devComm)
{
    /*
     * Assumptions:
     * Grid is one dimensional
     * Thread block is one dimensional
     */
    const FP a = 0.0, b = 1.0;

    unsigned int state[6];
    for (int i = 0; i < 6; i++) state[i] = d_seed[i];
```
// Generation offset
int offset = ((blockIdx.x * blockDim.x) + threadIdx.x)*npts;

// Storage offset
int store = threadIdx.x + npts*blockIdx.x*blockDim.x;

FP x = 0;

// Initialise state for this thread
naggpudevMrg32k3aInitA(0, 0, 0, 0, offset, state, devComm);

// Compute only as many points as requested
for (int i = 0; i < npts; i++)
{
    if (store < N)
    {
        naggpudevMrg32k3aUniformA(x, a, b, state);
        d_buff[store] = x;
        store += blockDim.x;
    }
    else
    {
        break;
    }
}

int main(int argc, char **argv)
{
    const int N = 2621440;
    FP *h_buff = 0, *d_buff = 0;
    unsigned int seed[] = {1, 2, 3, 1, 2, 3};
    unsigned int *d_seed;
    unsigned long long offset = 1;
    NagGpuMrg32k3aDeviceComm *devComm;
    NagGpuRandComm comm;
    NagGpuError error;
    cudaError_t cuError;

    // Print the title
    cout << "NAG GPU Example Program: ";
    cout << "naggpudevMrg32k3aUniformA, ";
    if (sizeof(FP)==sizeof(float))
        cout << "single precision";
    else
        cout << "double precision";
    cout << endl;

    // Allocate CPU and GPU memory
    h_buff = new FP[N];
    cuError = cudaMemcpy((void **)&d_buff, sizeof(FP)*N);
    checkCudaError(cuError);
    cuError = cudaMemcpy((void **)&d_seed, sizeof(unsigned int) * 6);
    checkCudaError(cuError);

    // Host initialisation call
    naggpuMrg32k3aDeviceInitA(0, 0, 0, 0, offset, seed,
                               &devComm, &comm, &error);
    checkNagError(&error);

    // Copy seed to device
    cuError = cudaMemcpy(d_seed, seed, sizeof(unsigned int) * 6,
                          cudaMemcpyHostToDevice);
checkCudaError(cuError);

// Launch GPU computation asynchronously. Feel free to experiment
// with nblks=#thread blocks, nthds=#threads/block & ppt=#points/thread
const int ppt = 1024;
const int nthds = 64;
const int nblks = N/(ppt*nthds) + 1;
mrg32k3a_device_functions_uniform_test<<<nblks, nthds>>>(ppt, N, d_buff, d_seed, devComm);

/*
 * One can now launch other kernels to operate on the random numbers,
 * or copy the numbers to the host and operate on them there.
 * Here we simply copy them to the host in order to print them
 */
cuError = cudaMemcpy(h_buff, d_buff, sizeof(FP)*N, cudaMemcpyDeviceToHost);
checkCudaError(cuError);

// Print random numbers
for(int row = 0; row < 10; row++) {
    for(int col = 0; col < 5; col++) {
        cout << h_buff[row*10 + col] << "\t";
    }
    cout << endl;
}

// Call cleanup for the NAG routine
naggpuMrg32k3aDeviceCleanupA(devComm, &comm, &error);
checkNagError(&error);

// Free CPU and GPU memory
delete[] h_buff;
if (d_buff) {
    cuError = cudaFree(d_buff);
    checkCudaError(cuError);
}
if (d_seed) {
    cuError = cudaFree(d_seed);
    checkCudaError(cuError);
}
return 0;

void checkNagError(NagGpuError *error) {
    if (error->code != 0) {
        char *buff;
        buff = new char[error->msgLength];
        naggpuErrorCopyMsg(buff, error);
        cout << buff << endl;
        delete[] buff;
        exit(1);
    }
}
void checkCudaError(cudaError_t cuError)
{
    if (cuError != cudaSuccess)
    {
        cout << cudaGetErrorString(cuError) << endl;
        exit(1);
    }
}

7.2 Program Data

None.

7.3 Program Results

NAG GPU Example Program: naggpudevMrg32k3aUniformA, single precision

The first 50 GPU random numbers:
1.0000 0.4958 0.6982 0.3558 0.9332
0.1354 0.2118 0.0943 0.5330 0.1159
0.3711 0.3951 0.3668 0.0217 0.6455
0.7909 0.7520 0.1526 0.4191 0.9747
0.8531 0.3999 0.7963 0.5187 0.8593
0.8437 0.4848 0.3127 0.8003 0.6967
0.7751 0.5720 0.9975 0.5525 0.0004
0.2339 0.3310 0.2257 0.6644 0.7754
0.4182 0.5760 0.9669 0.3899 0.5719
0.1206 0.2498 0.0371 0.2809 0.2462
NAG Numerical Routines for GPUs Function Document

naggpuMrg32k3aDeviceCleanupA

1 Purpose

naggpuMrg32k3aDeviceCleanupA frees system resources that were allocated by a previous call to naggpuMrg32k3aDeviceInitA.

2 Specification

```c
#include <nag_gpu.h>
extern "C"
cudaError_t naggpuMrg32k3aDeviceCleanupA(NagGpuMrg32k3aDeviceComm *devComm,
                                         NagGpuRandComm *comm,
                                         NagGpuError *error)
```

3 Description

3.1 Synchronization

This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.

3.2 Return Value

Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References

None.

5 Arguments

1: `devComm` – NagGpuMrg32k3aDeviceComm *  
   `Input`  
   On entry: the GPU memory address obtained from a prior call to naggpuMrg32k3aDeviceInitA.

2: `comm` – NagGpuRandComm *  
   `Communication Data`  
   On entry: the pointer that was passed to a previous call to naggpuMrg32k3aDeviceInitA.

3: `error` – NagGpuError *  
   `Error Reporting`  
   This parameter contains error information and should not be modified directly. Errors are indicated through the value of `error → code` which should be inspected after each call to this function. If `error → code = 0` then no error occurred. If `error → code ≠ 0` then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.
6 Error Indicators and Warnings

error → code = 1

On entry: the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

error → code = 2

During execution: a CUDA runtime error was detected.

error → code = 100

On entry: the value of comm is NULL.

error → code = 101

On entry: comm has not been initialized, or the internal state of comm is corrupted.

error → code = 110

On entry: devComm is NULL.

7 Example

There is no example program specifically for this function. For examples of how this function should be used, please see the example program for naggpudevMrg32k3aUniformA.
1 Purpose

naggpuSobolDeviceInitA creates initialization data on the GPU for the Sobol’ device function generators. It must be called prior to a call to naggpudevSobolInitA and must ultimately be followed by a call to naggpuSobolDeviceCleanupA to free allocated system resources.

2 Specification

#include <nag_gpu.h>

extern "C"
cudaError_t naggpuSobolDeviceInitA(NagGpuScramTypes stype, int maxDevGenDim,
                                          NagGpuSobolDeviceComm **devComm, NagCPURandComm *pseudoComm,
                                          NagGpuQuasiRandComm *comm, NagGpuError *error)

3 Description

To use the Sobol’ device function generators, the memory address of a NagGpuSobolDeviceComm structure must be obtained from the device generator initialization function naggpuSobolDeviceInitA. This structure will reside in the GPU memory space and will contain communication data for use by the device function generator. This GPU memory address must then be passed to the Sobol’ device generator initialization function naggpudevSobolInitA. Once all values have been obtained from the device function generators, the same NagGpuSobolDeviceComm structure memory address must be passed to naggpuSobolDeviceCleanupA to free allocated system resources.

This function will create the NagGpuSobolDeviceComm communication structure in the GPU memory space and will assign its address to devComm. It also requires the maximum dimension maxDevGenDim of any Sobol’ sequence which is to be generated by the GPU device function generators. Using the device function generators to generate sequences of dimension higher than maxDevGenDim will result in undefined behaviour.

3.1 Synchronization

This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.

3.2 Return Value

Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

4 References

None.

5 Arguments

1: stype – NagGpuScramTypes

\textit{Input}

\textit{On entry}: the type of scrambling to be used:

\begin{itemize}
  \item stype = NAGGPUSCRAMTYPES_NONE
  \item stype = NAGGPUSCRAMTYPES_Owen
  \item stype = NAGGPUSCRAMTYPES_Faure_Tezuka
\end{itemize}
**naggpuSobolDeviceInitA**

Please see NagGpuScramTypes for some of the benefits of scrambling and details about each of available scrambling types.

*Constraint: stype = NAGGPUSCRAMTYPES_OwenFAURE_TEZUKA*

2: **maxDevGenDim** – int

*Input*

*On entry:* the largest dimension of any quasi-random sequence which will be generated by the Sobol' device function generators such as naggpuSovolUniformA.

*Constraint: 1 ≤ maxDevGenDim ≤ 50000.*

3: **devComm** – NagGpuSobolDeviceComm **

*Communication Data*

NagGpuSobolDeviceComm is a structure which holds state and communication information and must not be modified in any way. A structure will be allocated in the GPU memory space and its address will be assigned to the location pointed to by devComm. This address must be passed to the Sobol device generator initialization function naggpuSovolInitA. Once all required points have been obtained, this address must be passed to naggpuSobolDeviceCleanupA to free allocated system resources.

4: **pseudoComm** – NagCPURandComm *

*Input*

*On entry:* a pointer to a NagCPURandComm structure which has already been initialized by the function nagCPURandInitA.

*Constraint: pseudoComm must be initialized before being passed to this function .*

5: **comm** – NagGpuQuasiRandComm *

*Communication Data*

NagGpuQuasiRandComm is a structure which holds state and communication information and must not be modified in any way. Once all required points have been obtained from the Sobol' device function generators, comm must be passed to naggpuSobolDeviceCleanupA to free allocated system resources.

6: **error** – NagGpuError *

*Error Reporting*

This parameter contains error information and should not be modified directly. Errors are indicated through the value of error → code which should be inspected after each call to this function. If error → code = 0 then no error occurred. If error → code ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 **Error Indicators and Warnings**

**error → code = 1**

*On entry:* the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

**error → code = 2**

*During execution:* a CUDA runtime error was detected.

**error → code = 100**

*On entry:* the value of comm is NULL.
error → code = 111

On entry: stype does not specify a valid scrambling type. See NagGpuScramTypes for permitted values.

error → code = 114

On entry: the value of pseudoComm is NULL.

error → code = 115

On entry: the pseudorandom generator nagCPURandUniformA returned an error when called by this function: pseudoComm is not initialized, or the internal state of pseudoComm is corrupted.

error → code = 116

On entry: the value of devComm is NULL.

error → code = 117

On entry: the value of maxDevGenDim does not satisfy the constraint listed above.

7 Example

There is no example program specifically for this function. For examples of how this function should be used, please see the example program for naggpudevSobolUniformA.
1 Purpose

naggpudevSobolInitA initializes the Sobol’ device function generator. It must be preceded by a call to naggpuSobolDeviceInitA and must ultimately be followed by a call to naggpuSobolDeviceCleanupA to free allocated system resources. naggpudevSobolInitA must be called prior to calling any of the Sobol’ device generator functions such as naggpudevSobolUniformA.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_sobolDevFuncs.h>
__device__ void naggpudevSobolInitA(int dim, unsigned int offset, int &comm1,
        unsigned int &comm2, unsigned int *comm3,
        const NagGpuSobolDeviceComm *devComm)
```

3 Description

The Sobol’ generator is an efficient quasi-random generator which is relatively light on resources. As such it is natural to try to embed it within a GPU kernel. A per-thread parallelization strategy has been adopted, whereby each CUDA thread has an independent Sobol’ generator. Starting points are controlled through the `offset` parameter.

High dimensional Sobol’ sequences require a lot of storage for each quasi-random point. Since shared memory on the GPU is limited, it may be necessary to store these points in local memory. On the older NVIDIA hardware this will result in relatively poor performance, however on the newer ‘Fermi’ architecture the performance should be much better. Please see the section on Local Memory in the CUDA Programming Guide.

4 Error Handling

This is a GPU device function, and no error handling is performed.

5 References

None.

6 Arguments

1: `dim` – int

   Input

   `On entry:` the dimension of the Sobol’ sequence to be generated.

   `Constraint:` $1 \leq \text{dim} \leq D$ where $D$ denotes the value of the `maxDevGenDim` parameter in the initialization call to naggpuSobolDeviceInitA.

2: `offset` – unsigned int

   Input

   `On entry:` the offset into the Sobol’ sequence at which to start generating.

3: `comm1` – int &

   Communication Data

   the parameter will be initialized with communication information and must be passed to subsequent calls to the device function generators such as naggpudevSobolUniformA. It must not be modified in any way.
4: **comm2** – unsigned int &  
**Communication Data**
the parameter will be initialized with communication information and must be passed to subsequent calls to the device function generators such as naggpudevSobolUniformA. It must not be modified in any way.

5: **comm3[dim]** – unsigned int *  
**Communication Array**
the parameter will be initialized with communication information and must be passed to subsequent calls to the device function generators such as naggpudevSobolUniformA. It must not be modified in any way. This array will contain state information which will be accessed frequently.

6: **devComm** – const NagGpuSobolDeviceComm *  
**Communication Data**
The NagGpuSobolDeviceComm memory address obtained from the host initialization function naggpuSobolDeviceInitA.

7  **Error Indicators and Warnings**
None.

8  **Example**
There is no example program specifically for this function. For examples of how this function should be used, please see the example program for naggpudevSobolUniformA.
1 Purpose

naggpudevSobolExpA generates the next point from a quasi-random exponential distribution with parameter $\lambda$.

The initialization function naggpudevSobolInitA must be called prior to the first call to naggpudevSobolExpA. Thereafter, this function may be called repeatedly to generate the next point in the sequence. Care should be taken to ensure that sequences from successive CUDA threads do not overlap.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_sobolDevFuncs.h>

template <typename FP>
__device__ void naggpudevSobolExpA(FP *x, FP lambda, const int comm1,
                                    unsigned int &comm2, unsigned int *comm3,
                                    const NagGpuSobolDeviceComm *devComm)
```

3 Description

Sobol’ sequences are made up of one or more multidimensional points, with each point composed of several one dimensional values. The dimensionality of the sequence is specified when calling naggpudevSobolInitA to initialize the generator. Below we will consider a $d$-dimensional Sobol’ sequence $x^0, x^1, \ldots$ so that each point $x^i = \left(x^i_1, x^i_2, \ldots, x^i_d\right)$ is composed of $d$ one dimensional values.

The exponential distribution has probability density function given by

$$f(x) = \begin{cases} \frac{1}{\lambda} e^{-x/\lambda} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\lambda > 0$. This function returns the next point $x = (x_1, x_2, \ldots, x_d)$ where

$$x_i = -\lambda \ln(y_i + 2^{-32})$$

for each $i = 1, 2, \ldots, d$ and $y = (y_1, y_2, \ldots, y_d) \in [0, 1]^d$ is the next point in the Sobol’ sequence.

4 Error Handling

This is a GPU device function, and no error handling is performed.

5 References

None.

6 Arguments

1: $x[d] - FP *$
   
   The template argument parameter FP can take type double or float.

   The value $d$ is the dimension of the sequence as specified to the initialization function naggpudevSobolInitA.

   On exit: the next $d$-dimensional point from the specified distribution.
2: \( \lambda \) – FP

The template argument parameter FP can take type double or float.

*On entry*: the mean, \( \lambda \), of the exponential distribution.

*Constraint*: \( \lambda > 0 \).

3: comm1 – const int

The value of comm1 from the initialization call naggPudevSobolInitA.

4: comm2 – unsigned int &

The value of comm2 obtained from the previous call to any of the Sobol’ device function generators, or if this is the first call to any of the Sobol’ device function generators, the value of comm2 from the initialization call naggPudevSobolInitA.

5: comm3[d] – unsigned int *

The value \( d \) is the dimension of the sequence as specified to the initialization function naggPudevSobolInitA.

The value of comm3 obtained from the previous call to any of the Sobol’ device function generators, or if this is the first call to any of the Sobol’ device function generators, the value of comm3 from the initialization call naggPudevSobolInitA.

6: devComm – const NagGpuSobolDeviceComm *

The NagGpuSobolDeviceComm memory address obtained from the host initialization function naggpuSobolDeviceInitA.

7 Error Indicators and Warnings

None.

8 Example

This example program uses naggPudevSobolExpA to print 5 quasi-random numbers of dimension 10 from an exponential distribution using the Sobol’ device function generator. The first point in the sequence is skipped and generation starts at the second point.

8.1 Program Text

```c
/*
 * Example Program: naggPudevSobolExpA
 * Copyright 2009, Numerical Algorithms Group Ltd, Oxford, UK.
 * Version 0.4, 2011.
 */
#include using namespace std;
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
#include <nag_gpu_sobolDevFuncs.h>

// precision defined by Makefile
ifdef SINGLEPRECISION
#define FP float
else
#define FP double
endif
```
void checkNagError(NagGpuError *error);
void checkCudaError(cudaError_t cuError);

#define MAX_SOBOL_DIMENSION 40

/*
* Test kernel to try out device level functions
* Note: this will store numbers in TRANSPOSE order
* to naggpuQuasiRandExpA
*/
__global__ void sobol_device_functions_exp_test(
    int n, int ppt, int dim, FP *d_buff,
    NagGpuSobolDeviceComm *devComm, int offset)
{
    /* Assumptions:
    * Grid is two dimensional - run along x dimension first, then along y
    * Thread block is one dimensional with number of threads in blockDim.x.
    * Numbers are stored as npts rows of dim columns each. Have no thread
    * coalescing *at all* in this kernel
    */

    // Compute thread number for this thread
    int blockNum = blockIdx.y*gridDim.x + blockIdx.x;
    int thdNum = blockNum*blockDim.x + threadIdx.x;

    // Not shared among threads - jbeing used as a cache to speed up reads
    extern __shared__ unsigned int comm3[];

    const FP lambda = 1.0;

    // Index of point we are currently creating. From 0 to npts-1
    int idx = thdNum*ppt;
    // Storage offset
    d_buff += idx*dim;

    if (idx < n)
    {
        // Workspace variable for device Sobol functions
        int comm1 = 0;
        unsigned int comm2 = 0;

        // Initialise stream generator
        naggpuDevSobolInitA(dim, offset + idx, comm1, comm2,
            &comm3[threadIdx.x*dim], devComm);

        // Loop over all points we’re required to create
        for(int i = 0; i < ppt && idx + i < n; i++)
        {
            naggpuDevSobolExpA(d_buff, lambda, comm1, comm2,
                &comm3[threadIdx.x*dim], devComm);
            d_buff += dim;
        }
    }
}

int main(int argc, char **argv)
{
    const int dim = 40;
    const int n = 101;
    FP *h_buff = 0, *d_buff = 0;
    int offset = 1;
    unsigned int pseudoSeed[] = {1, 2, 3, 4, 5, 6};
    NagGpuSobolDeviceComm *devComm;
    NagCPURandComm pseudoComm;
# NAG GPU Example Program: \texttt{naggpudevSobolExpA}

*single precision* or *double precision*

```cpp
// Allocate CPU and GPU memory
h_buff = new FP[n*dim];
cuError = cudaMalloc((void **)&d_buff, sizeof(FP)*n*dim);
checkCudaError(cuError);
```

```cpp
// Initialise the CPU pseudo-random generator
nagCPURandInitA(NAGGPURANDGEN_MRG32K3A, 0, 0, 0, 0, pseudoSeed, 
&pseudoComm, &error);
checkNagError(&error);
```

```cpp
// Initialise GPU Sobol generator
naggpuSobolDeviceInitA(NAGGPUSCRAMTYPES_NONE, dim, &devComm, 
&pseudoComm, &comm, &error);
checkNagError(&error);
```

```cpp
// Launch GPU computation asynchronously
// Use 96 threads per block and 2 points per thread.
const int ppt = 2;
const int nthds = 96;
const int nblks = n/(ppt*nthds) + 1;
sobol_device_functions_exp_test 
<<<nblks, nthds, nthds*MAX_SOBOL_DIMENSION*sizeof(unsigned int)>>>
(n, ppt, dim, d_buff, devComm, offset);
```

```cpp
// Copy results to host once completed
cuError = cudaMemcpy(h_buff, d_buff, sizeof(FP)*n*dim, 
cudaMemcpyDeviceToHost);
checkCudaError(cuError);
```

```cpp
// Print random numbers
cout << "The 5 GPU numbers from dimensions 1 to 10:" << endl;
cout.setf(ios::fixed,ios::floatfield);
cout.precision(3);
for(int d = 0; d < 10; d++)
{
    cout << "dim" << d+1 << "\t";
    for(int i = 0; i < 5; i++)
    {
        cout << h_buff[i*dim + d] << "\t";
    }
    cout << endl;
}
```

```cpp
// Call cleanup for the NAG routine
naggpuSobolDeviceCleanupA(devComm, &comm, &error);
checkNagError(&error);
```
Free CPU and GPU memory
delete[](h_buff);
if (d_buff)
{
    cuError = cudaFree(d_buff);
    checkCudaError(cuError);
}
return 0;
}

void checkNagError(NagGpuError *error)
{
    if (error->code != 0)
    {
        char *buff;
        buff = new char[error->msgLength];
        naggpuErrorCopyMsg(buff, error);
        cout << buff << endl;
        delete[] buff;
        exit(1);
    }
}

void checkCudaError(cudaError_t cuError)
{
    if (cuError != cudaSuccess)
    {
        cout << cudaGetErrorString(cuError) << endl;
        exit(1);
    }
}

8.2 Program Data
None.

8.3 Program Results
NAG GPU Example Program: naggpudevSobolExpA, single precision

The 5 GPU numbers from dimensions 1 to 10:
dim1 0.693 0.288 1.386 0.981 0.134
dim2 0.693 1.386 0.288 0.981 0.134
dim3 0.693 1.386 0.288 0.470 2.079
dim4 0.693 1.386 0.288 0.134 0.981
dim5 0.693 0.288 1.386 0.981 0.134
dim6 0.693 0.288 1.386 2.079 0.470
dim7 0.693 1.386 0.288 0.981 0.134
dim8 0.693 0.288 1.386 0.134 0.981
dim9 0.693 0.288 1.386 0.134 0.981
dim10 0.693 0.288 1.386 0.470 2.079
1 Purpose

naggpudevSobolNormalA generates the next point from a quasi-random Normal distribution with mean \( \mu \) and variance \( \sigma^2 \).

The initialization function naggpudevSobolInitA must be called prior to the first call to naggpudevSobolNormalA. Thereafter, this function may be called repeatedly to generate the next point in the sequence. Care should be taken to ensure that sequences from successive CUDA threads do not overlap.

2 Specification

```cpp
#include <nag_gpu.h>
#include <nag_gpu_sobolDevFuncs.h>

template <typename FP>
__device__ void naggpudevSobolNormalA(FP *x, FP mu, FP sigma, const int comm1,
                                       unsigned int &comm2, unsigned int *comm3,
                                       const NagGpuSobolDeviceComm *devComm)
```

3 Description

Sobol’ sequences are made up of one or more multidimensional points, with each point composed of several one dimensional values. The dimensionality of the sequence is specified when calling naggpudevSobolInitA to initialize the generator. Below we will consider a \( d \)-dimensional Sobol’ sequence \( x^0, x^1, \ldots \) so that each point \( x^k = (x^k_1, x^k_2, \ldots, x^k_d) \) is composed of \( d \) one dimensional values.

The Normal distribution has probability density function given by

\[
    f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left( -\frac{(x - \mu)^2}{2\sigma^2} \right)
\]

where \( \sigma > 0 \) and \( \mu \in \mathbb{R} \). This function returns the next point \( x = (x_1, x_2, \ldots, x_d) \) where

\[
    x_i = \mu + \sigma \sqrt{2} \text{erfinv}(z_i)
\]

for each \( i = 1, 2, \ldots, d \) and \( \text{erfinv} \) is the inverse error function. Here \( z = (z_1, z_2, \ldots, z_d) \) is a low discrepancy point in the interval \((-1, 1)^d\).

4 Error Handling

This is a GPU device function, and no error handling is performed.

5 References

None.

6 Arguments

1: \( x[d] \) \( \rightarrow \) FP *  

The template argument parameter FP can take type double or float.
The value \( d \) is the dimension of the sequence as specified to the initialization function naggpudevSobolInitA.

\textit{On exit}: the next \( d \)-dimensional point from the specified distribution.

2: \( \mu \) – FP 

\textit{Input}
The template argument parameter FP can take type double or float.

\textit{On entry}: the mean, \( \mu \), of the Normal distribution.

3: \( \sigma \) – FP 

\textit{Input}
The template argument parameter FP can take type double or float.

\textit{On entry}: the standard deviation, \( \sigma \), of the Normal distribution.

\textit{Constraint}: \( \sigma > 0 \).

4: \( \text{comm1} \) – const int 

\textit{Communication Data}
The value of \( \text{comm1} \) from the initialization call naggpudevSobolInitA.

5: \( \text{comm2} \) – unsigned int & 

\textit{Communication Data}
The value of \( \text{comm2} \) obtained from the previous call to any of the Sobol’ device function generators, or if this is the first call to any of the Sobol’ device function generators, the value of \( \text{comm2} \) from the initialization call naggpudevSobolInitA.

6: \( \text{comm3}[d] \) – unsigned int * 

\textit{Communication Array}
The value \( d \) is the dimension of the sequence as specified to the initialization function naggpudevSobolInitA.

The value of \( \text{comm3} \) obtained from the previous call to any of the Sobol’ device function generators, or if this is the first call to any of the Sobol’ device function generators, the value of \( \text{comm3} \) from the initialization call naggpudevSobolInitA.

7: \( \text{devComm} \) – const NagGpuSobolDeviceComm * 

\textit{Communication Data}
The NagGpuSobolDeviceComm memory address obtained from the host initialization function naggpuSobolDeviceInitA.

7 \ Error Indicators and Warnings
None.

8 \ Example
This example program uses naggpudevSobolNormalA to print 5 quasi-random numbers of dimension 10 from a Normal distribution using the Sobol’ device function generator. The first point in the sequence is skipped and generation starts at the second point.

8.1 \ Program Text
/*
 * Example Program: naggpudevSobolNormalA
 * Copyright 2009, Numerical Algorithms Group Ltd, Oxford, UK.
 * Version 0.4, 2011.
 */
#include
using namespace std;
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
#include <nag_gpu_sobolDevFuncs.h>
#include <nag_gpu_serial.h>

// precision defined by Makefile
#ifndef SINGLEPRECISION
#define FP float
#else
#define FP double
#endif

void checkNagError(NagGpuError *error);
void checkCudaError(cudaError_t cuError);

#define MAX_SOBOL_DIMENSION 40

/*
 Test kernel to try out device level functions
 Note: this will store numbers in TRANSPOSE order
 to naggpuQuasiRandNormalA
*/
__global__ void sobol_device_functions_normal_test(
    int n, int ppt, int dim, FP *d_buff,
    NagGpuSobolDeviceComm *devComm, int offset)
{
    /* Assumptions:
       * Grid is two dimensional - run along x dimension first, then along y
       * Thread block is one dimensional with number of threads in blockDim.x.
       * Numbers are stored as npts rows of dim columns each. Have no thread
       * coalescing *at all* in this kernel
     */

    int blockNum = blockIdx.y*gridDim.x + blockIdx.x;
    int thdNum = blockNum*blockDim.x + threadIdx.x;

    extern __shared__ unsigned int comm3[];

    const FP mu = 0.0, sigma = 1.0;
    int idx = thdNum*ppt;

    if (idx < n)
    {
        // Workspace variable for device Sobol functions
        int comm1 = 0;
        unsigned int comm2 = 0;

        // Initialise stream generator
        naggpuSobolInitA(dim, offset + idx, comm1, comm2,
                         &comm3[threadIdx.x*dim], devComm);

        // Loop over all points we’re required to create
        for(int i = 0; i < ppt && idx + i < n; i++)
        {
            naggpuSobolNormalA(d_buff, mu, sigma, comm1, comm2,
                               &comm3[threadIdx.x*dim], devComm);
            d_buff += dim;
        }
    }
}

int main(int argc, char **argv)
{

const int dim = 40;
const int n = 101;
FP *h_buff = 0, *d_buff = 0;
int offset = 1;
unsigned int pseudoSeed[] = {1, 2, 3, 4, 5, 6};

NagGpuSobolDeviceComm *devComm;
NagCPURandComm pseudoComm;
NagGpuQuasiRandComm comm;
NagGpuError error;
cudaError_t cuError;

// Print the title
cout << "NAG GPU Example Program: ";
if (sizeof(FP)==sizeof(float))
    cout << "single precision"
else
    cout << "double precision"
cout << endl << endl;

// Allocate CPU and GPU memory
h_buff = new FP[n*dim];
cuError = cudaMalloc((void **)&d_buff, sizeof(FP)*n*dim);
checkCudaError(cuError);

// Initialise the CPU pseudo-random generator
nagCPURandInitA(NAGGPURANDGEN_MRG32K3A, 0, 0, 0, 0, 0, pseudoSeed, &pseudoComm, &error);
checkNagError(&error);

// Initialise GPU Sobol generator
naggpuSobolDeviceInitA(NAGGpuscramTypes_NONE, dim, &devComm, &pseudoComm, &comm, &error);
checkNagError(&error);

// Launch GPU computation asynchronously
// Use 96 threads per block and 2 points per thread.
const int ppt = 2;
const int nthds = 96;
const int nbllks = n/(ppt*nthds) + 1;
sobol_device_functions_normal_test
<<<nbllks, nthds, nthds*MAX_SOBOL_DIMENSION*sizeof(unsigned int)>>>
(n, ppt, dim, d_buff, devComm, offset);

/*
 * One can now launch other kernels to operate on the Sobol numbers,
 * or copy the numbers to the host and operate on them there.
 * Here we simply copy them to the host in order to print them
 */

// Copy results to host once completed
cuError = cudaMemcpy(h_buff, d_buff, sizeof(FP)*n*dim, cudaMemcpyDeviceToHost);
checkCudaError(cuError);

// Print random numbers
cout << "The 5 GPU numbers from dimensions 1 to 10:" << endl;
cout.setf(ios::fixed, ios::floatfield);
cout.precision(3);
for(int d = 0; d < 10; d++)
{
    cout << "dim" << d+1 << "\t"
    for(int i = 0; i < 5; i++)
    { 
8.2 Program Data
None.

8.3 Program Results
NAG GPU Example Program: naggpudevSobolNormalA, single precision

The 5 GPU numbers from dimensions 1 to 10:
dim1 0.000 0.674 -0.674 -0.319 1.150
dim2 0.000 -0.674 0.674 -0.319 1.150
dim3 0.000 -0.674 0.674 0.319 -1.150
dim4 0.000 -0.674 0.674 1.150 -0.319
dim5 0.000 0.674 -0.674 -0.319 1.150
dim6 0.000 0.674 -0.674 -1.150 0.319
<table>
<thead>
<tr>
<th>dim7</th>
<th>0.000</th>
<th>-0.674</th>
<th>0.674</th>
<th>-0.319</th>
<th>1.150</th>
</tr>
</thead>
<tbody>
<tr>
<td>dim8</td>
<td>0.000</td>
<td>0.674</td>
<td>-0.674</td>
<td>1.150</td>
<td>-0.319</td>
</tr>
<tr>
<td>dim9</td>
<td>0.000</td>
<td>0.674</td>
<td>-0.674</td>
<td>1.150</td>
<td>-0.319</td>
</tr>
<tr>
<td>dim10</td>
<td>0.000</td>
<td>0.674</td>
<td>-0.674</td>
<td>0.319</td>
<td>-1.150</td>
</tr>
</tbody>
</table>
1 Purpose
naggpudevSobolUniformA generates the next point from a quasi-random uniform distribution over the interval \([a, b)\) for specified constants \(a\) and \(b\).

The initialization function naggpudevSobolInitA must be called prior to the first call to naggpudevSobolUniformA. Thereafter, this function may be called repeatedly to generate the next point in the sequence. Care should be taken to ensure that sequences from successive CUDA threads do not overlap.

2 Specification
#include <nag_gpu.h>
#include <nag_gpu_sobolDevFuncs.h>

template <typename FP>
__device__ void naggpudevSobolUniformA(FP *x, FP a, FP b, const int comm1,
unsigned int &comm2, unsigned int *comm3,
const NagGpuSobolDeviceComm *devComm)

3 Description
Sobol’ sequences are made up of one or more multidimensional points, with each point composed of several one dimensional values. The dimensionality of the sequence is specified when calling naggpudevSobolInitA to initialize the generator. Below we will consider a \(d\)-dimensional Sobol’ sequence \(x^0, x^1, \ldots \) so that each point \(x^j = (x^j_1, x^j_2, \ldots, x^j_d)\) is composed of \(d\) one dimensional values.

If \(a = 0\) and \(b = 1\), this function returns the next point \(y \in [0, 1)^d\) from the Sobol’ sequence. For other values of \(a\) and \(b\), the function applies the transformation

\[ x_i = a + (b - a)y_i \]

for each \(i = 1, 2, \ldots, d\) to produce a quasi-random point \(x\) in the interval \([a, b)^d\).

4 Error Handling
This is a GPU device function, and no error handling is performed.

5 References
None.

6 Arguments
1: \(x[d] - \text{FP *}
\)

The template argument parameter FP can take type double or float.

The value \(d\) is the dimension of the sequence as specified to the initialization function naggpudevSobolInitA.

On exit: the next \(d\)-dimensional point from the specified distribution.
2: a – FP  
   Input  
   The template argument parameter FP can take type double or float.  
   On entry: the lower bound for the quasi-random values.

3: b – FP  
   Input  
   The template argument parameter FP can take type double or float.  
   On entry: the upper bound for the quasi-random values.  
   Constraint: b > a.

4: comm1 – const int  
   Communication Data  
   The value of comm1 from the initialization call naggpudevSobolInitA.

5: comm2 – unsigned int &  
   Communication Data  
   The value of comm2 obtained from the previous call to any of the Sobol’ device function generators, or if this is the first call to any of the Sobol’ device function generators, the value of comm2 from the initialization call naggpudevSobolInitA.

6: comm3[d] – unsigned int *  
   Communication Array  
   The value d is the dimension of the sequence as specified to the initialization function naggpudevSobolInitA.  
   The value of comm3 obtained from the previous call to any of the Sobol’ device function generators, or if this is the first call to any of the Sobol’ device function generators, the value of comm3 from the initialization call naggpudevSobolInitA.

7: devComm – const NagGpuSobolDeviceComm *  
   Communication Data  
   The NagGpuSobolDeviceComm memory address obtained from the host initialization function naggpuSobolDeviceInitA.

7  Error Indicators and Warnings
None.

8  Example
This example program uses naggpudevSobolUniformA to print 5 quasi-random numbers of dimension 10 from a uniform distribution using the Sobol’ device function generator. The first point in the sequence is skipped and generation starts at the second point.

8.1  Program Text
/*
 * Example Program: naggpudevSobolUniformA
 * Copyright 2009, Numerical Algorithms Group Ltd, Oxford, UK.
 * Version 0.4, 2011.
 */
#include
using namespace std;
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
#include <nag_gpu_sobolDevFuncs.h>
// precision defined by Makefile
#ifndef SINGLEPRECISION
#define FP float
#else
#define FP double
#endif

void checkNagError(NagGpuError *error);
void checkCudaError(cudaError_t cuError);

#define MAX_SOBOL_DIMENSION 40

/*
 * Test kernel to try out device level functions
 * Note: this will store numbers in TRANSPOSE order
 * to naggpuQuasiRandUniformA
 */
__global__ void sobol_device_functions_uniform_test(
    int n, int ppt, int dim, FP *d_buff,
    NagGpuSobolDeviceComm *devComm, int offset)
{
    /* Assumptions:
     * Grid is two dimensional - run along x dimension first, then along y
     * Thread block is one dimensional with number of threads in blockDim.x.
     * Numbers are stored as npts rows of dim columns each. Have no thread
     * coalescing *at all* in this kernel
     */

    // Compute thread number for this thread
    int blockNum = blockIdx.y*gridDim.x + blockIdx.x;
    int thdNum = blockNum*blockDim.x + threadIdx.x;

    // Not shared among threads - jbeing used as a cache to speed up reads
    extern __shared__ unsigned int comm3[
]
    const FP a = 0.0, b = 1.0;

    // Index of point we are currently creating. From 0 to npts-1
    int idx = thdNum*ppt;
    // Storage offset
    d_buff += idx*dim;

    if (idx < n)
    {
        // Workspace variable for device Sobol functions
        int comm1 = 0;
        unsigned int comm2 = 0;

        // Initialise stream generator
        naggpuDevSobolInitA(dim, offset + idx, comm1, comm2,
            &comm3[threadIdx.x*dim], devComm);

        // Loop over all points we’re required to create
        for(int i = 0; i < ppt && idx + i < n; i++)
        {
            naggpuDevSobolUniformA(d_buff, a, b, comm1, comm2,
                &comm3[threadIdx.x*dim], devComm);
            d_buff += dim;
        }
    }
}

int main(int argc, char **argv)
{
    const int dim = 40;
    const int n = 101;

    FP *h_buff = 0, *d_buff = 0;
int offset = 1;

unsigned int pseudoSeed[] = {1, 2, 3, 4, 5, 6};

NagGpuSobolDeviceComm *devComm;
NagCPURandComm pseudoComm;
NagGpuQuasiRandComm comm;
NagGpuError error;
cudaError_t cuError;

// Print the title
cout << "NAG GPU Example Program: ";
cout << "naggpudevSobolUniformA, ";
if (sizeof(FP)==sizeof(float))
  cout << "single precision";
else
  cout << "double precision";
cout << endl << endl;

// Allocate CPU and GPU memory
h_buff = new FP[n*dim];
cuError = cudaMalloc((void **)&d_buff, sizeof(FP)*n*dim);
checkCudaError(cuError);

// Initialise the CPU pseudo-random generator
nagCPURandInitA(NAGGPURANDGEN_MRG32K3A, 0, 0, 0, 0, 0, pseudoSeed, &pseudoComm, &error);
checkNagError(&error);

// Initialise GPU Sobol generator
naggpuSobolDeviceInitA(NAGGPUSCRAMTYPES_NONE, dim, &devComm, &pseudoComm, &comm, &error);
checkNagError(&error);

// Launch GPU computation asynchronously
// Use 96 threads per block and 2 points per thread.
const int ppt = 2;
const int nthds = 96;
const int nblks = n/(ppt*nthds) + 1;
sobol_device_functions_uniform_test
  <<<nblks, nthds, nthds*MAX_SOBOL_DIMENSION*sizeof(unsigned int)>>>
    (n, ppt, dim, d_buff, devComm, offset);

/*
 * One can now launch other kernels to operate on the Sobol numbers,
 * or copy the numbers to the host and operate on them there.
 * Here we simply copy them to the host in order to print them
 */

// Copy results to host once completed
cuError = cudaMemcpy(h_buff, d_buff, sizeof(FP)*n*dim, cudaMemcpyDeviceToHost);
checkCudaError(cuError);

// Print random numbers
cout << "The 5 GPU numbers from dimensions 1 to 10:" << endl;
cout.setf(ios::fixed,ios::floatfield);
cout.precision(3);
for(int d = 0; d < 10; d++)
{
  cout << "dim" << d+1 << "\t";
  for(int i = 0; i < 5; i++)
  {
    cout << h_buff[i*dim + d] << "\t";
  }
  cout << endl;
}
cout << endl;
// Call cleanup for the NAG routine
naggpuSobolDeviceCleanupA(devComm, &comm, &error);
checkNagError(&error);

// Free CPU and GPU memory
delete[] h_buff;
if (d_buff)
    {
        cuError = cudaFree(d_buff);
        checkCudaError(cuError);
    }
return 0;
}

void checkNagError(NagGpuError *error)
{
    if (error->code != 0)
    {
        char *buff;
        buff = new char[error->msgLength];
        naggpuErrorCopyMsg(buff, error);
        cout << buff << endl;
        delete[] buff;
        exit(1);
    }
}

void checkCudaError(cudaError_t cuError)
{
    if (cuError != cudaSuccess)
    {
        cout << cudaGetErrorString(cuError) << endl;
        exit(1);
    }
}

8.2 Program Data
None.

8.3 Program Results
NAG GPU Example Program: naggpuSobolUniformA, single precision

The 5 GPU numbers from dimensions 1 to 10:
dim1 0.500 0.750 0.250 0.375 0.875
dim2 0.500 0.250 0.750 0.375 0.875
dim3 0.500 0.250 0.750 0.625 0.125
dim4 0.500 0.250 0.750 0.875 0.375
dim5 0.500 0.750 0.250 0.375 0.875
dim6 0.500 0.750 0.250 0.125 0.625
dim7 0.500 0.250 0.750 0.375 0.875
dim8 0.500 0.750 0.250 0.875 0.375
dim9 0.500 0.750 0.250 0.875 0.375
dim10 0.500 0.750 0.250 0.625 0.125
**1 Purpose**
naggpuSobolDeviceCleanupA frees system resources that were allocated by a previous call to naggpuSobolDeviceInitA.

**2 Specification**
#include <nag_gpu.h>
extern "C"
cudaError_t naggpuSobolDeviceCleanupA(NagGpuSobolDeviceComm *devComm,
NagGpuQuasiRandComm *comm, NagGpuError *error)

**3 Description**

3.1 Synchronization
This function is blocking and will force synchronization between host and device. Control will not return to the calling program before this function has terminated.

3.2 Return Value
Any CUDA runtime errors that were encountered, or cudaSuccess if no CUDA runtime errors were encountered. Please see the Error Handling Chapter Introduction for further details on error handling.

**4 References**
None.

**5 Arguments**

1: **devComm** – NagGpuSobolDeviceComm *  
   *Input*
   
   On entry: the GPU memory address obtained from a prior call to naggpuSobolDeviceInitA.

2: **comm** – NagGpuQuasiRandComm *  
   *Communication Data*
   
   On entry: the pointer that was passed to a previous call to naggpuSobolDeviceInitA.

3: **error** – NagGpuError *  
   *Error Reporting*
   
   This parameter contains error information and should not be modified directly. Errors are indicated through the value of **error** → **code** which should be inspected after each call to this function. If **error** → **code** = 0 then no error occurred. If **error** → **code** ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.
6 Error Indicators and Warnings

error → code = 1

On entry: the CUDA runtime error status has not been cleared, indicating a previous CUDA error. Call cudaGetLastError() in the CUDA runtime library to clear the runtime error status.

error → code = 2

During execution: a CUDA runtime error was detected.

error → code = 100

On entry: the value of comm is NULL.

error → code = 101

On entry: comm has not been initialized, or the internal state of comm is corrupted.

error → code = 110

On entry: the value of devComm is NULL.

7 Example

There is no example program specifically for this function. For examples of how this function should be used, please see the example program for naggpudevSobolUniformA.
1 Purpose

nagCPURandInitA initializes a serial, host-only pseudorandom number generator to a give a repeatable sequence of pseudorandom numbers. This function must be called before any call to the serial generator functions (such as nagCPURandUniformA) and must ultimately be followed by a call to the cleanup function nagCPURandCleanupA to release system resources. A base generator is selected through the genid parameter and initialized with the values given in the seed array.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPURandInitA(NagGpuRandGen genid, int a1, int b1, int a2, int b2,
    long long c, const unsigned int *seed, NagCPURandComm *comm,
    NagGpuError *error)
```

3 Description

For different values of seed, a given generator will yield different sequences of random numbers. Alternatively, the same sequence of random numbers will be generated if the same value of seed is used. In general there is no guarantee of statistical properties between sequences, only within sequences. This is important when generators are used in parallel. This function can ‘skip ahead’ or advance the seed by an amount

$$s = a_1 2^{b_1} + a_2 2^{b_2} + c$$

so that the generator will produce the sequence of random numbers \(X_s, X_{s+1}, X_{s+2}, \ldots\) instead of the original sequence \(X_0, X_1, X_2, \ldots\). This technique is useful to produce independent generators, often also called independent streams and substreams. Please see the Random Number Generators Chapter Introduction for further information.

Independent generators will be important mostly to applications which use multiple CPU cores simultaneously. In this case, each generator will have its own NagCPURandComm structure which encapsulates all the information the generator requires. An array of NagCPURandComm structures with judiciously chosen skip aheads (see the Random Number Generators Chapter Introduction) represents an array of independent generators. Each structure must be initialized by a call to nagCPURandInitA.

For the most common task of generating a block of pseudorandom numbers on a single CPU core, users will typically only have a single generator (i.e. only one communication structure) and the skip ahead \(s\) above can be set to zero. The comments about arrays of communication structures can be ignored.

3.1 Synchronization

As this function only uses the CPU there is no requirement for synchronization.

4 References

None.
5 Arguments

1: \texttt{genid} – \texttt{NagGpuRandGen} \hspace{1cm} \textit{Input}

\textit{On entry:} the type of generator to be used:

\texttt{genid = NAGGPRURANDGEN\_MRG32K3A}  
\texttt{genid = NAGGPRURANDGEN\_MT19937}

Please see the Random Number Generators Chapter Introduction for details about each of these base generators.

\textit{Constraint:} \texttt{genid = NAGGPRURANDGEN\_MRG32K3A} or \texttt{NAGGPRURANDGEN\_MT19937}.

2: \texttt{a1} – int \hspace{1cm} \textit{Input}

\textit{On entry:} the value of $a_1$ in the skip ahead equation (1) above.

\textit{Constraint:} $a_1 \geq 0$.

3: \texttt{b1} – int \hspace{1cm} \textit{Input}

\textit{On entry:} the value of $b_1$ in the skip ahead equation (1) above.

\textit{Constraints:}

if \texttt{genid = NAGGPRURANDGEN\_MRG32K3A}, $0 \leq b_1 \leq 191$;  
if \texttt{genid = NAGGPRURANDGEN\_MT19937}, $0 \leq b_1 \leq 19937$.

4: \texttt{a2} – int \hspace{1cm} \textit{Input}

\textit{On entry:} the value of $a_2$ in the skip ahead equation (1) above.

\textit{Constraint:} $a_2 \geq 0$.

5: \texttt{b2} – int \hspace{1cm} \textit{Input}

\textit{On entry:} the value of $b_2$ in the skip ahead equation (1) above.

\textit{Constraints:}

if \texttt{genid = NAGGPRURANDGEN\_MRG32K3A}, $0 \leq b_2 \leq 191$;  
if \texttt{genid = NAGGPRURANDGEN\_MT19937}, $0 \leq b_2 \leq 19937$.

6: \texttt{c} – long long \hspace{1cm} \textit{Input}

\textit{On entry:} the value of $c$ in the skip ahead equation (1) above.

\textit{Constraint:} $c \geq 0$.

7: \texttt{seed[\texttt{n}]} – const unsigned int * \hspace{1cm} \textit{Input}

\textit{On entry:} an array of $n$ 32-bit unsigned integers to initialize the generator.

\textit{Constraints:}

if \texttt{genid = NAGGPRURANDGEN\_MRG32K3A},

\begin{align*}
  n &= 6  
  &\text{for } i = 0, 1, 2, \text{ seed}[i] < 2^{32} - 209 \text{ and seed}(i) \neq 0 \text{ for at least one } i  
  &\text{for } i = 3, 4, 5, \text{ seed}[i] < 2^{32} - 22853 \text{ and seed}(i) \neq 0 \text{ for at least one } i;  
\end{align*}

if \texttt{genid = NAGGPRURANDGEN\_MT19937},

\begin{align*}
  n &= 624  
  &\text{for } i = 0, 1, 2, \ldots, 623, \text{ seed}(i) \neq 0 \text{ for at least one } i.  
\end{align*}
NagCPURandComm is a structure which holds state and communication information and must not be modified in any way. The structure will be initialized and must be passed to the generator functions (such as nagCPURandUniformA). Once all required points have been obtained, comm must be passed to nagCPURandCleanupA to free allocated system resources.

This parameter contains error information and should not be modified directly. Errors are indicated through the value of error → code which should be inspected after each call to this function. If error → code = 0 then no error occurred. If error → code ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

error → code = 100

On entry: the value of comm is NULL.

error → code = 110

On entry: genid does not specify a valid pseudorandom number generator. See NagGpuRandGen for permitted values.

error → code = 111

On entry: the value of a1 is negative.

error → code = 112

On entry: the value of b1 does not satisfy the constraints listed above.

error → code = 113

On entry: the value of a2 is negative.

error → code = 114

On entry: the value of b2 does not satisfy the constraints listed above.

error → code = 115

On entry: the value of c is negative.

error → code = 116

On entry: the value of seed is NULL.

error → code = 117

On entry: the values in the seed array do not satisfy the constraints listed above.

7 Example

None.
NAG Numerical Routines for GPUs Function Document

nagCPURandExpA

1 Purpose

nagCPURandExpA generates $n$ values $x_i$ from an exponential distribution with mean $\lambda$.

The initialization function nagCPURandInitA must be called prior to the first call to nagCPURandExpA. Thereafter, this function may be called repeatedly to generate additional sets of random values. Once all desired values have been obtained, the function nagCPURandCleanupA must be called to free allocated system resources.

Note: To obtain the same values from nagCPURandExpA as from the function nag_CPU_mrg32-k3a_exp(N, P) in release 0.3 of the library, please see Section 2.1.1.1 in the Random Number Generators Chapter Introduction.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPURandExpA_sp(int n, float lambda, float *buff, NagCPURandComm *comm, 
                        NagGpuError *error)
extern "C"
void nagCPURandExpA(int n, double lambda, double *buff, NagCPURandComm *comm, 
                     NagGpuError *error)
```

3 Description

The exponential distribution has probability density function given by

$$f(x) = \begin{cases} \frac{1}{\lambda}e^{-x/\lambda} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\lambda > 0$. This function returns

$$X_i = -\lambda \ln Y_i$$

where $Y_i$ are the next $n$ values generated by the underlying uniform $[0,1]$ generator.

3.1 Synchronization

As this function only uses the CPU there is no requirement for synchronization.

4 References

None.

5 Arguments

1: $n$ – int  

   Input

   On entry: the number of random values to be generated.

   Constraint: $n \geq 1$. 
**nagCPURandExpA**  

**Input**  

2: \( \texttt{lambda} \) – float  

3: \( \texttt{lambda} \) – double  

This parameter has type float or double depending on whether the single or double precision version of this function is called.

*On entry*: the mean, \( \lambda \), of the distribution.

*Constraint*: \( \texttt{lambda} > 0 \).

**Output**  

4: \( \texttt{buff[n]} \) – float *  

5: \( \texttt{buff[n]} \) – double *  

This parameter has type float or double depending on whether the single or double precision version of this function is called.

*On exit*: the \( n \) pseudorandom numbers from the specified distribution.

**Communication Data**  

6: \( \texttt{comm} \) – NagCPURandComm *  

NagCPURandComm is a structure which holds state and communication information and must not be modified in any way. Once all required points have been obtained, \( \texttt{comm} \) must be passed to nagCPURandCleanupA to free allocated system resources.

**Error Reporting**  

7: \( \texttt{error} \) – NagGpuError *  

This parameter contains error information and should not be modified directly. Errors are indicated through the value of \( \texttt{error} \rightarrow \texttt{code} \) which should be inspected after each call to this function. If \( \texttt{error} \rightarrow \texttt{code} = 0 \) then no error occurred. If \( \texttt{error} \rightarrow \texttt{code} \neq 0 \) then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

### 6 Error Indicators and Warnings

- **error \( \rightarrow \texttt{code} = 100 \)**  
  *On entry*: the value of \( \texttt{comm} \) is NULL.

- **error \( \rightarrow \texttt{code} = 101 \)**  
  *On entry*: \( \texttt{comm} \) has not been initialized, or the internal state of \( \texttt{comm} \) is corrupted.

- **error \( \rightarrow \texttt{code} = 110 \)**  
  *On entry*: \( n \leq 0 \).

- **error \( \rightarrow \texttt{code} = 112 \)**  
  *On entry*: \( \texttt{buff} \) is NULL.

- **error \( \rightarrow \texttt{code} = 114 \)**  
  *On entry*: \( \texttt{lambda} \leq 0 \).

### 7 Example

None.
NAG Numerical Routines for GPUs Function Document

nagCPURandGammaA

1 Purpose

nagCPURandGammaA generates \( n \) values \( X_i \) from a gamma distribution with shape parameter \( \alpha \) and scale parameter \( \beta \).

The initialization function nagCPURandInitA must be called prior to the first call to nagCPURandGammaA. Thereafter, this function may be called repeatedly to generate additional sets of random values. Once all desired values have been obtained, the function nagCPURandCleanupA must be called to free allocated system resources.

Note: currently only the MRG32k3a base generator is supported. Support for MT19937 will be added in future releases.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPURandGammaA_sp(int n, float alpha, float beta, float *buff,
                      NagCPURandComm *comm, NagGpuError *error)
extern "C"
void nagCPURandGammaA(int n, double alpha, double beta, double *buff,
                     NagCPURandComm *comm, NagGpuError *error)
```

3 Description

The gamma distribution has probability density function given by

\[
 f(x) = \begin{cases} 
  \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} & \text{if } x \geq 0 \\
  0 & \text{otherwise} 
\end{cases}
\]

where \( \alpha, \beta > 0 \). The rejection algorithm described in Marsaglia and Tsang (2000) is used to generate the gamma pseudorandom variates when \( \alpha \geq 1 \). When \( 0 < \alpha < 1 \), the scaling

\[
 \gamma_a = \gamma_{1+a} U^a
\]

is used where \( U \) denotes a uniform random variable in the interval \([0, 1]\) and \( \gamma_a \) denotes a gamma random variable with shape parameter \( \alpha \) and scale parameter \( \beta = 1 \). Note that currently only the MRG32k3a base generator is supported.

Note: rejection algorithms are extremely sensitive to computational accuracy. When a variate is generated close to the rejection envelope, small differences in numerical values can lead to it being accepted in double precision while it is rejected in single precision (or vice versa). From this point on, the single and double precision sequences will be different. The same behaviour is seen when comparing single precision sequences generated on the CPU and the GPU: differences in the floating point calculations will lead to the sequences diverging after a certain number of variates. In double precision, the CPU and GPU sequences will take much longer (on average) before they diverge, agreeing to tens or even hundreds of millions of variates before numerical differences cause a variate to be accepted on one platform while it is rejected on the other.

3.1 Synchronization

As this function only uses the CPU there is no requirement for synchronization.
4 References


5 Arguments

1: \( n \) – int 
\( \text{Input} \)

On entry: the number of random values to be generated.

Constraint: \( n \geq 1 \).

2: \( \text{alpha} \) – float 
\( \text{Input} \)

This parameter has type float or double depending on whether the single or double precision version of this function is called.

On entry: the shape parameter, \( \alpha \), of the distribution.

Constraint: \( \text{alpha} > 0 \).

3: \( \text{alpha} \) – double 
\( \text{Input} \)

4: \( \text{beta} \) – float 
\( \text{Input} \)

This parameter has type float or double depending on whether the single or double precision version of this function is called.

On entry: the scale parameter, \( \beta \), of the distribution.

Constraint: \( \text{beta} > 0 \).

5: \( \text{beta} \) – double 
\( \text{Input} \)

6: \( \text{buff}[n] \) – float * 
\( \text{Output} \)

This parameter has type float or double depending on whether the single or double precision version of this function is called.

On exit: the \( n \) pseudorandom numbers from the specified distribution.

7: \( \text{buff}[n] \) – double * 
\( \text{Output} \)

8: \( \text{comm} \) – NagCPURandComm * 
\( \text{Communication Data} \)

NagCPURandComm is a structure which holds state and communication information and must not be modified in any way. Once all required points have been obtained, \( \text{comm} \) must be passed to nagCPURandCleanupA to free allocated system resources.

9: \( \text{error} \) – NagGpuError * 
\( \text{Error Reporting} \)

This parameter contains error information and should not be modified directly. Errors are indicated through the value of \( \text{error} \rightarrow \text{code} \) which should be inspected after each call to this function. If \( \text{error} \rightarrow \text{code} = 0 \) then no error occurred. If \( \text{error} \rightarrow \text{code} \neq 0 \) then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

\( \text{error} \rightarrow \text{code} = 100 \)

On entry: the value of \( \text{comm} \) is NULL.

\( \text{error} \rightarrow \text{code} = 101 \)

On entry: \( \text{comm} \) has not been initialized, or the internal state of \( \text{comm} \) is corrupted.
error \rightarrow \text{code} = 110  
\text{On entry: } n \leq 0.

error \rightarrow \text{code} = 112  
\text{On entry: } \text{buff} \text{ is NULL.}

error \rightarrow \text{code} = 113  
\text{On entry: } \text{alpha} \leq 0

error \rightarrow \text{code} = 114  
\text{On entry: } \text{beta} \leq 0

error \rightarrow \text{code} = 115  
\text{On entry: } \text{the MT19937 base generator is selected (see nagCPURandInitA for further details). Currently only the MRG32k3a base generator is supported.}

7 Example
None.
NAG Numerical Routines for GPUs Function Document

nagCPURandNormalA

1 Purpose

nagCPURandNormalA generates \( n \) values \( x_i \) from a Normal distribution with mean \( \mu \) and variance \( \sigma^2 \).

The initialization function nagCPUrandInitA must be called prior to the first call to nagCPURandNormalA. Thereafter, this function may be called repeatedly to generate additional sets of random values. Once all desired values have been obtained, the function nagCPUrandCleanupA must be called to free allocated system resources.

Note: To obtain the same values from nagCPURandNormalA as from the function \nag_CPU_mrg32k3a_normal(N, P)\ in release 0.3 of the library, please see Section 2.1.1.1 in the Random Number Generators Chapter Introduction.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>

extern "C"
void nagCPURandNormalA_sp(int n, float mu, float sigma, float *buff,
                            NagCPURandComm *comm, NagGpuError *error);

extern "C"
void nagCPURandNormalA(int n, double mu, double sigma, double *buff,
                        NagCPURandComm *comm, NagGpuError *error);
```

3 Description

The Normal distribution has probability density function given by

\[
f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right)
\]

where \( \sigma > 0 \) and \( \mu \in \mathbb{R} \). This function uses a Box-Muller transform to convert a pair of uniform \((0, 1)\) random numbers into a pair of Normal random numbers. Let \( X_0, X_1, X_2, \ldots \) denote the sequence of uniform \((0, 1)\) pseudorandom variates as specified by the base generator algorithm. This function uses successive pairs of uniform variates in the Box-Muller transform to produce successive pairs of Normal variates, i.e. \((X_0, X_1)\)\(\rightarrow\)(\(Z_0, Z_1\)), \((X_2, X_3)\)\(\rightarrow\)(\(Z_2, Z_3\)) where \(Z_0, Z_1, Z_2, \ldots\) denotes the output sequence of Normal variates.

3.1 Synchronization

As this function only uses the CPU there is no requirement for synchronization.

4 References

None.

5 Arguments

1. \( n \) – int

\( \text{Input} \)

\( \text{On entry:} \) the number of random values to be generated.

\( \text{Constraint:} \ n \geq 1. \)
mu – float  
**Input**

This parameter has type float or double depending on whether the single or double precision version of this function is called.

On entry: the mean, \( \mu \), of the distribution.

sigma – float  
**Input**

This parameter has type float or double depending on whether the single or double precision version of this function is called.

On entry: the standard deviation, \( \sigma \), of the distribution

\[ \sigma > 0. \]

buff[n] – float *  
**Output**

This parameter has type float or double depending on whether the single or double precision version of this function is called.

On exit: the \( n \) pseudorandom numbers from the specified distribution.

com – NagCPURandComm *  
**Communication Data**

NagCPURandComm is a structure which holds state and communication information and must not be modified in any way. Once all required points have been obtained, \( \text{com} \) must be passed to nagCPURandCleanupA to free allocated system resources.

error – NagGpuError *  
**Error Reporting**

This parameter contains error information and should not be modified directly. Errors are indicated through the value of \( \text{error} \to \text{code} \) which should be inspected after each call to this function. If \( \text{error} \to \text{code} = 0 \) then no error occurred. If \( \text{error} \to \text{code} \neq 0 \) then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6  **Error Indicators and Warnings**

\( \text{error} \to \text{code} = 100 \)

On entry: the value of \( \text{com} \) is NULL.

\( \text{error} \to \text{code} = 101 \)

On entry: \( \text{com} \) has not been initialized, or the internal state of \( \text{com} \) is corrupted.

\( \text{error} \to \text{code} = 110 \)

On entry: \( n \leq 0. \)

\( \text{error} \to \text{code} = 112 \)

On entry: \( \text{buff} \) is NULL.

\( \text{error} \to \text{code} = 115 \)

On entry: \( \sigma \leq 0 \)

7  **Example**

None.
1 Purpose

nagCPURandUniformA generates \( n \) values \( X_i \) from a uniform distribution over the interval \([a, b]\) for specified constants \( a \) and \( b \).

The initialization function nagCPURandInitA must be called prior to the first call to nagCPURandUniformA. Thereafter, this function may be called repeatedly to generate additional sets of random values. Once all desired values have been obtained, the function nagCPURandCleanupA must be called to free allocated system resources.

**Note:** To obtain the same values from nagCPURandUniformA as from the function `nag_CPU_mrg32k3a_uniform(N, P)` in release 0.3 of the library, please see Section 2.1.1.1 in the Random Number Generators Chapter Introduction.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPURandUniformA_sp(int n, float a, float b, float *buff,
                           NagCPURandComm *comm, NagGpuError *error)
extern "C"
void nagCPURandUniformA(int n, double a, double b, double *buff,
                        NagCPURandComm *comm, NagGpuError *error)
```

3 Description

If \( a = 0 \) and \( b = 1 \), this function returns the next \( n \) values \( Y_i \) from a uniform \([0, 1]\) generator. For other values of \( a \) and \( b \), the function applies the transformation

\[
X_i = a + (b - a)Y_i
\]

to produce random numbers from the interval \([a, b]\).

3.1 Synchronization

As this function only uses the CPU there is no requirement for synchronization.

4 References

None.

5 Arguments

1: \( n \) – int

   **Input**

   **On entry:** the number of random values to be generated.

   **Constraint:** \( n \geq 1 \).

2: \( a \) – float

3: \( a \) – double

   **Input**

   This parameter has type float or double depending on whether the single or double precision version of this function is called.
On entry: The lower bound for the uniform random values.

4: \( b \) – float \hspace{2cm} Input
5: \( b \) – double \hspace{2cm} Input

This parameter has type float or double depending on whether the single or double precision version
of this function is called.

On entry: The upper bound for the uniform random values.

Constraint: \( b > a \).

6: \( \text{buff} \) – float * \hspace{2cm} Output
7: \( \text{buff} \) – double * \hspace{2cm} Output

This parameter has type float or double depending on whether the single or double precision version
of this function is called.

On exit: the \( n \) pseudorandom numbers from the specified distribution.

8: \( \text{comm} \) – NagCPURandComm * \hspace{2cm} Communication Data

NagCPURandComm is a structure which holds state and communication information and must not
be modified in any way. Once all required points have been obtained, \( \text{comm} \) must be passed to
nagCPURandCleanupA to free allocated system resources.

9: \( \text{error} \) – NagGpuError * \hspace{2cm} Error Reporting

This parameter contains error information and should not be modified directly. Errors are indicated
through the value of \( \text{error} \rightarrow \text{code} \) which should be inspected after each call to this function. If
\( \text{error} \rightarrow \text{code} = 0 \) then no error occurred. If \( \text{error} \rightarrow \text{code} \neq 0 \) then an error was detected and a
call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error.
Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

\( \text{error} \rightarrow \text{code} = 100 \)

On entry: the value of \( \text{comm} \) is NULL.

\( \text{error} \rightarrow \text{code} = 101 \)

On entry: \( \text{comm} \) has not been initialized, or the internal state of \( \text{comm} \) is corrupted.

\( \text{error} \rightarrow \text{code} = 110 \)

On entry: \( n \leq 0 \).

\( \text{error} \rightarrow \text{code} = 112 \)

On entry: \( \text{buff} \) is NULL.

\( \text{error} \rightarrow \text{code} = 113 \)

On entry: \( b \leq a \)

7 Example

None.
1 Purpose

nagCPURandCleanupA frees system resources that were allocated by a previous call to nagCPURandInitA.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPURandCleanupA(NagCPURandComm *comm, NagGpuError *error)
```

3 Description

3.1 Synchronization

As this function only uses the CPU there is no requirement for synchronization.

4 References

None.

5 Arguments

1: comm – NagCPURandComm *  
   Communication Data  
   On entry: the pointer that was passed to a previous call to nagCPURandInitA.

2: error – NagGpuError *  
   Error Reporting  
   This parameter contains error information and should not be modified directly. Errors are indicated through the value of error → code which should be inspected after each call to this function. If error → code = 0 then no error occurred. If error → code ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

- error → code = 100  
  On entry: the value of comm is NULL.

- error → code = 101  
  On entry: comm has not been initialized, or the internal state of comm is corrupted.

7 Example

None.
1 Purpose

nagCPUQuasiRandInitA initializes a serial, host-only quasi-random number generator. This function must be called before any call to the serial quasi-random generator functions (such as nagCPUQuasiRandUniformA) and must ultimately be followed by a call to the cleanup function nagCPUQuasiRandCleanupA to release system resources.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPUQuasiRandInitA(NagGpuQuasiGen genid, NagGpuScramTypes stype, int dim,
                          int offset, NagCPURandComm *pseudoComm, NagCPUQuasiRandComm *comm,
                          NagGpuError *error)
```

3 Description

Low discrepancy (quasi-random) sequences are used in numerical integration, simulation and optimization. Like pseudorandom numbers they are uniformly distributed, but they are not statistically independent. Quasi-random sequences are designed to give a more even distribution in multidimensional space (uniformity), and are often more efficient than pseudorandom numbers in multidimensional Monte Carlo methods.

Let \( x_1, x_2, \ldots, x_N \) be a sequence of \( d \)-dimensional points in the unit cube \( I^d = [0, 1]^d \). Let \( G \) be a subset of \( I^d \) and define the counting function \( S_N(G) \) as the number of \( d \)-dimensional points \( x \in G \). For each point \( x = (x_1, x_2, \ldots, x_d) \in I^d \), let \( G_x \) be the rectangular \( d \)-dimensional region

\[
G_x = [0, x_1) \times [0, x_2) \times \cdots \times [0, x_d)
\]

with volume \( x_1 \cdot x_2 \cdots x_d = \prod_{i=1}^d x_i \). Then one measure of the uniformity of the points \( x_1, x_2, \ldots, x_N \) is the so-called star discrepancy:

\[
D^*(x_1, x_2, \ldots, x_N) = \sup_{x \in I^d} \left| S_N(G_x) - N \prod_{i=1}^d x_i \right|
\]

which satisfies the inequality

\[
D^*(x_1, x_2, \ldots, x_N) \leq C_d (\log N)^d + O\left((\log N)^{d-1}\right) \quad \text{for all} \quad N \geq 2.
\]

The principal aim in the construction of low-discrepancy sequences is to find sequences of points in \( I^d \) with a bound of this form where the constant \( C_d \) is as small as possible.

The type of low-discrepancy sequence generated by nagCPUQuasiRandInitA depends on the value of genid, and the sequence can optionally be scrambled through the parameter stype. See NagGpuQuasiGen and NagGpuScramTypes respectively for further information.

3.1 Synchronization

As this function only uses the CPU there is no requirement for synchronization.

4 References

None.
5 Arguments

1: \texttt{genid} – NagGpuQuasiGen \hspace{1cm} \textit{Input}

\textit{On entry:} the type of generator to be used:
\begin{verbatim}
genid = NAGGPUQUASIGEN_SOBL\end{verbatim}

\textit{Constraint:} \texttt{genid} = NAGGPUQUASIGEN_SOBL.

2: \texttt{stype} – NagGpuScramTypes \hspace{1cm} \textit{Input}

\textit{On entry:} the type of scrambling to be used:
\begin{verbatim}
stype = NAGGPUSCRAMTYPES_NONE
stype = NAGGPUSCRAMTYPES_OWEN
stype = NAGGPUSCRAMTYPES_FAURE_TEZUKA
stype = NAGGPUSCRAMTYPES_OWEN_FAURE_TEZUKA\end{verbatim}

Please see NagGpuScramTypes for some of the benefits of scrambling and details about each of available scrambling types.

\textit{Constraint:} \texttt{stype} = NAGGPUSCRAMTYPES_NONE or NAGGPUSCRAMTYPES_OWEN or NAGGPUSCRAMTYPES_FAURE_TEZUKA or NAGGPUSCRAMTYPES_OWEN_FAURE_TEZUKA

3: \texttt{dim} – int \hspace{1cm} \textit{Input}

\textit{On entry:} the dimension of the quasi-random sequence.

\textit{Constraint:} \(1 \leq \texttt{dim} \leq 50000.

4: \texttt{offset} – int \hspace{1cm} \textit{Input}

\textit{On entry:} the offset into the sequence at which to start generating.

\textit{Constraint:} \texttt{offset} \geq 0.

5: \texttt{pseudoComm} – NagCPURandComm * \hspace{1cm} \textit{Input}

\textit{On entry:} a pointer to a NagCPURandComm structure which has already been initialized by the function nagCPURandInitA.

\textit{Constraint:} \texttt{pseudoComm} must be initialized before being passed to this function.

6: \texttt{comm} – NagCPUQuasiRandComm * \hspace{1cm} \textit{Communication Data}

\textit{NagCPUQuasiRandComm is a structure which holds state and communication information and must not be modified in any way. The structure will be initialized and must be passed to the generator functions (such as \texttt{nagCPUQuasiRandUniformA}). Once all required points have been obtained, \texttt{comm} must be passed to \texttt{nagCPUQuasiRandCleanupA} to free allocated system resources.}

7: \texttt{error} – NagGpuError * \hspace{1cm} \textit{Error Reporting}

This parameter contains error information and should not be modified directly. Errors are indicated through the value of \texttt{error} \rightarrow \texttt{code} which should be inspected after each call to this function. If \texttt{error} \rightarrow \texttt{code} = 0 then no error occurred. If \texttt{error} \rightarrow \texttt{code} \neq 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.
6 Error Indicators and Warnings

error → code = 100

  On entry: the value of comm is NULL.

error → code = 110

  On entry: genid does not specify a valid quasi-random number generator. See
  NagGpuQuasiGen for permitted values.

error → code = 111

  On entry: stype does not specify a valid scrambling type. See NagGpuScramTypes for
  permitted values.

error → code = 112

  On entry: the value of dim does not satisfy the constraint listed above.

error → code = 113

  On entry: the value of offset is negative.

error → code = 114

  On entry: the value of pseudoComm is NULL.

error → code = 115

  On entry: the pseudorandom generator nagCPRandUniformA returned an error when called
  by this function: pseudoComm is not initialized, or the internal state of pseudoComm is
  corrupted.

7 Example

None.
1 Purpose

nagCPUQuasiRandExpA generates \( n \) points \( x_i \) from a quasi-random exponential distribution with mean \( \lambda \).

The initialization function nagCPUQuasiRandInitA must be called prior to the first call to nagCPUQuasiRandExpA. Thereafter, this function may be called repeatedly to generate additional sets of quasi-random points. Once all desired points have been obtained, the function nagCPUQuasiRandCleanupA must be called to free allocated system resources.

Note: Concerns were raised about the set of Sobol’ direction numbers that were used in release 0.3 of the NAG Numerical Routines for GPUs. These concerns have been addressed by an amended set of direction numbers in Joe and Kuo (2008) which are used in this release. Consequently, the higher dimensions of this Sobol’ generator may not match the higher dimensions of the generator in release 0.3 since the direction numbers are different.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPUQuasiRandExpA_sp(int n, NagGpuQuasiOrient orient, float lambda,
                           float *buff, NagCPUQuasiRandComm *comm, NagGpuError *error)
extern "C"
void nagCPUQuasiRandExpA(int n, NagGpuQuasiOrient orient, double lambda,
                          double *buff, NagCPUQuasiRandComm *comm, NagGpuError *error)
```

3 Description

Quasi-random sequences are made up of one or more multidimensional points, with each point composed of several one dimensional values. The dimensionality of the sequence is specified when calling nagCPUQuasiRandInitA to initialize the generator. Below we will consider a \( d \)-dimensional quasi-random sequence \( x^0, x^1, \ldots \) so that each point \( x^j = (x^j_1, x^j_2, \ldots, x^j_d) \) is composed of \( d \) one dimensional values.

The exponential distribution has probability density function given by

\[
f(x) = \begin{cases} \frac{1}{\lambda} e^{-x/\lambda} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}
\]

where \( \lambda > 0 \). This function returns the next \( n \) points \( x^j = (x^j_1, x^j_2, \ldots, x^j_d) \) for \( j = 1, 2, \ldots, n \) where

\[
x^j_i = -\lambda \ln\left(y^j_i + 2^{-32}\right)
\]

for each \( i = 1, 2, \ldots, d \). Here \( y^j = (y^j_1, y^j_2, \ldots, y^j_d) \in [0, 1]^d \) are the next \( n \) points from the quasi-random generator.

3.1 Synchronization

As this function only uses the CPU there is no requirement for synchronization.
4 References

5 Arguments
1: \( n \) – int \( \quad \) Input
On entry: the number of quasi-random points to be generated.
Constraint: \( n \geq 1 \).

2: \( \text{orient} \) – NagGpuQuasiOrient \( \quad \) Input
On entry: specifies the orientation with which the generator will store the output points. See NagGpuQuasiOrient for further details.

\( \text{orient} = \text{NAGGPUQUASIORIENT\_DIMVALS\_CONSEC} \)
\( \text{orient} = \text{NAGGPUQUASIORIENT\_DIMVALS\_SCATT} \)
Constraint: \( \text{orient} = \text{NAGGPUQUASIORIENT\_DIMVALS\_CONSEC} \) or \( \text{NAGGPUQUASIORIENT\_DIMVALS\_SCATT} \)

3: \( \text{lambda} \) – float \( \quad \) Input
4: \( \text{lambda} \) – double \( \quad \) Input
This parameter has type float or double depending on whether the single or double precision version of this function is called.
On entry: The mean, \( \lambda \), of the exponential distribution.
Constraint: \( \text{lambda} > 0 \).

5: \( \text{buff}[n \times d] \) – float * \( \quad \) Output
6: \( \text{buff}[n \times d] \) – double * \( \quad \) Output
This parameter has type float or double depending on whether the single or double precision version of this function is called.
The value \( d \) is the dimension \( \text{dim} \) of the sequence as specified to the initialization function nagCPUQuasiRandInitA.
On exit: the \( n \) quasi-random points from the specified distribution.

When \( \text{orient} = \text{NAGGPUQUASIORIENT\_DIMVALS\_CONSEC} \), the \( i \)-th dimension \( x_{ji} \) of the \( j \)-th quasi-random point will be stored at location \( \text{buff}[j \times d + (i - 1)] \) for every \( 0 \leq j < n \) and \( 1 \leq i \leq d \).

When \( \text{orient} = \text{NAGGPUQUASIORIENT\_DIMVALS\_SCATT} \), the \( i \)-th dimension \( x_{ji} \) of the \( j \)-th quasi-random point will be stored at location \( \text{buff}[(i - 1) \times n + j] \) for every \( 0 \leq j < n \) and \( 1 \leq i \leq d \).

7: \( \text{comm} \) – NagCPUQuasiRandComm * \( \quad \) Communication Data
NagCPUQuasiRandComm is a structure which holds state and communication information and must not be modified in any way. Once all required points have been obtained, \( \text{comm} \) must be passed to nagCPUQuasiRandCleanupA to free allocated system resources.

8: \( \text{error} \) – NagGpuError * \( \quad \) Error Reporting
This parameter contains error information and should not be modified directly. Errors are indicated through the value of \( \text{error} \rightarrow \text{code} \) which should be inspected after each call to this function. If \( \text{error} \rightarrow \text{code} = 0 \) then no error occurred. If \( \text{error} \rightarrow \text{code} \neq 0 \) then an error was detected and a
call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

- **error → code = 100**
  
  *On entry:* the value of `comm` is NULL.

- **error → code = 101**
  
  *On entry:* `comm` has not been initialized, or the internal state of `comm` is corrupted.

- **error → code = 110**
  
  *On entry:* `n ≤ 0`.

- **error → code = 111**
  
  *On entry:* `orient` does not satisfy the constraint listed above.

- **error → code = 112**
  
  *On entry:* `buff` is NULL.

- **error → code = 114**
  
  *On entry:* `lambda ≤ 0`.

7 Example

None.
NAG Numerical Routines for GPUs Function Document

nagCPUQuasiRandNormalA

1 Purpose

nagCPUQuasiRandNormalA generates \( n \) points \( x_i \) from a quasi-random Normal distribution with mean \( \mu \) and variance \( \sigma^2 \).

The initialization function nagCPUQuasiRandInitA must be called prior to the first call to nagCPUQuasiRandNormalA. Thereafter, this function may be called repeatedly to generate additional sets of quasi-random points. Once all desired points have been obtained, the function nagCPUQuasiRandCleanupA must be called to free allocated system resources.

Note: Concerns were raised about the set of Sobol’ direction numbers that were used in release 0.3 of the NAG Numerical Routines for GPUs. These concerns have been addressed by an amended set of direction numbers in Joe and Kuo (2008) which are used in this release. Consequently, the higher dimensions of this Sobol’ generator may not match the higher dimensions of the generator in release 0.3 since the direction numbers are different.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPUQuasiRandNormalA_sp(int n, NagGpuQuasiOrient orient, float mu,
                                float sigma, float *buff, NagCPUQuasiRandComm *comm, NagGpuError *error)
extern "C"
void nagCPUQuasiRandNormalA(int n, NagGpuQuasiOrient orient, double mu,
                            double sigma, double *buff, NagCPUQuasiRandComm *comm, NagGpuError *error)
```

3 Description

Quasi-random sequences are made up of one or more multidimensional points, with each point composed of several one dimensional values. The dimensionality of the sequence is specified when calling nagCPUQuasiRandInitA to initialize the generator. Below we will consider a \( d \)-dimensional quasi-random sequence \( x^0, x^1, \ldots \) so that each point \( x^j = (x^j_1, x^j_2, \ldots, x^j_d) \) is composed of \( d \) one dimensional values.

The Normal distribution has probability density function given by

\[
f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(x - \mu)^2}{2\sigma^2} \right)
\]

where \( \sigma > 0 \) and \( \mu \in \mathbb{R} \). This function returns the next \( n \) points \( x^j = (x^j_1, x^j_2, \ldots, x^j_d) \) for \( j = 1, 2, \ldots, n \) where

\[
x^j_i = \mu + \sigma \sqrt{2} \text{erfinv} (z_i^j)
\]

for each \( i = 1, 2, \ldots, d \) and \( \text{erfinv} \) is the inverse error function. Here each \( z^j = (z^j_1, z^j_2, \ldots, z^j_d) \) is a low discrepancy point in the interval \((-1, 1)^d\).

3.1 Synchronization

As this function only uses the CPU there is no requirement for synchronization.
4 References

5 Arguments

1: \( n \) – int 
   
   Input
   
   On entry: the number of quasi-random points to be generated.
   
   Constraint: \( n \geq 1 \).

2: \( \text{orient} \) – NagGpuQuasiOrient 
   
   Input
   
   On entry: specifies the orientation with which the generator will store the output points. See NagGpuQuasiOrient for further details.
   
   \( \text{orient} = \text{NAGGPUQUASIORIENT\_DIMVALS\_CONSEC} \)
   
   \( \text{orient} = \text{NAGGPUQUASIORIENT\_DIMVALS\_SCATT} \)
   
   Constraint: \( \text{orient} = \text{NAGGPUQUASIORIENT\_DIMVALS\_CONSEC} \) or \( \text{NAGGPUQUASIORIENT\_DIMVALS\_SCATT} \)

3: \( \mu \) – float 
   
   Input
   
   On entry: the mean, \( \mu \), of the distribution.

4: \( \mu \) – double 
   
   Input
   
   This parameter has type float or double depending on whether the single or double precision version of this function is called.
   
   On entry: the mean, \( \mu \), of the distribution.

5: \( \sigma \) – float 
   
   Input
   
   On entry: the standard deviation, \( \sigma \), of the distribution.
   
   Constraint: \( \sigma > 0 \).

6: \( \sigma \) – double 
   
   Input
   
   This parameter has type float or double depending on whether the single or double precision version of this function is called.
   
   On entry: the standard deviation, \( \sigma \), of the distribution.

7: \( \text{buff}\{n \times d\} \) – float * 
   
   Output
   
   This parameter has type float or double depending on whether the single or double precision version of this function is called.
   
   The value \( d \) is the dimension \( \text{dim} \) of the sequence as specified to the initialization function nagCPUQuasiRandInitA.
   
   On exit: the \( n \) quasi-random points from the specified distribution.

   When \( \text{orient} = \text{NAGGPUQUASIORIENT\_DIMVALS\_CONSEC} \), the \( i \)-th dimension \( x_j^i \) of the \( j \)-th quasi-random point will be stored at location \( \text{buff}[j \times d + (i - 1)] \) for every \( 0 \leq j < n \) and \( 1 \leq i \leq d \).

   When \( \text{orient} = \text{NAGGPUQUASIORIENT\_DIMVALS\_SCATT} \), the \( i \)-th dimension \( x_j^i \) of the \( j \)-th quasi-random point will be stored at location \( \text{buff}[(i - 1) \times n + j] \) for every \( 0 \leq j < n \) and \( 1 \leq i \leq d \).

nagCPUQuasiRandNormalA.2

Mark 0.6
Comm

NagCPUQuasiRandComm is a structure which holds state and communication information and must not be modified in any way. Once all required points have been obtained, comm must be passed to nagCPUQuasiRandCleanupA to free allocated system resources.

10: error – NagGpuError *

Error Reporting

This parameter contains error information and should not be modified directly. Errors are indicated through the value of error -> code which should be inspected after each call to this function. If error -> code = 0 then no error occurred. If error -> code ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

error -> code = 100
On entry: the value of comm is NULL.

error -> code = 101
On entry: comm has not been initialized, or the internal state of comm is corrupted.

error -> code = 110
On entry: n ≤ 0.

error -> code = 111
On entry: orient does not satisfy the constraint listed above.

error -> code = 112
On entry: buff is NULL.

error -> code = 115
On entry: sigma ≤ 0.

7 Example

None.
1 Purpose

nagCPUQuasiRandUniformA generates \( n \) points \( x_i \) from a quasi-random uniform distribution over the interval \( [a, b) \) for specified constants \( a \) and \( b \).

The initialization function nagCPUQuasiRandInitA must be called prior to the first call to nagCPUQuasiRandUniformA. Thereafter, this function may be called repeatedly to generate additional sets of quasi-random points. Once all desired points have been obtained, the function nagCPUQuasiRandCleanupA must be called to free allocated system resources.

**Note:** Concerns were raised about the set of Sobol’ direction numbers that were used in release 0.3 of the NAG Numerical Routines for GPUs. These concerns have been addressed by an amended set of direction numbers in Joe and Kuo (2008) which are used in this release. Consequently, the higher dimensions of this Sobol’ generator may not match the higher dimensions of the generator in release 0.3 since the direction numbers are different.

2 Specification

```
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPUQuasiRandUniformA_sp(int n, NagGpuQuasiOrient orient, float a, float b, float *buff, NagCPUQuasiRandComm *comm, NagGpuError *error)

extern "C"
void nagCPUQuasiRandUniformA(int n, NagGpuQuasiOrient orient, double a, double b, double *buff, NagCPUQuasiRandComm *comm, NagGpuError *error)
```

3 Description

Quasi-random sequences are made up of one or more multidimensional points, with each point composed of several one dimensional values. The dimensionality of the sequence is specified when calling nagCPUQuasiRandInitA to initialize the generator. Below we will consider a \( d \)-dimensional quasi-random sequence \( x^0, x^1, ... \) so that each point \( x^i = (x^i_1, x^i_2, ..., x^i_d) \) is composed of \( d \) one dimensional values.

If \( a = 0 \) and \( b = 1 \), this function returns the next \( n \) points \( y^i \in [0,1)^d \) from the quasi-random generator. For other values of \( a \) and \( b \), the function applies the transformation

\[
x^i_j = a + (b - a)y^i_j
\]

for each \( i = 1, 2, ..., d \) to produce quasi-random points \( x^i \) from the interval \( [a, b)^d \) for each \( j = 1, 2, ..., n \).

3.1 Synchronization

As this function only uses the CPU there is no requirement for synchronization.

4 References

5 Arguments

1: \( n \) – int \hspace{1cm} Input

*On entry:* the number of quasi-random points to be generated.

*Constraint:* \( n \geq 1 \).

2: \( \text{orient} \) – NagGpuQuasiOrient \hspace{1cm} Input

*On entry:* specifies the orientation with which the generator will store the output points. See NagGpuQuasiOrient for further details.

\( \text{orient} = \text{NAGGPUQUASIORIENT\_DIMVALS\_CONSEC} \)

\( \text{orient} = \text{NAGGPUQUASIORIENT\_DIMVALS\_SCATT} \)

*Constraint:* \( \text{orient} = \text{NAGGPUQUASIORIENT\_DIMVALS\_CONSEC} \) or \( \text{NAGGPUQUASIORIENT\_DIMVALS\_SCATT} \)

3: \( a \) – float \hspace{1cm} Input

4: \( a \) – double \hspace{1cm} Input

This parameter has type float or double depending on whether the single or double precision version of this function is called.

*On entry:* The lower bound for the uniform random values.

5: \( b \) – float \hspace{1cm} Input

6: \( b \) – double \hspace{1cm} Input

This parameter has type float or double depending on whether the single or double precision version of this function is called.

*On entry:* The upper bound for the uniform random values.

*Constraint:* \( b > a \).

7: \( \text{buff}[n \times d] \) – float * \hspace{1cm} Output

8: \( \text{buff}[n \times d] \) – double * \hspace{1cm} Output

This parameter has type float or double depending on whether the single or double precision version of this function is called.

The value \( d \) is the dimension \( \text{dim} \) of the sequence as specified to the initialization function nagCPUQuasiRandInitA.

*On exit:* the \( n \) quasi-random points from the specified distribution.

When \( \text{orient} = \text{NAGGPUQUASIORIENT\_DIMVALS\_CONSEC} \), the \( i \)-th dimension \( x^i_j \) of the \( j \)-th quasi-random point will be stored at location \( \text{buff}[j \times d + (i - 1)] \) for every \( 0 \leq j < n \) and \( 1 \leq i \leq d \).

When \( \text{orient} = \text{NAGGPUQUASIORIENT\_DIMVALS\_SCATT} \), the \( i \)-th dimension \( x^i_j \) of the \( j \)-th quasi-random point will be stored at location \( \text{buff}[(i - 1) \times n + j] \) for every \( 0 \leq j < n \) and \( 1 \leq i \leq d \).

9: \( \text{comm} \) – NagCPUQuasiRandComm * \hspace{1cm} Communication Data

NagCPUQuasiRandComm is a structure which holds state and communication information and must not be modified in any way. Once all required points have been obtained, \( \text{comm} \) must be passed to nagCPUQuasiRandCleanupA to free allocated system resources.

10: \( \text{error} \) – NagGpuError * \hspace{1cm} Error Reporting

This parameter contains error information and should not be modified directly. Errors are indicated through the value of \( \text{error} \to \text{code} \) which should be inspected after each call to this function. If
error → code = 0 then no error occurred. If error → code ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

error → code = 100

On entry: the value of comm is NULL.

error → code = 101

On entry: comm has not been initialized, or the internal state of comm is corrupted.

error → code = 110

On entry: n ≤ 0.

error → code = 111

On entry: orient does not satisfy the constraint listed above.

error → code = 112

On entry: buff is NULL.

error → code = 113

On entry: b ≤ a

7 Example

None.
NAG Numerical Routines for GPUs Function Document
nagCPUQuasiRandCleanupA

1 Purpose
nagCPUQuasiRandCleanupA frees system resources that were allocated by a previous call to
nagCPUQuasiRandInitA.

2 Specification
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPUQuasiRandCleanupA(NagCPUQuasiRandComm *comm, NagGpuError *error)

3 Description
3.1 Synchronization
As this function only uses the CPU there is no requirement for synchronization.

4 References
None.

5 Arguments
1: comm – NagCPUQuasiRandComm * Communication Data
   On entry: the pointer that was passed to a previous call to nagCPUQuasiRandInitA.

2: error – NagGpuError * Error Reporting
   This parameter contains error information and should not be modified directly. Errors are indicated
   through the value of error \rightarrow code which should be inspected after each call to this function. If
   error \rightarrow code = 0 then no error occurred. If error \rightarrow code \neq 0 then an error was detected and a
   call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error.
   Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings
   error \rightarrow code = 100
   On entry: the value of comm is NULL.

   error \rightarrow code = 101
   On entry: comm has not been initialized, or the internal state of comm is corrupted.

7 Example
None.
NAG Numerical Routines for GPUs Function Document

nagCPUBBInitA

1 Purpose

nagCPUBBInitA initializes the serial, host-only Brownian bridge generator function nagCPUBBA. It must be called before any calls to nagCPUBBA and must finally be followed by a call to nagCPUBBCleanupA before it can be called a second time.

Note: after the first call to nagCPUBBInitA, all subsequent calls (for example, to change time points) must be preceded by a call to nagCPUBBCleanupA.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>

extern "C"
void nagCPUBBInitA(double tStart, double tEnd, const double *times, int nTimes,
                     NagCPUBBComm *comm, NagGpuError *error)
```

3 Description

3.1 Brownian Bridge Algorithm

Fix two times $t_0 < T$ and let $(t_i)_{1 \leq i \leq N}$ be any set of time points satisfying $t_0 < t_1 < t_2 < \ldots < t_N < T$. A popular method for constructing a $d$ dimensional Brownian sample path $(X_{t_i})_{1 \leq i \leq N}$ at these times is via the Brownian bridge algorithm (see Glasserman (2004)). From any two known points $X_{t_i}$ at time $t_i$ and $X_{t_k}$ at time $t_k$ with $t_i < t_k$, we can interpolate a new point $X_{t_j}$ at any time $t_j \in (t_i, t_k)$ by setting

$$X_{t_j} = \frac{X_{t_i}(t_k - t_j) + X_{t_k}(t_j - t_i)}{t_k - t_i} + CZ \sqrt{\frac{(t_k - t_j)(t_j - t_i)}{(t_k - t_i)}}$$

where $Z$ is a $d$ dimensional standard Normal random variable and $C$ is any $d \times d$ matrix such that $CC^T$ is the desired covariance structure for the Brownian motion $X$. Clearly this algorithm is iterative in nature. All that is needed to complete the specification is to fix the start point $X_{t_0}$ and end point $X_T$, and to specify how successive interpolation times $t_j$ are chosen. For $X$ to behave like a Brownian motion we should set $X_{t_0}$ equal to some value $x \in \mathbb{R}^d$ and then set $X_T = x + C\sqrt{T-t_0}Z$ where $Z$ is any $d$ dimensional standard Normal random variable. However when it comes to deciding how the successive interpolation times $t_j$ should be chosen, there is virtually no restriction. Any method of choosing which $t_j \in (t_i, t_k)$ to interpolate next will yield a statistically correct Brownian motion, provided $t_i$ is the nearest known point to the left of $t_j$ and $t_k$ is the nearest known point to the right of $t_j$. In other words, the interpolation interval $(t_i, t_k)$ must not contain any other known points. If it does, the covariance structure of the process will be incorrect.

The order in which the successive interpolation times $t_j$ are chosen is called the bridge construction order. Since all construction orders will yield a statistically correct Brownian motion, the question arises whether one construction order should be preferred over another. When the $Z$ values are drawn from a pseudorandom generator, the answer is typically no. However the bridge algorithm is frequently used with quasi-random numbers, and in this case the bridge construction order can be important.

3.2 Bridge Construction Order and Quasi-Random Sequences

Consider the one dimensional case so that $d = C = 1$. The Brownian bridge is frequently combined with low-discrepancy (quasi-random) sequences to perform quasi-Monte Carlo integration. Quasi-random points $Z^1, Z^2, Z^3, \ldots$ are generated from the standard Normal distribution, where each quasi-random point

Mark 0.6
\( \mathbf{Z}' = (\mathbf{Z}_1', \mathbf{Z}_2', \ldots, \mathbf{Z}_D') \) consists of \( D \) one dimensional values. The process \( X \) starts at \( X_{t_0} = x \) which is known. There remain \( N + 1 \) time points at which the bridge is to be computed, namely \( (X_i)_{1 \leq i \leq N} \) and \( X_T \). In this case \( D \) is set equal to \( N + 1 \), so that \( N + 1 \) dimensional quasi-random points are generated. A single quasi-random point is used to construct one Brownian sample path.

The question is how to use the dimension values of each \( N + 1 \) dimensional quasi-random point. Often the ‘lower’ dimension values \((\mathbf{Z}_1', \mathbf{Z}_2', \ldots, \mathbf{Z}_D')\) display better uniformity properties than the ‘higher’ dimension values \((\mathbf{Z}_{N+1}', \mathbf{Z}_N', \ldots, \mathbf{Z}_D')\) so that the ‘lower’ dimension values should be used to construct the most important sections of the Brownian sample path. For example, consider a model which is particularly sensitive to the behaviour of the underlying Brownian motion at time 3. When constructing the Brownian sample path, one would therefore ensure that time 3 was one of the interpolation points of the bridge, and that a ‘lower’ dimension value was used in (1) to construct the corresponding bridge point \( X_3 \). Indeed, one would most likely also ensure that time 3 was one of the first bridge points that was constructed: ‘lower’ dimension values would be used to construct both the left and right bridge points used in (1) to interpolate \( X_3 \), so that the distribution of \( X_3 \) benefits as much as possible from the uniformity properties of the quasi-random sequence. For further discussions in this regard we refer to Glasserman (2004).

### 3.3 Implementation

The bridge construction order is given by the array \texttt{times}. Suppose we require \( P \) Brownian sample paths each of dimension \( d \). We therefore have corresponding quasi-random points \( \mathbf{Z}^1, \mathbf{Z}^2, \ldots, \mathbf{Z}^P \) where each point \( \mathbf{Z}^p = (\mathbf{Z}_1^p, \mathbf{Z}_2^p, \ldots, \mathbf{Z}_{d(N+1)}^p) \) has dimension \( d(N+1) \). The starting value \( \mathbf{X}_{t_0} \) of the Brownian motion is assumed to be a known constant.

When \texttt{nagCPUBBBA} is called, the \( p \)-th sample path for \( 1 \leq p \leq P \) is constructed as follows: the end value \( \mathbf{X}_T \) is first constructed as

\[
\mathbf{X}_T = \mathbf{X}_{t_0} + C \sqrt{T - t_0} \begin{bmatrix} \mathbf{Z}_1^p \\ \vdots \\ \mathbf{Z}_d^p \end{bmatrix}
\]

where \( C \) is the matrix from Section 3.1. The array \texttt{times} holds the remaining time points \( t_1, t_2, \ldots, t_N \) in the order in which the bridge is to be constructed. For each \( j = 1, \ldots, N \) set \( \mathbf{t} = \texttt{times}(j-1) \) and construct the point \( \mathbf{X}_r \) as

\[
\mathbf{X}_r = \frac{\mathbf{X}_s(s - r) + \mathbf{X}_q(r - q)}{s - q} + C \sqrt{\frac{(s - r)(r - q)}{(s - q)}} \begin{bmatrix} \mathbf{Z}_{p_d+1}^p \\ \vdots \\ \mathbf{Z}_{p_d+d}^p \end{bmatrix}
\]

where \( q = \max\{t_0, \texttt{times}[i] : 0 \leq i < j - 1, \texttt{times}[i] < r \} \) and \( s = \min\{T, \texttt{times}[i] : 0 \leq i < j - 1, \texttt{times}[i] > r \} \). Note that in our discussion \( j \) is indexed from 1, and so \( X_i \) is interpolated between the nearest (in time) Brownian points which have already been constructed.

The function \texttt{naggpuMakeBridgeOrderA} can be used to initialize the \texttt{times} array for several pre-defined bridge construction orders.

### 3.4 Bridge Construction Order and Working Stack

The efficient implementation of a Brownian bridge algorithm requires the use of a workspace array called the working stack. Since previously computed points will be used to interpolate new points, they should be kept close to the compute units so that the data can be accessed quickly. Ideally the whole stack should be held in hardware cache. Different bridge construction orders may require different amounts of working stack. Indeed, a naive bridge algorithm may require a stack of size \( \frac{N}{4} \) or even \( \frac{N}{2} \), which could be very inefficient when \( N \) is large.

For this reason, \texttt{nagCPUBBInitA} performs a detailed analysis of the bridge construction order specified by \texttt{times}. Heuristics are used to find an execution strategy which requires a small working stack, while still constructing the bridge in the order required. Upon a successful return from \texttt{nagCPUBBInitA}, the amount of working stack per bridge dimension required to construct the bridge can be observed through the \texttt{stackSize} member of the \texttt{NagCPUComm} structure \texttt{comm}. If this number is large, performance might be affected and a different bridge construction order could be considered.
3.5 Synchronization
As this function only uses the CPU there is no requirement for synchronization.

4 References

5 Arguments
1: \( t_{\text{Start}} \) – double
   \( \text{Input} \)
   \( \text{On entry:} \) the starting value \( t_0 \) of the time interval.

2: \( t_{\text{End}} \) – double
   \( \text{Input} \)
   \( \text{On entry:} \) the end value \( T \) of the time interval.
   \( \text{Constraint:} \ t_{\text{End}} > t_{\text{Start}}. \)

3: \( \text{times}[^{\text{nTimes}}] \) – const double *
   \( \text{Input} \)
   \( \text{On entry:} \) the points in the time interval \((t_{\text{Start}}, t_{\text{End}})\) at which the Brownian motion is to be constructed. The order in which points are listed in \text{times} determines the bridge construction order. The function \text{naggpuMakeBridgeOrderA} can be used to create pre-defined bridge construction orders from a set of input times.
   \( \text{Constraints:} \)
   \[ t_{\text{Start}} < \text{times}[i] < t_{\text{End}} \text{ for all } i = 0, 1, \ldots, \text{nTimes} - 1; \]
   \[ \text{times}[i] \neq \text{times}[j] \text{ for all } i, j = 0, 1, \ldots, \text{nTimes} - 1 \text{ and } i \neq j. \]

4: \( \text{nTimes} \) – int
   \( \text{Input} \)
   \( \text{On entry:} \) the number of elements in the array \text{times}, denoted by \( N \) in Section 3.1.
   \( \text{Constraint:} \ 1 \leq \text{nTimes} \leq 65535. \)

5: \text{comm} – NagCPUBBComm *
   \( \text{Communication Data} \)
   NagCPUBBComm is a structure which holds state and communication information and must not be modified in any way. The structure will be initialized and must be passed to the generator function \text{nagCPUBBA}. Once all required sample paths have been obtained, \text{comm} must be passed to \text{nagCPUBBCleanupA} to free allocated system resources.
   Upon successful return from this function, the size of the working stack per bridge dimension required to construct the bridge may be observed through \text{comm}. Please see the NagCPUBBComm documentation for further details. Note that this parameter is no longer observable after calling \text{nagCPUBBCleanupA}.

6: \text{error} – NagGpuError *
   \( \text{Error Reporting} \)
   This parameter contains error information and should not be modified directly. Errors are indicated through the value of \text{error} \rightarrow \text{code} which should be inspected after each call to this function. If \text{error} \rightarrow \text{code} = 0 \text{ then no error occurred. If } \text{error} \rightarrow \text{code} \neq 0 \text{ then an error was detected and a call to \text{naggpuErrorCopyMsg} will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.}
6 Error Indicators and Warnings

error → code = 100

On entry: the value of comm is NULL.

error → code = 110

On entry: the value of times is NULL.

error → code = 111

On entry: the value of nTimes does not satisfy the constraint listed above.

error → code = 112

On entry: the value of tEnd does not satisfy the constraint listed above.

error → code = 113

On entry: the values in the times array do not satisfy the constraints listed above.

7 Example

None.
1 Purpose

nagCPUBBA is a serial, host-only function which uses a Brownian bridge algorithm to construct sample paths for a Brownian motion. It must be preceded by a call to the initialization function nagCPUBBInitA, and must finally be followed by a call to nagCPUBBCleanupA. For further details on the Brownian bridge algorithm and the bridge construction order, please see the documentation for nagCPUBBInitA.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPUBBA_sp(int nPaths, int dim, const float *bgStart, const float *z,
                    const float *cholCov, float *bgVals, NagCPUBBComm *comm,
                    NagGpuError *error)
extern "C"
void nagCPUBBA(int nPaths, int dim, const double *bgStart, const double *z,
                const double *cholCov, double *bgVals, NagCPUBBComm *comm,
                NagGpuError *error)
```

3 Description

3.1 Synchronization

As this function only uses the CPU there is no requirement for synchronization.

4 References

None.

5 Arguments

1: **nPaths** – int  
   
   *Input*
   
   *On entry:* the number of Brownian sample paths to create.
   
   *Constraint:* \( nPaths \geq 1 \).

2: **dim** – int  
   
   *Input*
   
   *On entry:* the dimension of each Brownian sample path.
   
   *Constraint:* \( 1 \leq dim \leq 4 \).

3: **bgStart[dim]** – const float *  
   
   *Input*
   
   This parameter has type float or double depending on whether the single or double precision version of this function is called.
   
   *On entry:* the starting value of the Brownian motion. If \( bgStart \) is set equal to NULL, the Brownian motion will be taken to start at 0.

4: **bgStart[dim]** – const double *  
   
   *Input*
   
   This parameter has type float or double depending on whether the single or double precision version of this function is called.
   
   *On entry:* the starting value of the Brownian motion. If \( bgStart \) is set equal to NULL, the Brownian motion will be taken to start at 0.
This parameter has type float or double depending on whether the single or double precision version of this function is called.

The variable \( N \) denotes the length \( \text{nTimes} \) of the \( \text{times} \) array passed to the initialization function \text{nagCPUBBInitA}.

\textbf{On entry:} the Normal random numbers used to construct the bridge.

\textit{Constraint:} \( z \) must contain \( \text{dim} \times (N + 1) \times \text{nPaths} \) values. The values should be laid out as a matrix with \( \text{nPaths} \) rows and \( \text{dim} \times (N + 1) \) columns. If quasi-random numbers are to be used, successive \( \text{dim} \times (N + 1) \) dimensional points should be stored in successive rows of the matrix, i.e. an ordering corresponding to \text{NAGGPUQUASIORIENT\_DIMVALS\_CONSEC}.

\textbf{On entry:} the lower triangular Cholesky factorisation \( C \) so that \( CC^T \) gives the covariance matrix of the Brownian motion. The elements must be stored in column major order (see the Linear Equations Chapter Introduction for details on matrix storage schemes), so that \( c_{ij} \) for \( i, j = 1, 2, \ldots, \text{dim} \) are stored in \( \text{cholCov}[(j - 1) \times \text{dim} + i - 1] \). Elements of the matrix above the diagonal are not referenced.

\textbf{On exit:} the values of the Brownian bridge. If \( X_{p,i}^d \) denotes the \( d \)-th dimension of the \( i \)-th point of the \( p \)-th sample path where \( 0 \leq d < \text{dim} \), \( 0 \leq i < N + 1 \) and \( 0 \leq p < \text{nPaths} \), then \( X_{p,i}^d \) will be stored at \( \text{bgVals}[d + i \times \text{dim} + p \times \text{dim} \times (N + 1)] \). The starting value \( \text{bgStart} \) is never stored.

\textbf{Communication Data} \text{nagCPUBBComm} is a structure which holds state and communication information and must not be modified in any way. Once all required Brownian sample paths have been obtained, \text{comm} must be passed to \text{nagCPUBBCleanupA} to free allocated system resources.

\textbf{Error Reporting} This parameter contains error information and should not be modified directly. Errors are indicated through the value of \text{error} \rightarrow \text{code} which should be inspected after each call to this function. If \text{error} \rightarrow \text{code} = 0 \) then no error occurred. If \text{error} \rightarrow \text{code} \neq 0 \) then an error was detected and a call to \text{nagGpuErrorCopyMsg} will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

\section{Error Indicators and Warnings}

\textbf{On entry:} the value of \text{comm} is NULL.

\textbf{On entry:} \text{comm} has not been initialized, or the internal state of \text{comm} is corrupted.
error → code = 110

On entry: the value of nPaths does not satisfy the constraint listed above.

error → code = 111

On entry: the value of dim does not satisfy the constraint listed above.

error → code = 112

On entry: the value of z is NULL.

error → code = 113

On entry: the value of cholCov is NULL.

error → code = 115

On entry: the value of bgVals is NULL.

7 Example

None.
1  Purpose

nagCPUBBCleanupA frees system resources which were allocated by a previous call to nagCPUBBInitA.

2  Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPUBBCleanupA(NagCPUBBComm *comm, NagGpuError *error)
```

3  Description

3.1  Synchronization

As this function only uses the CPU there is no requirement for synchronization.

4  References

None.

5  Arguments

1:  `comm` – NagCPUBBComm *

   *Communication Data*

   The structure which was initialized by a previous call to nagCPUBBInitA.

2:  `error` – NagGpuError *

   *Error Reporting*

   This parameter contains error information and should not be modified directly. Errors are indicated through the value of `error → code` which should be inspected after each call to this function. If `error → code = 0` then no error occurred. If `error → code ≠ 0` then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6  Error Indicators and Warnings

   `error → code = 100`

   *On entry:* the value of `comm` is NULL.

   `error → code = 101`

   *On entry:* `comm` has not been initialized, or the internal state of `comm` is corrupted.

7  Example

None.
1 Purpose

nagCPUBBIncInitA initializes the serial, host-only Brownian bridge increments generator function nagCPUBBIncA. It must be called before any calls to nagCPUBBIncA and must finally be followed by a call to nagCPUBBIncCleanupA before it can be called a second time. 

Note: after the first call to nagCPUBBIncInitA, all subsequent calls (for example, to change time points) must be preceded by a call to nagCPUBBIncCleanupA.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>

extern "C"
void nagCPUBBIncInitA(double tStart, double tEnd, const double *times, 
int nTimes, NagCPUBBIncComm *comm, NagGpuError *error)
```

3 Description

3.1 Brownian Bridge Algorithm

Fix two times \( t_0 < T \) and let \((t_i)_{1\leq i \leq N}\) be any set of time points satisfying \( t_0 < t_1 < t_2 < \ldots < t_N < T \). A popular method for constructing a \( d \) dimensional Brownian sample path \((X_i)_{1\leq i \leq N}\) at these times is via the Brownian bridge algorithm (see Glasserman (2004)). From any two known points \( X_i \) at time \( t_i \) and \( X_k \) at time \( t_k \) with \( t_i < t_k \), we can interpolate a new point \( X_j \) at any time \( t_j \in (t_i,t_k) \) by setting

\[
X_j = X_i \frac{(t_k - t_j)}{t_k - t_i} + X_k \frac{(t_j - t_i)}{t_k - t_i} + CZ \sqrt{\frac{(t_k - t_j)(t_j - t_i)}{(t_k - t_i)}}
\]

where \( Z \) is a \( d \) dimensional standard Normal random variable and \( C \) is any \( d \times d \) matrix such that \( CC^T \) is the desired covariance structure for the Brownian motion \( X \). The Brownian bridge increments generator uses the Brownian bridge algorithm to construct the Brownian sample path \( X \), and then uses this to compute the scaled Brownian increments

\[
\frac{X_{i+1} - X_i}{t_{i+1} - t_i}, \frac{X_{i+2} - X_{i+1}}{t_{i+2} - t_{i+1}}, \ldots, \frac{X_{N} - X_{N-1}}{t_N - t_{N-1}}, \frac{X_T - X_N}{T - t_N}
\]

Such increments can be useful in computing numerical solutions to stochastic differential equations driven by Brownian motion.

For further details about the Brownian bridge algorithm, the working stack, bridge construction orders and the connection with quasi-random sequences, please see the nagCPUBBInitA documentation.

3.2 Synchronization

As this function only uses the CPU there is no requirement for synchronization.

4 References

5 Arguments

1: \( t\text{Start} \) – double\(^{\text{ Input}} \)

\( \text{On entry:} \) the starting value \( t_0 \) of the time interval.

2: \( t\text{End} \) – double\(^{\text{ Input}} \)

\( \text{On entry:} \) the end value \( T \) of the time interval.

\text{Constraint:} \ t\text{End} > t\text{Start}.

3: \( \text{times}[\text{nTimes}] \) – const double *\(^{\text{ Input}} \)

\( \text{On entry:} \) the points in the time interval \( (t\text{Start}, t\text{End}) \) at which the Brownian motion is to be constructed. The order in which points are listed in \( \text{times} \) determines the bridge construction order. The function naggpuMakeBridgeOrderA can be used to create pre-defined bridge construction orders from a set of input times.

\text{Constraints:}

\( t\text{Start} < \text{times}[i] < t\text{End} \) for all \( i = 0, 1, \ldots, \text{nTimes} - 1 \);

\( \text{times}[i] \neq \text{times}[j] \) for all \( i, j = 0, 1, \ldots, \text{nTimes} - 1 \) and \( i \neq j \).

4: \( \text{nTimes} \) – int\(^{\text{ Input}} \)

\( \text{On entry:} \) the number of elements in the array \( \text{times} \), denoted by \( N \) in Section 3.1.

\text{Constraint:} \ 1 \leq \text{nTimes} \leq 65535.

5: \( \text{comm} \) – nagCPUBBIncComm *\(^{\text{ Communication Data}} \)

\( \text{NagCPUBBIncComm} \) is a structure which holds state and communication information and must not be modified in any way. The structure will be initialized and must be passed to the generator function nagCPUBBIncA. Once all required sample paths have been obtained, \( \text{comm} \) must be passed to nagCPUBBIncCleanupA to free allocated system resources.

Upon successful return from this function, the size of the working stack per bridge dimension required to construct the bridge may be observed through \( \text{comm} \). Please see the NagCPUBBIncComm documentation for further details. Note that this parameter is no longer observable after calling nagCPUBBIncCleanupA.

6: \( \text{error} \) – NagGpuError *\(^{\text{ Error Reporting}} \)

This parameter contains error information and should not be modified directly. Errors are indicated through the value of \( \text{error} \rightarrow \text{code} \) which should be inspected after each call to this function. If \( \text{error} \rightarrow \text{code} = 0 \) then no error occurred. If \( \text{error} \rightarrow \text{code} \neq 0 \) then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

\( \text{error} \rightarrow \text{code} = 100 \)

\( \text{On entry:} \) the value of \( \text{comm} \) is NULL.

\( \text{error} \rightarrow \text{code} = 110 \)

\( \text{On entry:} \) the value of \( \text{times} \) is NULL.

\( \text{error} \rightarrow \text{code} = 111 \)

\( \text{On entry:} \) the value of \( \text{nTimes} \) does not satisfy the constraint listed above.
error → code = 112
  On entry: the value of tEnd does not satisfy the constraint listed above.

error → code = 113
  On entry: the values in the times array do not satisfy the constraints listed above.

7 Example
None.
NAG Numerical Routines for GPUs Function Document

nagCPUBBIncA

1 Purpose

nagCPUBBIncA is a serial, host-only function which computes scaled increments of Brownian sample paths that are constructed using a Brownian bridge algorithm. It must be preceded by a call to the initialization function nagCPUBBIncInitA, and must finally be followed by a call to nagCPUBBIncCleanupA. For further details on the Brownian bridge algorithm and the bridge construction order, please see the documentation for nagCPUBBInitA and nagCPUBBIncInitA.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPUBBIncA_sp(int nPaths, int dim, const float *z, const float *cholCov,
                      float *bgIncs, NagCPUBBIncComm *comm, NagGpuError *error)
extern "C"
void nagCPUBBIncA(int nPaths, int dim, const double *z, const double *cholCov,
                   double *bgIncs, NagCPUBBIncComm *comm, NagGpuError *error)
```

3 Description

3.1 Synchronization

As this function only uses the CPU there is no requirement for synchronization.

4 References

None.

5 Arguments

1: nPaths – int  
   On entry: the number of Brownian sample paths to create.  
   Constraint: nPaths ≥ 1.

2: dim – int  
   On entry: the dimension of each Brownian sample path.  
   Constraint: 1 ≤ dim ≤ 4.

3: z[dim × (N + 1) × nPaths] – const float *  
   On entry: the Normal random numbers used to construct the bridge.  
   Constraint: z must contain dim × (N + 1) × nPaths values. The values should be laid out as a matrix with nPaths rows and dim × (N + 1) columns. If quasi-random numbers are to be used,
successive \( \text{dim} \times (N + 1) \) dimensional points should be stored in successive rows of the matrix, i.e. an ordering corresponding to \text{NAGGPUQUASIORIENT\_DIMVALS\_CONSEC}.

5: \text{cholCov[\dim \times \dim]} \rightarrow \text{const float} \hspace{1cm} \text{Input}
6: \text{cholCov[\dim \times \dim]} \rightarrow \text{const double} \hspace{1cm} \text{Input}

This parameter has type float or double depending on whether the single or double precision version of this function is called.

\text{On entry:} the lower triangular Cholesky factorisation \( C \) so that \( CC^T \) gives the covariance matrix of the Brownian motion. The elements must be stored in column major order (see the Linear Equations Chapter Introduction for details on matrix storage schemes), so that \( c_{i,j} \) for \( i, j = 1, 2, \ldots, \text{dim} \) are stored in \text{cholCov[(j-1) \times \dim + i - 1]}. Elements of the matrix above the diagonal are not referenced.

7: \text{bgIncs[\dim \times (N + 1) \times \text{nPaths}]} \rightarrow \text{float} \hspace{1cm} \text{Output}
8: \text{bgIncs[\dim \times (N + 1) \times \text{nPaths}]} \rightarrow \text{double} \hspace{1cm} \text{Output}

This parameter has type float or double depending on whether the single or double precision version of this function is called.

The variable \( N \) denotes the length \text{nTimes} of the \text{times} array passed to the initialization function \text{nagCPUBBIncInitA}.

\text{On exit:} the scaled increments of the Brownian motion. If \( X_{p,i}^d \) denotes the \( d \)-th dimension of the \( i \)-th point of the \( p \)-th sample path where \( 0 \leq d < \text{dim}, 0 \leq i < N + 1 \) and \( 0 \leq p < \text{nPaths} \), then the scaled increment \( (X_{p,i+1}^d - X_{p,i}^d)/(t_{i+1} - t_i) \) will be stored at \text{bgIncs[d + i * \text{dim} + p * \text{dim} * (N + 1)]}.

9: \text{comm} \rightarrow \text{NagCPUBBIncComm} \hspace{1cm} \text{Communication Data}

\text{NagCPUBBIncComm} is a structure which holds state and communication information and must not be modified in any way. Once all required Brownian sample paths have been obtained, \text{comm} must be passed to \text{nagCPUBBIncCleanupA} to free allocated system resources.

10: \text{error} \rightarrow \text{NagGpuError} \hspace{1cm} \text{Error Reporting}

This parameter contains error information and should not be modified directly. Errors are indicated through the value of \text{error \rightarrow code} which should be inspected after each call to this function. If \text{error \rightarrow code} = 0 then no error occurred. If \text{error \rightarrow code} \neq 0 then an error was detected and a call to \text{naggpuErrorCopyMsg} will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

\text{error \rightarrow code} = 100

\text{On entry:} the value of \text{comm} is NULL.

\text{error \rightarrow code} = 101

\text{On entry:} \text{comm} has not been initialized, or the internal state of \text{comm} is corrupted.

\text{error \rightarrow code} = 110

\text{On entry:} the value of \text{nPaths} does not satisfy the constraint listed above.

\text{error \rightarrow code} = 111

\text{On entry:} the value of \text{dim} does not satisfy the constraint listed above.

\text{error \rightarrow code} = 112

\text{On entry:} the value of \text{z} is NULL.
The value of **cholCov** is NULL.

error → code = 113

On entry: the value of **cholCov** is NULL.

error → code = 114

On entry: the value of **bgIncs** is NULL.

7 Example

None.
NAG Numerical Routines for GPUs Function Document

nagCPUBBIncCleanupA

1 Purpose

nagCPUBBIncCleanupA frees system resources which were allocated by a previous call to
nagCPUBBIncInitA.

2 Specification

#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPUBBIncCleanupA(NagCPUBBIncComm *comm, NagGpuError *error)

3 Description

3.1 Synchronization

As this function only uses the CPU there is no requirement for synchronization.

4 References

None.

5 Arguments

1: comm – NagCPUBBIncComm *
   Communication Data
   The structure which was initialized by a previous call to nagCPUBBIncInitA.

2: error – NagGpuError *
   Error Reporting
   This parameter contains error information and should not be modified directly. Errors are indicated
   through the value of error → code which should be inspected after each call to this function. If
   error → code = 0 then no error occurred. If error → code ≠ 0 then an error was detected and a
   call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error.
   Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

   error → code = 100
   On entry: the value of comm is NULL.

   error → code = 101
   On entry: comm has not been initialized, or the internal state of comm is corrupted.

7 Example

None.
NAG Numerical Routines for GPUs Function Document

nagCPUDepthBBInitA

1 Purpose
nagCPUDepthBBInitA initializes the serial, host-only depth-order Brownian bridge generator nagCPUDepthBBA. It must be called before any calls to nagCPUDepthBBA and must finally be followed by a call to nagCPUDepthBBCleanupA.

Note: after the first call to nagCPUDepthBBInitA, all subsequent calls (for example, to change the time points) must be preceded by a call to nagCPUDepthBBCleanupA.

2 Specification
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPUDepthBBInitA(float tStart, const float *times, int nTimes,
   bool isBridgeFree, NagCPUDepthBBComm *comm, NagGpuError *error)

3 Description
3.1 Synchronization
As this function only uses the CPU there is no requirement for synchronization.

4 References
None.

5 Arguments
1:  tStart – float
    Input
    On entry: the starting value of the time interval.

2:  times[nTimes] – const float *
    Input
    On entry: the vector of times at which to compute the Brownian bridge.
    Constraint: the values in times must be in increasing order, and each must be greater than tStart.

3:  nTimes – int
    Input
    On entry: the length of the vector times.
    Constraint: 1 ≤ nTimes ≤ 4095.

4:  isBridgeFree – bool
    Input
    On entry: specifies whether a free or 'pinned' Brownian bridge is to be constructed. See nagCPUDepthBBA for more details.
    If isBridgeFree = true, nagCPUDepthBBA will construct a free Brownian motion via a depth-order Brownian bridge algorithm.
    If isBridgeFree = false, nagCPUDepthBBA will construct a non-free or 'pinned' Brownian motion.
NagCPU DepthBBComm is a structure which holds state and communication information and must not be modified in any way. The structure will be initialized and must be passed to the generator function nagCPU DepthBBA. Once all required bridge sample paths have been obtained, comm must be passed to nagCPU DepthBBCleanupA to free allocated system resources.

error – NagGpuError *

This parameter contains error information and should not be modified directly. Errors are indicated through the value of error → code which should be inspected after each call to this function. If error → code = 0 then no error occurred. If error → code ≠ 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

Error Indicators and Warnings

error → code = 100

On entry: the value of comm is NULL.

error → code = 110

On entry: the value of times is NULL.

error → code = 111

On entry: the value of nTimes does not satisfy the constraint listed above.

error → code = 112

On entry: the values in the times array do not satisfy the constraints listed above.

Example

None.
NAG Numerical Routines for GPUs Function Document

nagCPUDepthBBA

1 Purpose

nagCPUDepthBBA is a serial, host-only function which constructs sample paths for a Brownian bridge or for a free Brownian motion using a depth-order bridge interpolation algorithm. It must be preceded by a call to the initialization function nagCPUDepthBBInitA, and must finally be followed by a call to nagCPUDepthBBCleanupA.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPUDepthBBA(int nPaths, int dim, float bgStart, float bgEnd,
    const float *z, const float *cholCov, float *bgVals,
    NagCPUDepthBBComm *comm, NagGpuError *error)
```

3 Description

3.1 Background

Fix $T > 0$ and let $W = (W_t)_{0 \leq t \leq T}$ be a standard $d$-dimensional Wiener process. A standard $d$-dimensional Brownian bridge $B = (B_t)_{0 \leq t \leq T}$ is defined (see Revuz and Yor (1999)) as

$$B_t = W_t - \frac{t}{T}W_T$$

for all $t \in [0, T]$. This process is continuous, starts at zero at time 0 and ends at zero at time $T$. It is Gaussian, has zero mean and has a covariance structure given by

$$\mathbb{E}(B_sB_t') = s\left(1 - \frac{t}{T}\right)I_d = \frac{s(T-t)}{T}I_d$$

for any $s \leq t$ in $[0, T]$ where $I_d$ is the $d$-dimensional identity matrix. The Brownian bridge is often called a non-free or ‘pinned’ Brownian motion, since it is forced to be equal to 0 at time $T$ but is otherwise very similar to a standard Brownian motion.

We can generalize this construction as follows. Fix points $x, w \in \mathbb{R}^d$, let $\Sigma$ be a $d \times d$ covariance matrix and choose any $d \times d$ matrix $C$ such that $CC' = \Sigma$. We will define the generalized $d$-dimensional Brownian bridge $X = (X_t)_{0 \leq t \leq T}$ by setting

$$X_t = \frac{tw + (T-t)x}{T} + CB_t = \frac{tw + (T-t)x}{T} + CW_t - \frac{t}{T}CW_T$$

for all $t \in [0, T]$. The process $X$ is therefore continuous, starts at $x$ at time zero and ends at $w$ at time $T$. It has time-dependent mean $(tw + (T-t)x)/T$ and has the covariance structure

$$\mathbb{E}(X_s - \mathbb{E}X_s)(X_t - \mathbb{E}X_t') = \mathbb{E}(CB_sB_tC') = \frac{s(T-t)}{T}CC' = \frac{s(T-t)}{T}\Sigma$$

for all $s \leq t$ in $[0, T]$. This is a non-free bridge since it is forced to be equal to $w$ at time $T$. However if we set $w = x + CW_T$, then $X$ simplifies to

$$X_t = x + CW_t$$

for all $t \in [0, T]$ which is a free $d$-dimensional Brownian motion with covariance given by $\Sigma$. 
3.2 Implementation

The bridge is generated in a modified depth-first order. Suppose there are \( N \) time points \( t_1, \ldots, t_N \) at which the bridge is to be computed. The algorithm starts by taking the known values \( X_{t_0} = x \) and \( X_{t_N} = w \) and then generating 

\[ X_{t_{(N/2)}}, X_{t_{(N/4)}}, X_{t_{(N/8)}}, \ldots, X_{t_i} \]

according to the standard Brownian bridge interpolation formula (see Glasserman (2004)). Once \( X_{t_i} \) is reached, the algorithm moves upwards from \( t_1 \) searching for an interval \( [t_i, t_k] \) such that both \( X_{t_i} \) and \( X_{t_k} \) are already known, but all \( X_{t_j} \) for \( i < j < k \) are not. This interval is then treated in the same way as the interval \( [t_0, t_N] \), and the process repeats until all points are computed.

The main input to the bridge algorithm is an array of standard Normal random numbers. If these come from a quasi-random generator (e.g., Sobol numbers), then the order in which these numbers are used becomes important. Suppose that the bridge is one-dimensional and that we have an \( N \)-dimensional quasi-random point. Roughly speaking, the algorithm uses the dimensions in this point in breadth-first order: the first dimension is used to compute \( X_{t_{(N/2)}} \), the second dimension is used to compute \( X_{t_{(N/4)}} \), the third to compute \( X_{t_{(N/8)}} \), the fourth to compute \( X_{t_{(N/16)}} \) and so on. For a \( d \)-dimensional bridge, and corresponding \( N \times d \) dimensional quasi-random point, the first \( d \) dimensions are used to compute \( X_{t_{(N/2)}} \), the second \( d \) to compute \( X_{t_{(N/4)}} \), the third \( d \) to compute \( X_{t_{(N/8)}} \), and so on. If the bridge is free, in other words \( X_t = x + CW_t \), then the first \( d \) dimensions are used to compute \( X_{t_N} \), the second \( d \) to compute \( X_{t_{(N/2)}} \), the third \( d \) to compute \( X_{t_{(N/4)}} \), and so on.

The boolean parameter \( \text{isBridgeFree} \) in the initialization function \( \text{nagCPUDepthBBInitA} \) whether a free or non-free Brownian sample path is created. Note that the final value \( w \) of the bridge is always stored, whereas the starting value \( x \) is never stored. The algorithm therefore only produces the values \( X_{t_1}, X_{t_2}, X_{t_3}, \ldots, X_{t_N} \).

3.3 Synchronization

As this function only uses the CPU there is no requirement for synchronization.

4 References


Revuz D and Yor M (1999) *Continuous Martingales and Brownian Motion* Springer

5 Arguments

1: \( \text{nPaths} \) – int

\( \text{Input} \)

\( \text{On entry:} \) the number of Brownian bridge sample paths to create.

\( \text{Constraint:} \ \text{nPaths} \geq 1. \)

2: \( \text{dim} \) – int

\( \text{Input} \)

\( \text{On entry:} \) the dimension of each Brownian bridge sample path.

\( \text{Constraint:} \ 1 \leq \text{dim} \leq 8. \)

3: \( \text{bgStart} \) – float

\( \text{Input} \)

\( \text{On entry:} \) the starting value \( x \) of the bridge.

4: \( \text{bgEnd} \) – float

\( \text{Input} \)

\( \text{On entry:} \) the final value \( w \) of the bridge. If \( \text{nagCPUDepthBBInitA} \) was called with \( \text{isBridgeFree} = \text{true} \), this value is ignored and \( w \) is set equal to \( x + CW_T \).
The variable $N$ denotes the length $nTimes$ of the $times$ array passed to the initialization function nagCPUDepthBBInitA.

On entry: the Normal random numbers used to construct the bridge.

Constraints:
If nagCPUDepthBBInitA was called with isBridgeFree = true, then $z$ must contain $N \times \text{dim} \times \text{nPaths}$ values. The values should be laid out as a matrix with $\text{nPaths}$ rows and $\text{dim} \times N$ columns. If quasi-random numbers are to be used, successive $\text{dim} \times N$-dimensional points should be stored in successive rows of the matrix, i.e. an ordering corresponding to NAGGPUQUASIORIENT_DIMVALS_CONSEC;
If nagCPUDepthBBInitA was called with isBridgeFree = false, then $z$ must contain $(N - 1) \times \text{dim} \times \text{nPaths}$ values. The values should be laid out as a matrix with $\text{nPaths}$ rows and $\text{dim} \times (N - 1)$ columns. If quasi-random numbers are to be used, successive $\text{dim} \times N$-dimensional points should be stored in successive rows of the matrix, i.e. an ordering corresponding to NAGGPUQUASIORIENT_DIMVALS_CONSEC.

On entry: the matrix $C$ which specifies the correlation structure of the Brownian bridge. $C$ should be chosen such that $CC^T = \Sigma$ where $\text{Cov}(X_s, X_t) = s(T - t)/T\Sigma$ for all $s \leq t$ in $[0, T]$.

The variable $N$ denotes the length $nTimes$ of the $times$ array passed to the initialization function nagCPUDepthBBInitA.

On exit: the values of the Brownian bridge. If $x_{p,i}^d$ denotes the $d$-th dimension of the $i$-th point of the $p$-th sample path where $0 \leq d < \text{dim}$, $0 \leq i < N$ and $0 \leq p < \text{nPaths}$, then $x_{p,i}^d$ will be stored at $\text{bgVals}[d + i \times \text{dim} + p \times \text{dim} \times N]$.

Note: output is transposed relative to the equivalent GPU function.

NagCPUDepthBBComm is a structure which holds state and communication information and must not be modified in any way. Once all required bridge sample paths have been obtained, $\text{comm}$ must be passed to nagCPUDepthBBCleanupA to free allocated system resources.

This parameter contains error information and should not be modified directly. Errors are indicated through the value of $\text{error} \rightarrow \text{code}$ which should be inspected after each call to this function. If $\text{error} \rightarrow \text{code} = 0$ then no error occurred. If $\text{error} \rightarrow \text{code} \neq 0$ then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

On entry: the value of $\text{comm}$ is NULL.

On entry: $\text{comm}$ has not been initialized, or the internal state of $\text{comm}$ is corrupted.

On entry: the value of $\text{nPaths}$ does not satisfy the constraint listed above.
error → code = 111
On entry: the value of dim does not satisfy the constraint listed above.

error → code = 112
On entry: the value of z is NULL.

error → code = 113
On entry: the value of cholCov is NULL.

error → code = 115
On entry: the value of bgVals is NULL.

7 Example

None.
NAG Numerical Routines for GPUs Function Document

nagCPU DepthBB IncInitA

1 Purpose

nagCPU DepthBB IncInitA initializes the serial, host-only depth-order Brownian bridge increments generator nagCPU DepthBB IncA. This function must be called before any calls to nagCPU DepthBB IncA and must finally be followed by a call to nagCPU DepthBB CleanupA.

Note: after the first call to nagCPU DepthBB IncInitA, all subsequent calls (for example, to change the time points) must be preceded by a call to nagCPU DepthBB CleanupA.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>
extern "C"
void nagCPU DepthBB IncInitA(float tStart, const float *times, int nTimes,
                             bool isBridgeFree, NagCPU DepthBB Comm *comm, NagGpu Error *error)
```

3 Description

3.1 Synchronization

As this function only uses the CPU there is no requirement for synchronization.

4 References

None.

5 Arguments

1: `tStart` – float

   *Input*

   *On entry*: the starting value of the time interval.

2: `times[nTimes]` – const float *

   *Input*

   *On entry*: the vector of times at which to compute the Brownian bridge.

   *Constraint*: the values in `times` must be in increasing order, and each must be greater than `tStart`.

3: `nTimes` – int

   *Input*

   *On entry*: the length of the vector `times`.

   *Constraint*: 1 ≤ `nTimes` ≤ 4095.

4: `isBridgeFree` – bool

   *Input*

   *On entry*: specifies whether scaled increments for a free or ‘pinned’ Brownian bridge is to be constructed. See nagCPU DepthBB IncA for more details.

   If `isBridgeFree = true`, nagCPU DepthBB IncA will construct scaled increments of a free Brownian motion via a depth-order Brownian bridge algorithm.

   If `isBridgeFree = false`, nagCPU DepthBB IncA will construct scaled increments of a non-free or ‘pinned’ Brownian motion.
5: \textbf{comm} – NagCPUDepthBBComm *

\textit{Communication Data}

NagCPUDepthBBComm is a structure which holds state and communication information and must not be modified in any way. The structure will be initialized and must be passed to the generator function nagCPUDepthBBIncA. Once all required bridge increments have been obtained, \textbf{comm} must be passed to nagCPUDepthBBCleanupA to free allocated system resources.

6: \textbf{error} – NagGpuError *

\textit{Error Reporting}

This parameter contains error information and should not be modified directly. Errors are indicated through the value of \textbf{error} \rightarrow \textbf{code} which should be inspected after each call to this function. If \textbf{error} \rightarrow \textbf{code} = 0 then no error occurred. If \textbf{error} \rightarrow \textbf{code} \neq 0 then an error was detected and a call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 \textbf{Error Indicators and Warnings}

\textbf{error} \rightarrow \textbf{code} = 100

\textit{On entry:} the value of \textbf{comm} is NULL.

\textbf{error} \rightarrow \textbf{code} = 110

\textit{On entry:} the value of \textbf{times} is NULL.

\textbf{error} \rightarrow \textbf{code} = 111

\textit{On entry:} the value of \textbf{nTimes} does not satisfy the constraint listed above.

\textbf{error} \rightarrow \textbf{code} = 112

\textit{On entry:} the values in the \textbf{times} array do not satisfy the constraints listed above.

7 \textbf{Example}

None.
NAG Numerical Routines for GPUs Function Document

nagCPUDepthBBIncA

1 Purpose

nagCPUDepthBBIncA is a serial, host-only function which computes scaled increments of a depth-order Brownian bridge or free Brownian motion. It must be preceded by a call to the initialization function nagCPUDepthBBIncInitA, and must finally be followed by a call to nagCPUDepthBBCleanupA.

2 Specification

```c
#include <nag_gpu.h>
#include <nag_gpu_serial.h>

extern "C"
void nagCPUDepthBBIncA(int nPaths, int dim, float startEndDiff, const float *z,
                        const float *cholCov, float *bgIncs, NagCPUDepthBBComm *comm,
                        NagGpuError *error)
```

3 Description

Fix $T > 0$ and suppose that $0 = t_0 < t_1 < \cdots < t_N = T$. Conceptually, this algorithm first constructs a depth-order Brownian bridge $X = (X_t)_{0 \leq t \leq T}$ in the same way as nagCPUDepthBBA and then computes

$$\frac{X_{t_1} - X_{t_0}}{t_1 - t_0}, \frac{X_{t_2} - X_{t_1}}{t_2 - t_1}, \ldots, \frac{X_{t_N} - X_{t_{N-1}}}{t_N - t_{N-1}}.$$ 

Such increments can be useful when computing numerical solutions to stochastic differential equations driven by either a Brownian bridge or a free Brownian motion. For more details on the Brownian bridge and how it is constructed, see the discussion in nagCPUDepthBBA.

We briefly recall some notation: for further details see nagCPUDepthBBA. We let $W = (W_t)_{0 \leq t \leq T}$ be a standard $d$-dimensional Wiener process, we let $\Sigma$ be a $d \times d$ covariance matrix, we choose $C$ to be a $d \times d$ matrix such that $CC' = \Sigma$, and we fix two points $x$ and $w$ in $\mathbb{R}^d$. The generalized Brownian bridge $X = (X_t)_{0 \leq t \leq T}$ is defined as

$$X_t = tw + (T - t)x + CW_t - \frac{t}{T}CW_T$$

for all $t \in [0, T]$ so that $X_0 = x$, $X_T = w$ and $\text{Cov}(X_s, X_t) = s(T - t)/T \Sigma$ for all $s \leq t$ in $[0, T]$. This process is a non-free or ‘pinned’ Brownian motion since $X_T = w$. However if we set $w = x + CW_T$ then $X_t = x + CW_t$ becomes a standard, correlated $d$-dimensional Brownian motion. The boolean parameter `isBridgeFree` in the initialization routine nagCPUDepthBBIncInitA controls whether a free or non-free Brownian sample path is created.

3.1 Synchronization

As this function only uses the CPU there is no requirement for synchronization.

4 References

None.
5 Arguments

1: \( nPaths \) – int  
   \( \text{Input} \)
   \( \text{On entry:} \) the number of Brownian bridge sample paths that are created.
   \( \text{Constraint:} \ nPaths \geq 1. \)

2: \( \text{dim} \) – int  
   \( \text{Input} \)
   \( \text{On entry:} \) the dimension of each Brownian bridge sample path.
   \( \text{Constraint:} \ 1 \leq \text{dim} \leq 8. \)

3: \( \text{startEndDiff} \) – float  
   \( \text{Input} \)
   \( \text{On entry:} \) the difference between \( X_{t_N} \) and \( X_{t_0} \). If \( \text{nagCPUDepthBBIncInitA} \) was called with \( \text{isBridgeFree} = \text{true} \), this value is ignored and \( X_{t_N} \) is set equal to \( x + CW_T \).

4: \( z[[\text{dim} \times N \times nPaths]] \) – const float *  
   \( \text{Input} \)
   \( \text{The variable} \ N \text{denotes the length} nTimes \text{of the} \text{times} \text{array passed to the initialization function} \text{nagCPUDepthBBIncInitA}. \)
   \( \text{On entry:} \) the Normal random numbers used to construct the bridge.
   \( \text{Constraints:} \)
   
   If \( \text{nagCPUDepthBBIncInitA} \) was called with \( \text{isBridgeFree} = \text{true} \), then \( z \) must contain \( N \times \text{dim} \times nPaths \) values. The values should be laid out as a matrix with \( nPaths \) rows and \( \text{dim} \times N \) columns. If quasi-random numbers are to be used, successive \( \text{dim} \times N \)-dimensional points should be stored in successive rows of the matrix, i.e. an ordering corresponding to \text{NAGGPUQUASIORIENT_DIMVALS_CONSEC};

   If \( \text{nagCPUDepthBBIncInitA} \) was called with \( \text{isBridgeFree} = \text{false} \), then \( z \) must contain \( (N-1) \times \text{dim} \times nPaths \) values. The values should be laid out as a matrix with \( nPaths \) rows and \( \text{dim} \times (N-1) \) columns. If quasi-random numbers are to be used, successive \( \text{dim} \times N \)-dimensional points should be stored in successive rows of the matrix, i.e. an ordering corresponding to \text{NAGGPUQUASIORIENT_DIMVALS_CONSEC}.

5: \( \text{cholCov[\text{dim} \times \text{dim}]} \) – const float *  
   \( \text{Input} \)
   \( \text{On entry:} \) the matrix \( C \) which specifies the correlation structure of the Brownian bridge. \( C \) should be chosen such that \( CC^T = \Sigma \) where \( \text{Cov}(X_s, X_t) = s(T-t)/T\Sigma \) for all \( s \leq t \) in \([0,T]\).

6: \( \text{bgIncs[\text{dim} \times N \times nPaths]} \) – float *  
   \( \text{Output} \)
   \( \text{The variable} \ N \text{denotes the length} nTimes \text{of the} \text{times} \text{array passed to the initialization function} \text{nagCPUDepthBBIncInitA}. \)
   \( \text{On exit:} \) the scaled increments of the Brownian bridge. If \( x_{p,i}^d \) denotes the \( d \)-th dimension of the \( i \)-th point of the \( p \)-th sample path where \( 0 \leq d < \text{dim} \), \( 0 \leq i < N \) and \( 0 \leq p < nPaths \), then the scaled increment \( (x_{p,i+1}^d - x_{p,i}^d)/(t_{i+1} - t_i) \) will be stored at \( \text{bgIncs}[d + i \times \text{dim} + p \times \text{dim} \times N] \).
   \( \text{Note:} \) output is transposed relative to the equivalent GPU function.

7: \( \text{comm} \) – NagCPUDepthBBComm *  
   \( \text{Communication Data} \)
   \( \text{NagCPUDepthBBComm} \) is a structure which holds state and communication information and must not be modified in any way. Once all required bridge sample paths have been obtained, \( \text{comm} \) must be passed to \( \text{nagCPUDepthBBCleanupA} \) to free allocated system resources.

8: \( \text{error} \) – NagGpuError *  
   \( \text{Error Reporting} \)
   This parameter contains error information and should not be modified directly. Errors are indicated through the value of \( \text{error} \rightarrow \text{code} \) which should be inspected after each call to this function. If \( \text{error} \rightarrow \text{code} = 0 \) then no error occurred. If \( \text{error} \rightarrow \text{code} \neq 0 \) then an error was detected and a
call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error. Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings

error → code = 100
On entry: the value of comm is NULL.

error → code = 101
On entry: comm has not been initialized, or the internal state of comm is corrupted.

error → code = 110
On entry: the value of nPaths does not satisfy the constraint listed above.

error → code = 111
On entry: the value of dim does not satisfy the constraint listed above.

error → code = 112
On entry: the value of z is NULL.

error → code = 113
On entry: the value of cholCov is NULL.

error → code = 114
On entry: the value of bgIncs is NULL.

7 Example

None.
NAG Numerical Routines for GPUs Function Document  
nagCPUDepthBBCleanupA

1 Purpose  
nagCPUDepthBBCleanupA frees system resources which were allocated by a previous call to  
nagCPUDepthBBInitA or nagCPUDepthBBIncInitA.

2 Specification  
#include <nag_gpu.h>  
#include <nag_gpu_serial.h>  
extern "C"  
void nagCPUDepthBBCleanupA(NagCPUDepthBBComm *comm, NagGpuError *error)

3 Description  
3.1 Synchronization  
As this function only uses the CPU there is no requirement for synchronization.

4 References  
None.

5 Arguments  
1: comm – NagCPUDepthBBComm *  
   Communication Data  
The structure which was initialized by a previous call to nagCPUDepthBBInitA or  
nagCPUDepthBBIncInitA.
2: error – NagGpuError *  
   Error Reporting  
   This parameter contains error information and should not be modified directly. Errors are indicated  
   through the value of error \rightarrow code which should be inspected after each call to this function. If  
   error \rightarrow code = 0 then no error occurred. If error \rightarrow code \neq 0 then an error was detected and a  
call to naggpuErrorCopyMsg will retrieve a null terminated ANSI C string describing the error.  
   Please see the Error Handling Chapter Introduction for further details on error handling.

6 Error Indicators and Warnings  
error \rightarrow code = 100  
\textit{On entry:} the value of \texttt{comm} is NULL.
error \rightarrow code = 101  
\textit{On entry:} \texttt{comm} has not been initialized, or the internal state of \texttt{comm} is corrupted.

7 Example  
None.
NAG Numerical Routines for GPUs Data Type Document

NagGpuBBComm

1 Purpose
NagGpuBBComm is used by the library for communication between the GPU Brownian bridge generator functions naggpuBBInitA, naggpuBBA and naggpuBBCleanupA. It is for internal library use and should not be modified by the user in any way.

2 Specification
#include <nag_gpu.h>

struct NagGpuBBComm {
        NagGpuTuneOrigin tuneOrigin;
        NagGpuBBTune *tuneParamsUsed;
        int stackSize;
        int param3;
        void *param4;
    }

3 Description
3.1 Monitoring Launch Parameters
The first two members of NagGpuBBComm may be used to monitor the launch configuration which was used for a particular GPU kernel. This will typically only be of interest to users wanting to fine tune the performance of the library’s GPU functions.

Immediately following a successful call to naggpuBBA, the tuneParamsUsed member of the NagGpuBBComm structure will contain the parameters which were used to launch the kernel. See NagGpuBBTune for further details on tuning parameters and their meanings.

Note that the tuneParamsUsed pointer is no longer valid after calling naggpuBBCleanupA. Parameters must be observed before calling the cleanup function.

4 References
None.

5 Members
The full structure definition is provided so that the library can be called from languages other than C/C++. The members of this structure not documented below are private to the library and must not be modified in any way.

1: tuneOrigin – NagGpuTuneOrigin
   Indicates where the tuning data in tuneParamsUsed originated: was it supplied by the user (by passing a NagGpuBBTune structure to naggpuBBA), or was it a default value used by the library. This value must not be modified in any way.

2: tuneParamsUsed – NagGpuBBTune *
   Contains the members used to launch the GPU kernel. See NagGpuBBTune for further details. This value, and the values inside the NagGpuBBTune structure, must not be modified in any way.

   Note: tuneParamsUsed is no longer valid after calling naggpuBBCleanupA.
3:  **stackSize** – int

After a successful return from naggpuBBInitA, this contains the size of the working stack per bridge dimension which is needed to construct the Brownian bridge. Please see naggpuBBInitA and NagGpuBBTune for further details on the working stack. This value must not be modified in any way.

**Note**: **stackSize** is no longer valid after calling naggpuBBCleanupA.
NAG Numerical Routines for GPUs Data Type Document

NagGpuBBIncComm

1 Purpose

NagGpuBBIncComm is used by the library for communication between the GPU Brownian bridge generator functions naggpuBBIncInitA, naggpuBBIncA and naggpuBBIncCleanupA. It is for internal library use and should not be modified by the user in any way.

2 Specification

```c
#include <nag_gpu.h>

struct NagGpuBBIncComm {
    NagGpuTuneOrigin tuneOrigin;
    NagGpuBBTune *tuneParamsUsed;
    int stackSize;
    int param3;
    void *param4;
};
```

3 Description

3.1 Monitoring Launch Parameters

The first two members of NagGpuBBIncComm may be used to monitor the launch configuration which was used for a particular GPU kernel. This will typically only be of interest to users wanting to fine tune the performance of the library's GPU functions.

Immediately following a successful call to naggpuBBIncA, the `tuneParamsUsed` member of the NagGpuBBIncComm structure will contain the parameters which were used to launch the kernel. See NagGpuBBTune for further details on tuning parameters and their meanings.

Note that the `tuneParamsUsed` pointer is no longer valid after calling naggpuBBIncCleanupA. Parameters must be observed before calling the cleanup function.

4 References

None.

5 Members

The full structure definition is provided so that the library can be called from languages other than C/C++. The members of this structure not documented below are private to the library and must not be modified in any way.

1: `tuneOrigin` – NagGpuTuneOrigin

Indicates where the tuning data in `tuneParamsUsed` originated: was it supplied by the user (by passing a NagGpuBBTune structure to naggpuBBIncA), or was it a default value used by the library. This value must not be modified in any way.

2: `tuneParamsUsed` – NagGpuBBTune *

Contains the members used to launch the GPU kernel. See NagGpuBBTune for further details. This value, and the values inside the NagGpuBBTune structure, must not be modified in any way.

Note: `tuneParamsUsed` is no longer valid after calling naggpuBBIncCleanupA.
3: \(\text{stackSize} \rightarrow \text{int}\)

After a successful return from \text{naggpuBBIncInitA}, this contains the size of the working stack per bridge dimension which is needed to construct the Brownian bridge. Please see \text{naggpuBBInitA} and \text{NagGpuBBTune} for further details on the working stack. This value must not be modified in any way.

\textbf{Note:} \text{stackSize} is no longer valid after calling \text{naggpuBBIncCleanupA}.
NAG Numerical Routines for GPUs Data Type Document

NagGpuBBTune

1 Purpose
NagGpuBBTune provides parameters to tune the performance of the GPU Brownian bridge generators naggpuBBA and naggpuBBIncA.

The generators are based on parameterized kernels, and these parameters are given in this structure. This structure represents advanced features of the library which are optional: users do not need to provide launch parameters in order to use the GPU functions, since default values will be used.

2 Specification
#include <nag_gpu.h>

struct NagGpuBBTune {
    int bbAThdsPerBlk;
    int bbANumBlks;
    int bbAShmStackSz;
}

3 Description
The members of this structure control the launch parameters for the GPU Brownian bridge generator kernels. These parameters are passed to the kernels through the generator functions naggpuBBA and naggpuBBIncA. The parameters used in the subsequent kernel launch may be queried through the tuneParamsUsed member of the NagGpuBBComm or NagGpuBBIncComm communication structures respectively. If the call to the generator function succeeded, they will be the same as the parameters passed in by the user (if the user gave a negative value for bbAShmStackSz, the actual value used by the library will be reported). If an error occurred, the values in tuneParamsUsed are unset and should be ignored.

As a starting point in tuning a function, it may be useful to launch a kernel with a given problem size and specify a NULL NagGpuBBTune pointer. In this case the library will use default launch parameters, which can be queried through the tuneParamsUsed member of the NagGpuBBComm or NagGpuBBIncComm communication structures. This could give some indication of where a search could begin.

3.1 Background on Performance Tuning
CUDA compute kernels partition the computational load into independent blocks, each block being made up of a number of threads. Blocks cannot communicate, and all blocks have the same number of threads. The blocks can optionally be arranged in a 2D grid, although this is only to aid the programmer and does not impact performance. To launch a CUDA kernel on a graphics device, one specifies to the CUDA runtime system which kernel to launch, how many blocks to launch it with, and how many threads each block will have. The graphics device is (loosely speaking) made up of a number of processors, where each processor can run one or more blocks. Two or more processors cannot combine efforts to run a single block. The kernel launch can fail for a number of reasons, the most common being:

1. A badly written kernel – buffer overruns, underruns or segmentation faults
2. Requesting too many blocks. Graphics devices have a limited number of blocks that can be launched, which varies from card to card.
3. Requesting too many threads per block. Graphics devices have a limited number of threads each block can run, which varies from card to card.
4. Requesting too many resources. Each processor on the graphics device has a limited number of registers and shared memory. A CUDA kernel will use a given number of registers per thread (which depends on the kernel). If too many threads are launched, the block will need more registers than a processor can provide, and the launch will fail.

By specifying the number of blocks and the number of threads, the user is determining how the computational load is spread across the device and this will determine the runtime. For example, if only one block is launched on a device with several processors, the runtime performance will be poor since much of the device is idle. If too many blocks are launched, performance will also be poor due to the overhead of block scheduling and setup, and the fact that each block does very little work before being replaced by another.

It should be evident that tuning a CUDA kernel for performance is not always simple. In general the optimal launch parameters depend on

1. The particular CUDA kernel being considered, including the number of registers and shared memory it uses.
2. The graphics device the kernel is run on.
3. The computational load of the kernel.

The last point above means that optimal launch parameters can be dependent on problem size, e.g. launching a kernel with a smaller computational load may have different optimal launch parameters to the same kernel for a larger computational load.

3.2 Tuning and CUDA Errors

Note that tuning a kernel can produce CUDA errors: while some NAG functions will endeavour to indicate when a launch would have required too much shared memory, no attempt is made to determine whether sufficient registers are available for a launch to succeed. Passing a high number of threads to a register-heavy kernel (typically the double precision kernels) may result in a launch failure and a CUDA runtime error such as cudaErrorLaunchOutOfResources. When tuning a function, users should check the error codes to see whether a resource-related error occurred (such as a launch requiring too much shared memory or too many registers). If such an error occurred, the launch configuration should be discarded and cudaGetLastError() should be called to clear the CUDA error status. A new launch configuration can then be tried.

4 References

None.

5 Members

1: bbAThdsPerBlk – int

The number of threads per block.

Constraint: $1 \leq bbAThdsPerBlk \leq T$ where $T$ is the maximum number of threads per block as reported by the CUDA runtime system (see the CUDA runtime function cudaGetDeviceProperties()).

2: bbANumBlks – int

The number of thread blocks to be launched.

Constraint: $1 \leq bbANumBlks \leq B_{max}$, where $B_{max}$ is the maximum number of thread blocks in the $x$ dimension of the grid, as reported by the CUDA runtime system (see the CUDA runtime function cudaGetDeviceProperties()).

3: bbAShmStackSz – int

The amount of working stack per bridge dimension which is to be held in shared memory.
Let $S$ denote the size of the working stack per bridge dimension which is required in order to construct the bridge. The value of $S$ can be observed through NagGpuBBComm.stackSize after a successful call to naggpuBBInitA, or through NagGpuBBIncComm.stackSize after a successful call to naggpuBBIncInitA.

Suppose a bridge of dimension $d$ is required. Then the total amount of working stack needed is $dSF$ where $F = 4$ in single precision and $F = 8$ in double precision. The Brownian bridge kernels allow for the working stack to be split between shared memory and local memory. Each streaming multiprocessor (SM) has a limited amount of shared memory available. If a kernel requests a lot of shared memory, the number of active thread blocks per SM will be reduced, which could impact performance. For this reason the amount of shared memory devoted to working cache is configurable by the user. The remaining working cache will be held in local memory. Since the kernels are compiled with L1 caching disabled, some of the local memory will be held in L1 cache and should not be affected by global memory traffic.

Each thread block will use a certain amount of shared memory regardless of the bridge dimension or the working stack size. The size of this fixed overhead is implementation dependent, but is on the order of 3KB. If $dSF$ is small, then `bbAShmStackSz` can be set equal to $S$ so that the entire stack is held in shared memory. If $dSF$ is large, it may be better to reduce `bbAShmStackSz` so that more than one block can run on each SM. What is ‘small’ and what is ‘large’ will depend on the GPU on which the kernel is launched.

If a negative value is assigned to `bbAShmStackSz`, the library will calculate a default value to use, and it will launch the kernel with `bbAThdsPerBlk` threads and `bbANumBlks` blocks.

*Constraint:* $bbAShmStackSz \leq S$ where $S$ is defined above.
NAG Numerical Routines for GPUs Data Type Document

NagGpuDepthBBComm

1  Purpose

NagGpuDepthBBComm is used by the library for communication between the GPU Brownian bridge generator functions. It is for internal library use and should not be modified by the user in any way.

2  Specification

#include <nag_gpu.h>

struct NagGpuDepthBBComm {
    int param1;
    void * param2;
    int param3;
    int *param4;
    int *param5;
    float *param6;
    int param7;
    bool param8;
};

3  Description

3.1  Monitoring Launch Parameters

It is currently not possible to change or observe the launch parameters of the GPU Brownian bridge kernels. This will be added in a future release.

4  References

None.

5  Members

The structure definition is provided so that the library can be called from languages other than C/C++. All the members of this structure are private to the library and must not be modified in any way.
1 Purpose
NagGpuMrg32k3aDeviceComm is used by the library for communication between the host and the GPU MRG32k3a pseudorandom number generator functions such as naggpuDevMrg32k3aUniformA. It is for internal library use and should not be modified in any way.

2 Specification
#include <nag_gpu.h>
struct NagGpuMrg32k3aDeviceComm {
    int param1;
    int *param2;
    unsigned int *param3;
    unsigned int *param4;
    unsigned int *param5;
}

3 Description
To use the MRG32k3a device function generators, the memory address of a NagGpuMrg32k3aDeviceComm structure must be obtained from the device generator initialization function naggpuMrg32k3aDeviceInitA. This structure will reside in the GPU memory space and will contain communication data for use by the device function generator. This GPU memory address must then be passed to the MRG32k3a device generator initialization function naggpuDevMrg32k3aInitA. Once all values have been obtained from the device function generators, the same NagGpuMrg32k3aDeviceComm structure memory address must be passed to naggpuMrg32k3aDeviceCleanupA to free allocated system resources.

4 References
None.

5 Members
The structure definition is provided so that the library can be called from languages other than C/C++. All the members of this structure are private to the library and must not be modified in any way.
NAG Numerical Routines for GPUs Data Type Document

NagGpuQuasiRandComm

1 Purpose
NagGpuQuasiRandComm is used by the library for communication between the GPU quasi-random number generator functions. It is for internal library use and should not be modified by the user in any way.

2 Specification
#include <nag_gpu.h>

struct NagGpuQuasiRandComm {
    NagGpuTuneOrigin tuneOrigin;
    NagGpuQuasiRandTune *tuneParamsUsed;
    int param1;
    int param2;
    int param3;
    unsigned int param4;
    unsigned int *param5;
    unsigned int *param6;
    NagGpuSobolDeviceComm *param7;
}

3 Description
3.1 Monitoring Launch Parameters
The first two members of NagGpuQuasiRandComm may be used to monitor the launch configuration which was used for a particular GPU kernel. This will typically only be of interest to users wanting to fine tune the performance of the library’s GPU functions.

Immediately following a successful call to one of the GPU quasi-random generator functions (such as naggpuQuasiRandUniformA), the tuneParamsUsed member of the NagGpuQuasiRandComm structure will contain the parameters which were used to launch the kernel. Only the members of tuneParamsUsed relevant to the kernel that was launched should be inspected. See NagGpuQuasiRandTune for further details on these members and their meanings.

Note that the tuneParamsUsed pointer is no longer valid after calling naggpuQuasiRandCleanupA. Parameters must be observed before calling the cleanup function.

4 References
None.

5 Members
The full structure definition is provided so that the library can be called from languages other than C/C++. The members of this structure not documented below are private to the library and must not be modified in any way.

1: tuneOrigin – NagGpuTuneOrigin
   Indicates where the tuning data in tuneParamsUsed originated: was it supplied by the user (by passing a NagGpuQuasiRandTune to the generator routine), or was it a default value used by the library. This value must not be modified in any way.
2:  **tuneParamsUsed** – NagGpuQuasiRandTune *

Contains the members used to launch the GPU kernel. See NagGpuQuasiRandTune for further details. This value, and the values inside the NagGpuQuasiRandTune structure, must not be modified in any way.

**Note:** `tuneParamsUsed` is no longer valid after calling `naggpuQuasiRandCleanupA`. 
1 Purpose

NagGpuQuasiRandTune provides parameters to tune the performance of the GPU quasi-random generators (such as naggpuQuasiRandUniformA).

The generators are based on parameterized kernels, and these parameters are given in this structure. This structure represents advanced features of the library which are optional: users do not need to provide launch parameters in order to use the GPU functions, since default values will be used.

2 Specification

```c
#include <nag_gpu.h>

struct NagGpuQuasiRandTune {
  int sblAThdsPerBlk;
  int sblABlksPerDim;
};
```

3 Description

The members of this structure control the launch parameters for the GPU quasi-random generator kernels. These parameters are passed to the kernels through the generator functions such as naggpuQuasiRandUniformA. The parameters used in the subsequent kernel launch may be queried through the `tuneParamsUsed` member of the NagGpuQuasiRandComm communication structure. If the call to the generator function succeeded, they will be the same as the parameters passed in by the user. If an error occurred, the values in `tuneParamsUsed` are unset and should be ignored.

As a starting point in tuning a function, it may be useful to launch a kernel with a given problem size and specify a NULL NagGpuQuasiRandTune pointer. In this case the library will use default launch parameters, which can be queried through the `tuneParamsUsed` member of the NagGpuQuasiRandComm communication structure. This could give some indication of where a search could begin.

3.1 Background on Performance Tuning

CUDA compute kernels partition the computational load into independent blocks, each block being made up of a number of threads. Blocks cannot communicate, and all blocks have the same number of threads. The blocks can optionally be arranged in a 2D grid, although this is only to aid the programmer and does not impact performance. To launch a CUDA kernel on a graphics device, one specifies to the CUDA runtime system which kernel to launch, how many blocks to launch it with, and how many threads each block will have. The graphics device is (loosely speaking) made up of a number of processors, where each processor can run one or more blocks. Two or more processors cannot combine efforts to run a single block. The kernel launch can fail for a number of reasons, the most common being:

1. A badly written kernel – buffer overruns, underruns or segmentation faults
2. Requesting too many blocks. Graphics devices have a limited number of blocks that can be launched, which varies from card to card.
3. Requesting too many threads per block. Graphics devices have a limited number of threads each block can run, which varies from card to card.
4. Requesting too many resources. Each processor on the graphics device has a limited number of registers and shared memory. A CUDA kernel will use a given number of registers per thread (which depends on the kernel). If too many threads are launched, the block will need more registers than a processor can provide, and the launch will fail.
By specifying the number of blocks and the number of threads, the user is determining how the computational load is spread across the device and this will determine the runtime. For example, if only one block is launched on a device with several processors, the runtime performance will be poor since much of the device is idle. If too many blocks are launched, performance will also be poor due to the overhead of block scheduling and setup, and the fact that each block does very little work before being replaced by another.

It should be evident that tuning a CUDA kernel for performance is not always simple. In general the optimal launch parameters depend on

1. The particular CUDA kernel being considered, including the number of registers and shared memory it uses.
2. The graphics device the kernel is run on.
3. The computational load of the kernel.

The last point above means that optimal launch parameters can be dependent on problem size, e.g. launching a kernel with a smaller computational load may have different optimal launch parameters to the same kernel for a larger computational load.

3.2 Tuning and CUDA Errors

Note that tuning a kernel can produce CUDA errors: while some NAG functions will endeavour to indicate when a launch would have required too much shared memory, no attempt is made to determine whether sufficient registers are available for a launch to succeed. Passing a high number of threads to a register-heavy kernel (typically the double precision kernels) may result in a launch failure and a CUDA runtime error such as cudaErrorLaunchOutOfResources. When tuning a function, users should check the error codes to see whether a resource-related error occurred (such as a launch requiring too much shared memory or too many registers). If such an error occurred, the launch configuration should be discarded and cudaGetLastError() should be called to clear the CUDA error status. A new launch configuration can then be tried.

4 References

None.

5 Members

1: sblAThdsPerBlk – int

The number of threads per block for the Sobol’ generator.

Constraints:

\[ 32 \leq sblAThdsPerBlk \leq T \]

where \( T \) is the maximum number of threads per block as reported by the CUDA runtime system (see the CUDA runtime function cudaGetDeviceProperties());

\( sblAThdsPerBlk \) must be a power of 2.

2: sblABlksPerDim – int

The number of thread blocks which will cooperate to generate numbers from each dimension. By default the generator will launch one block for each dimension. If many points are generated from a low dimensional sequence, increasing \( sblABlksPerDim \) may lead to better work distribution on the GPU.

If \( d \) is the dimensionality of the sequence, the total number of blocks \( B \) that is launched is given by

\[ B = sblABlksPerDim \times d \]
Constraints:

\[ 1 \leq \text{sblABlksPerDim} \leq B_x \] where \( B_x \) is the maximum number of blocks in the \( x \) dimension of the grid, as reported by the CUDA runtime system (see the CUDA runtime function `cudaGetDeviceProperties()`);

\text{sblABlksPerDim} must be a power of 2.
1 Purpose
NagGpuRandComm is used by the library for communication between the GPU pseudorandom number generator functions. It is for internal library use and should not be modified by the user in any way.

2 Specification
#include <nag_gpu.h>
struct NagGpuRandComm {
    NagGpuTuneOrigin tuneOrigin;
    NagGpuRandTune *tuneParamsUsed;
    int param1;
    int param2;
    unsigned int *param3;
    unsigned int *param4;
    unsigned int *param5;
    unsigned int *param6;
    int *param7;
    int param8;
    NagGpuMrg32k3aDeviceComm *param9;
    void *param10;
}

3 Description
3.1 Monitoring Launch Parameters
The first two members of NagGpuRandComm may be used to monitor the launch configuration which was used for a particular GPU kernel. This will typically only be of interest to users wanting to fine tune the performance of the library’s GPU functions.

Immediately following a successful call to one of the GPU pseudorandom generator functions (such as naggpuRandUniformA), the tuneParamsUsed member of the NagGpuRandComm structure will contain the parameters which were used to launch the kernel. Only the members of tuneParamsUsed relevant to the kernel that was launched should be inspected. See NagGpuRandTune for further details on these members and their meanings.

Note that the tuneParamsUsed pointer is no longer valid after calling naggpuRandCleanupA. Parameters must be observed before calling the cleanup function.

4 References
None.

5 Members
The full structure definition is provided so that the library can be called from languages other than C/C++. The members of this structure not documented below are private to the library and must not be modified in any way.
1: **tuneOrigin** – NagGpuTuneOrigin

Indicates where the tuning data in **tuneParamsUsed** originated: was it supplied by the user (by passing a NagGpuRandTune to the generator routine), or was it a default value used by the library. This value must not be modified in any way.

2: **tuneParamsUsed** – NagGpuRandTune *

Contains the members used to launch the GPU kernel. See NagGpuRandTune for further details. This value, and the values inside the NagGpuRandTune structure, must not be modified in any way.

**Note**: **tuneParamsUsed** is no longer valid after calling naggpuRandCleanupA.
NAG Numerical Routines for GPUs Data Type Document

NagGpuRandTune

1 Purpose

NagGpuRandTune provides parameters to tune the performance of the GPU pseudorandom number generators (such as naggpuRandUniformA).

The generators are based on parameterized kernels, and these parameters are given in this structure. This structure represents advanced features of the library which are optional: users do not need to provide launch parameters in order to use the GPU functions, since default values will be used.

2 Specification

#include <nag_gpu.h>

struct NagGpuRandTune {
    int mrgOptAThdsPerBlk;
    int mrgOptAPtsPerThd;
    int mrgConAThdsPerBlk;
    int mrgConANumLoops;
    int mtANumBlks;
    int *mtAGen;
    int mrgRejOptAThdsPerBlk;
    int mrgRejOptANumBlks;
    void (*mrgRejConA)(int npts, int *nthds, int *nblks);
}

3 Description

The members of this structure control the launch parameters for the GPU pseudorandom number generator kernels. These parameters are passed to the kernels through the generator functions such as naggpuRandUniformA. The parameters used in the subsequent kernel launch may be queried through the tuneParamsUsed member of the NagGpuRandComm communication structure. If the call to the generator function succeeded, they will be the same as the parameters passed in by the user. If an error occurred, the values in tuneParamsUsed are unset and should be ignored.

Note that only the members of NagGpuRandTune relevant to the kernel that is to be launched need be set. For example, when launching the MRG32k3a consistent order kernel for uniform pseudorandom numbers, only mrgConAThdsPerBlk and mrgConANumLoops need be set.

As a starting point in tuning a function, it may be useful to launch a kernel with a given problem size and specify a NULL NagGpuRandTune pointer. In this case the library will use default launch parameters, which can be queried through the tuneParamsUsed member of the NagGpuRandComm communication structure. This could give some indication of where a search could begin.

3.1 Background on Performance Tuning

CUDA compute kernels partition the computational load into independent blocks, each block being made up of a number of threads. Blocks cannot communicate, and all blocks have the same number of threads. The blocks can optionally be arranged in a 2D grid, although this is only to aid the programmer and does not impact performance. To launch a CUDA kernel on a graphics device, one specifies to the CUDA runtime system which kernel to launch, how many blocks to launch it with, and how many threads each block will have. The graphics device is (loosely speaking) made up of a number of processors, where each
processor can run one or more blocks. Two or more processors cannot combine efforts to run a single block. The kernel launch can fail for a number of reasons, the most common being:

1. A badly written kernel – buffer overruns, underruns or segmentation faults
2. Requesting too many blocks. Graphics devices have a limited number of blocks that can be launched, which varies from card to card.
3. Requesting too many threads per block. Graphics devices have a limited number of threads each block can run, which varies from card to card.
4. Requesting too many resources. Each processor on the graphics device has a limited number of registers and shared memory. A CUDA kernel will use a given number of registers per thread (which depends on the kernel). If too many threads are launched, the block will need more registers than a processor can provide, and the launch will fail.

By specifying the number of blocks and the number of threads, the user is determining how the computational load is spread across the device and this will determine the runtime. For example, if only one block is launched on a device with several processors, the runtime performance will be poor since much of the device is idle. If too many blocks are launched, performance will also be poor due to the overhead of block scheduling and setup, and the fact that each block does very little work before being replaced by another.

It should be evident that tuning a CUDA kernel for performance is not always simple. In general the optimal launch parameters depend on

1. The particular CUDA kernel being considered, including the number of registers and shared memory it uses.
2. The graphics device the kernel is run on.
3. The computational load of the kernel.

The last point above means that optimal launch parameters can be dependent on problem size, e.g. launching a kernel with a smaller computational load may have different optimal launch parameters to the same kernel for a larger computational load.

### 3.2 Tuning and CUDA Errors

Note that tuning a kernel can produce CUDA errors: while some NAG functions will endeavour to indicate when a launch would have required too much shared memory, no attempt is made to determine whether sufficient registers are available for a launch to succeed. Passing a high number of threads to a register-heavy kernel (typically the double precision kernels) may result in a launch failure and a CUDA runtime error such as `cudaErrorLaunchOutOfResources`. When tuning a function, users should check the error codes to see whether a resource-related error occurred (such as a launch requiring too much shared memory or too many registers). If such an error occurred, the launch configuration should be discarded and `cudaGetLastError()` should be called to clear the CUDA error status. A new launch configuration can then be tried.

### 4 References

None.

### 5 Members

1. `mrgOptAThdsPerBlk` – int

   The number of threads per block for the MRG32k3a kernel with ordering `NAGGPURANDORDER_OPTIMAL`.


   Constraint: $1 \leq mrgOptAThdsPerBlk \leq T$ where $T$ is the maximum number of threads per block as reported by the CUDA runtime system (see the CUDA runtime function `cudaGetDeviceProperties()`).
2: \( \text{mrgOptAPtsPerThd} \) – int

The number of random variates each CUDA thread calculates in the MRG32k3a kernel with ordering \text{NAGGURANDORDER\_OPTIMAL}.

\text{Relevant distributions: naggpuRandUniformA, naggpuRandExpA and naggpuRandNormalA.}

Let \( X_0, X_1, X_2, \ldots \) be the random variates in \text{NAGGURANDORDER\_CONSISTENT} order. The parallelization strategy of this kernel is simply to split the sequence into adjacent, non-overlapping subsequences, and to assign each subsequence to a CUDA thread. Let \( T = \text{mrgOptAThdsPerBlk} \) and \( P = \text{mrgOptAPtsPerThd} \), let \( \text{d\_buff} \) be the storage buffer and let \( N \) be the number of random variates required. The number of blocks \( B_{xy} \) that is launched is approximately \( B_{xy} = N/TP \). Let \( B = \lfloor N/TP \rfloor \). Then for all integers \( 0 \leq t < T \) and \( 0 \leq p < P \) and \( 0 \leq b < B \), we have that

\[
\text{d\_buff}[t + pT + bPT] = X_{p+tP+bT}
\] (1)

The ordering of the variates \( \text{d\_buff}[i] \) for \( PTB \leq i < N \) is implementation specific. For the Normal distribution, the relation (1) will only hold if subsequences start at even offsets in the generator sequence: a sufficient condition for this is that \( \text{mrgOptAThdsPerBlk} \) be even.

\text{Constraint: } \text{mrgOptAPtsPerThd} \geq 1.

3: \( \text{mrgConAThdsPerBlk} \) – int

The number of threads per block for the MRG32k3a kernel with ordering \text{NAGGURANDORDER\_CONSISTENT}.

\text{Relevant distributions: naggpuRandUniformA, naggpuRandExpA and naggpuRandNormalA.}

\text{Constraints:}

\[ 1 \leq \text{mrgConAThdsPerBlk} \leq T \] where \( T \) is the maximum number of threads per block as reported by the CUDA runtime system (see the CUDA runtime function \text{cudaisGetDeviceProperties()});

\( \text{mrgConAThdsPerBlk} \) must be divisible by \( W \) where \( W = 16 \) on devices of compute capability 1.3 or lower, and \( W = 32 \) otherwise. See the CUDA runtime function \text{cudaisGetDeviceProperties()} for details of determining the compute capability of a given device.

4: \( \text{mrgConANumLoops} \) – int

Controls the number of random variates computed by each block of the MRG32k3a kernel with ordering \text{NAGGURANDORDER\_CONSISTENT}.

\text{Relevant distributions: naggpuRandUniformA, naggpuRandExpA and naggpuRandNormalA.}

For a given sample size \( N \), the number of blocks \( B_{xy} \) that is launched is approximately

\[ B_{xy} = \frac{N}{\text{mrgConAThdsPerBlk} \times \text{mrgConANumLoops} \times W} \]

where \( W = 16 \) on devices of compute capability 1.3 or lower, and \( W = 32 \) otherwise. See \text{cudaisGetDeviceProperties()} for details of determining the compute capability of a given device.

\text{The number of random variates generated by each block is therefore approximately } N/B_{xy}.

\text{Constraint: } \text{mrgConANumLoops} \geq 1.

5: \( \text{mtANumBlks} \) – int

The number of thread blocks to be launched for the MT19937 generator. This parameter is used for both optimal and consistent ordering.

\text{Relevant distributions: naggpuRandUniformA, naggpuRandExpA and naggpuRandNormalA.}

\text{Constraint: } 1 \leq \text{mtANumBlks} \leq B_{\text{max}}, \text{ where } B_{\text{max}} \text{ is the maximum number of thread blocks in the } x \text{ dimension of the grid, as reported by the CUDA runtime system (see the CUDA runtime function cudaisGetDeviceProperties()).}
6:  \texttt{mtnAGen[mtnANumBlks]} – int *

The number of random variates each CUDA thread block of the MT19937 generator will calculate.
This parameter is used for both optimal and consistent ordering.

\textit{Relevant distributions:} \texttt{naggpuRandUniformA}, \texttt{naggpuRandExpA} and \texttt{naggpuRandNormalA}.

Let \(X_0, X_1, X_2, \ldots\) denote the sequence of uniform pseudorandom variates specified by the MT19937 algorithm. The algorithm is based on a recurrence where generating \(X_n\) requires the values of \(X_{n-624}, X_{n-623}\) and \(X_{n-227}\) for each \(n \geq 0\) (we assume the seed has negative indices).
This means that a maximum of 226 new values can be updated in parallel before there is a need to wait for the previous values to be computed (a synchronization point). In this form there is an inherent limit on the amount of parallelization which is possible. To scale the problem beyond this limit, it must be subdivided into several subproblems. If \(N\) random variates are required, then the sequence can be split into \(B\) subsequences where the \(i\)th subsequence is of length \(n_i\), subject to the condition that \(\sum_{i=1}^{B} n_i = N\). The subsequences start at locations corresponding to \(X_0, X_{n_1}, X_{n_1+n_2}, \ldots\) in the original sequence.

For distributions where a single input uniform variate is mapped to a single output variate (e.g. through the inverse CDF method), the \texttt{NAGGPURANDORDER_OPTIMAL} and \texttt{NAGGPURANDORDER_CONSISTENT} orderings for MT19937 will agree. In other words, \textit{there is no distinction between optimal and consistent order} and both options will adhere to the order specified by the MT19937 algorithm. For more complex output distributions where two or more uniforms are mapped to one or more output values, there will typically be no mapping between the output for optimal and consistent ordering, such as there is for the MRG32k3a.

The CUDA kernel launches \texttt{mtnANumBlks} thread blocks where block \(i\) generates \texttt{mtnAGen[i]} variates for \(0 \leq i < \texttt{mtnANumBlks}\). Each thread block uses the skipahead functionality to advance the state of the generator to the location where it needs to start generating. Thus the first block can start generating immediately, while the second is performing a skip ahead of \texttt{mtnAGen[0]} before it is able to start, etc. The values in \texttt{mtnAGen} therefore control the skip ahead strategy followed by the generator.

The final requirement below that the members of \texttt{mtnAGen} be even is due to the Box-Muller transform that is used to compute Normal random variates.

\textit{Constraints:}
\[
\begin{align*}
\texttt{mtnAGen}(i) &\geq 0 \text{ for all } 0 \leq i \leq \texttt{mtnANumBlks} - 1; \\
\sum_{i=0}^{\texttt{mtnANumBlks}-1} \texttt{mtnAGen}(i) &= N \text{ where } N \text{ is the required number of random variates to be generated}; \\
\text{When calling } \texttt{naggpuRandNormalA} \text{ with } \texttt{order = NAGGPURANDORDER_CONSISTENT}, \texttt{mtnAGen}(i) \text{ must be even for all } 0 \leq i \leq \texttt{mtnANumBlks} - 2.
\end{align*}
\]

7:  \texttt{mrgRejOptAThdsPerBlk} – int *

The number of threads per block to be launched for the MRG32k3a rejection kernel with ordering \texttt{NAGGPURANDORDER_OPTIMAL}.

\textit{Relevant distributions:} \texttt{naggpuRandGammaA}.

\textit{Constraints:}
\[
1 \leq \texttt{mrgRejOptAThdsPerBlk} \leq T \text{ where } T \text{ is the maximum number of threads per block as reported by the CUDA runtime system (see the CUDA runtime function } \texttt{cudaGetDeviceProperties()}); \\
\texttt{mrgRejOptAThdsPerBlk} \text{ must be divisible by } W \text{ where } W = 16 \text{ on devices of compute capability 1.3 or lower, and } W = 32 \text{ otherwise. See the CUDA runtime function } \texttt{cudaGetDeviceProperties()} \text{ for details of determining the compute capability of a given device.}
\]
The number of thread blocks to be launched for the MRG32k3a rejection kernel with ordering **NAGGPURANDORDER_OPTIMAL**.

**Relevant distributions:** naggpuRandGammaA.

This kernel uses independent substreams (please see the Random Number Generators Chapter Introduction for a brief introduction to streams and substreams) to parallelise the rejection algorithm. Each CUDA thread takes the MRG32k3a state and advances it by $2^{76i}$ steps, where $i$ is the thread’s index. For performance reasons, warps of threads cooperate when writing variates to global memory. As such there is no simple mapping between the output in optimal and consistent order. The state of each CUDA thread is stored after the kernel completes so that calls to this kernel can be mixed safely with calls to the other MRG32k3a kernels.

**Constraint:** $1 \leq \text{mrgRejOptANumBlks} \leq B_{\text{max}}$, where $B_{\text{max}}$ is the maximum number of thread blocks in the $x$ dimension of the grid, as reported by the CUDA runtime system (see the CUDA runtime function `cudaGetDeviceProperties()`).

A function returning the number of threads per block and number of thread blocks to be used for the MRG32k3a rejection kernel with ordering **NAGGPURANDORDER_CONSISTENT**.

**Relevant distributions:** naggpuRandGammaA.

This kernel uses an iterative method to generate random variates in consistent order. An attempt is first made to generate the required number of variates, say $n$; when the generator completes, the actual number of variates $g$ that were generated is subtracted from the required number of variates, and an attempt is made to generate remaining $n-g$ variates. The process continues until all $n$ variates have been generated. This gives rise to the need for a tuning function since the generator will be called repeatedly with decreasing sample sizes. A single set of launch parameters used across all these different sample sizes will almost certainly be suboptimal.

The parameters for **mrgRejConA** are as follows:

- **npts**, input: the number of pseudorandom variates to be generated.
- **nthds**, output: the number of threads per block to be used.
- **nblks**, output: the number of threads blocks to launch.

**Constraints:**

- $\text{mrgRejConA} \neq \text{NULL}$;
- $1 \leq \text{nthds} \leq T$ where $T$ is the maximum number of threads per block as reported by the CUDA runtime system (see the CUDA runtime function `cudaGetDeviceProperties()`);
- **nthds** must be divisible by $W$ where $W = 16$ on devices of compute capability 1.3 or lower, and $W = 32$ otherwise. See the CUDA runtime function `cudaGetDeviceProperties()` for details of determining the compute capability of a given device;
- $1 \leq \text{nblks} \leq B_{\text{max}}$, where $B_{\text{max}}$ is the maximum number of thread blocks in the $x$ dimension of the grid, as reported by the CUDA runtime system (see the CUDA runtime function `cudaGetDeviceProperties()`).
NAG Numerical Routines for GPUs Data Type Document

NagGpuSobolDeviceComm

1 Purpose
NagGpuSobolDeviceComm is used by the library for communication between the host and the GPU Sobol' quasi-random generator functions such as naggpuSdevSobolUniformA. It is for internal use and should not be modified in any way.

2 Specification
#include <nag_gpu.h>
struct NagGpuSobolDeviceComm {
    int param1;
    const unsigned int *param2;
    const unsigned int *param3;
}

3 Description
To use the Sobol' device function generators, the memory address of a NagGpuSobolDeviceComm structure must be obtained from the device generator initialization function naggpuSobolDeviceInitA. This structure will reside in the GPU memory space and will contain communication data for use by the device function generator. This GPU memory address must then be passed to the Sobol' device generator initialization function naggpuSdevSobolInitA. Once all values have been obtained from the device function generators, the same NagGpuSobolDeviceComm structure memory address must be passed to naggpuSobolDeviceCleanupA to free allocated system resources.

4 References
None.

5 Members
The structure definition is provided so that the library can be called from languages other than C/C++. All the members of this structure are private to the library and must not be modified in any way.
NAG Numerical Routines for GPUs Data Type Document

NagCPUBBComm

1 Purpose
NagCPUBBComm is used by the library for communication between the serial, host-only Brownian bridge generator functions nagCPUBBInitA, nagCPUBBA and nagCPUBBCleanupA. It is for internal library use and should not be modified by the user in any way.

2 Specification
#include <nag_gpu.h>
struct NagCPUBBComm {
    int param1;
    int stackSize;
    void *param3;
}

3 Description
None.

4 References
None.

5 Members
1:  param1 – int
    This member is used internally by the library and must not be modified in any way.

2:  stackSize – int
    After a successful return from nagCPUBBInitA, this contains the size of the working stack per bridge dimension which is needed to construct the Brownian bridge. Please see nagCPUBBInitA for further details on the working stack. This value must not be modified in any way.

    Note: stackSize is no longer valid after calling nagCPUBBCleanupA.

3:  param3 – void *
    This member is used internally by the library and must not be modified in any way.
NAG Numerical Routines for GPUs Data Type Document

NagCPUBBIIncComm

1 Purpose
NagCPUBBIIncComm is used by the library for communication between the serial, host-only Brownian bridge increments generator functions nagCPUBBIIncInitA, nagCPUBBIIncA and nagCPUBBIIncCleanupA. It is for internal library use and should not be modified by the user in any way.

2 Specification
#include <nag_gpu.h>
struct NagCPUBBIIncComm {
    int param1;
    int stackSize;
    void *param3;
};

3 Description
None.

4 References
None.

5 Members
1: param1 – int
   This member is used internally by the library and must not be modified in any way.

2: stackSize – int
   After a successful return from nagCPUBBIIncInitA, this contains the size of the working stack per bridge dimension which is needed to construct the Brownian bridge. Please see nagCPUBBIIncA for further details on the working stack. This value must not be modified in any way.
   Note: stackSize is no longer valid after calling nagCPUBBIIncCleanupA.

3: param3 – void *
   This member is used internally by the library and must not be modified in any way.
NAG Numerical Routines for GPUs Data Type Document

NagCPUDepthBBComm

1 Purpose
NagCPUDepthBBComm is used by the library for communication between the serial, host-only Brownian bridge generator functions. It is for internal library use and should not be modified by the user in any way.

2 Specification
#include <nag_gpu.h>

struct NagCPUDepthBBComm {
    int param1;
    int *param2;
    int *param3;
    float *param4;
    int param5;
    int param6;
    bool param7;
};

3 Description
None.

4 References
None.

5 Members
The structure definition is provided so that the library can be called from languages other than C/C++. All the members of this structure are private to the library and must not be modified in any way.
1 Purpose

NagCPUQuasiRandComm is used by the library for communication between the serial, host-only quasi-random number generator functions. It is for internal library use and should not be modified by the user in any way.

2 Specification

```c
#include <nag_gpu.h>
struct NagCPUQuasiRandComm {
    int param1;
    int param2;
    int param3;
    unsigned int param4;
    unsigned int *param5;
    unsigned int *param6;
};
```

3 Description

None.

4 References

None.

5 Members

The structure definition is provided so that the library can be called from languages other than C/C++. All the members of this structure are private to the library and must not be modified in any way.
1 Purpose
NagCPURandComm is used by the library for communication between the serial, host-only pseudorandom number generator functions. It is for internal library use and should not be modified by the user in any way.

2 Specification
#include <nag_gpu.h>
struct NagCPURandComm {
    int param1;
    int param2;
    unsigned int *param3;
    int param4;
    int param5;
    int param6;
    int param7;
};

3 Description
None.

4 References
None.

5 Members
The structure definition is provided so that the library can be called from languages other than C/C++. All the members of this structure are private to the library and must not be modified in any way.
NAG Numerical Routines for GPUs Data Type Document

NagGpuBridgeOrder

1 Purpose

Identifies a number of pre-defined Brownian bridge construction orders which can be created by naggpuMakeBridgeOrderA. These can then be passed to the Brownian bridge initialization functions naggpuBBInitA and naggpuBBIncInitA.

2 Specification

```c
#include <nag_gpu.h>
enum NagGpuBridgeOrder {
    NAGGUBRIDGEORDER_BISECT_ASCEND_ROUNDDOWN = 800,
    NAGGUBRIDGEORDER_BISECT_ASCEND_ROUNDUP,
    NAGGUBRIDGEORDER_BISECT_DESCEND_ROUNDDOWN,
    NAGGUBRIDGEORDER_BISECT_DESCEND_ROUNDUP
};
```

3 Description

The Brownian bridge algorithm (see Glasserman (2004)) is a popular method for constructing a Brownian motion \( X \) on an interval \([t_0, T]\) at a set of discrete times \( t_0 < t_1 < t_2 < \ldots < t_N < T \) for some \( N \geq 1 \). Inherent in the algorithm is the notion of a bridge construction order which specifies the order in which points \( X_{t_i} \) for \( i = 1, 2, \ldots, N \) are generated. The value of \( X_{t_0} \) is always assumed known, and the first Brownian point to be generated is always the final time \( X_T \). Thereafter, successive points are generated iteratively by an interpolation formula, using points which were computed at previous iterations. For further details, please see the documentation for naggpuBBInitA.

The symbols in NagGpuBridgeOrder identify several well known bridge construction orders. For ease of exposition, let us set \( M = N + 1 \) and define \( t_M = T \). In addition, by \( \lfloor a \rfloor \) we will denote the largest integer less than \( a \), and by \( \lceil a \rceil \) we will denote the smallest integer larger than \( a \).

4 References


5 Symbols

1: **NAGGUBRIDGEORDER_BISECT_ASCEND_ROUNDDOWN**

Defines a bridge order based on bisection of the time indices interval \([0, M]\), proceeding from bottom to top and rounding all calculations down (where necessary).

The interval is processed in levels. The first construction point is \( t_M \). On the first level, \([0, M]\) is bisected so that the second construction point is \( t_{\lfloor M/2 \rfloor} \). On the second level, the interval \([0, \lfloor M/2 \rfloor]\) is bisected yielding a third construction point \( t_{\lfloor M/4 \rfloor} \), and then the interval \([\lfloor M/2 \rfloor, M]\) is bisected yielding a fourth construction point \( t_{\lceil M/2 \rceil + \lfloor (M - \lfloor M/2 \rfloor)/2 \rfloor} \). This process continues until all the times \( t_i \) for \( i = 1, \ldots, M \) have been processed. For example, on the set of discrete times

\[ t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9, t_{10}, t_{11}, t_{12}, t_{13} \]

the Brownian bridge will be constructed in the order

\[ t_{13}, t_6, t_3, t_9, t_1, t_4, t_7, t_{11}, t_2, t_5, t_8, t_{10}, t_{12} \]
2: **NAGPUBRIDGEORDER_BISECT_ASCEND_ROUNDDOWN**

Defines a bridge order based on bisection of the *time indices* interval \([0, M]\), proceeding from bottom to top and rounding all calculations down (where necessary).

The interval is processed in levels. The first construction point is \(t_M\). On the first level, \([0, M]\) is bisected so that the second construction point is \(t_{(M/2)}\). On the second level, the interval \([0, [M/2]]\) is bisected yielding a third construction point \(t_{(M/4)}\), and then the interval \([[M/2], M]\) is bisected yielding a fourth construction point \(t_{([M/2]+([M-(M/2)])/2]}\). This process continues until all the times \(t_i\) for \(i = 1, \ldots, M\) have been processed. For example, on the set of discrete times

\[
t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \quad t_6 \quad t_7 \quad t_8 \quad t_9 \quad t_{10} \quad t_{11} \quad t_{12} \quad t_{13}
\]

the Brownian bridge will be constructed in the order

\[
t_{13} \quad t_7 \quad t_4 \quad t_{10} \quad t_2 \quad t_6 \quad t_9 \quad t_{12} \quad t_1 \quad t_3 \quad t_5 \quad t_8 \quad t_{11}
\]

3: **NAGPUBRIDGEORDER_BISECT_DESCEND_ROUNDDOWN**

Defines a bridge order based on bisection of the *time indices* interval \([0, M]\), proceeding from top to bottom and rounding all calculations down (where necessary).

The interval is processed in levels. The first construction point is \(t_M\). On the first level, \([0, M]\) is bisected so that the second construction point is \(t_{(M/2)}\). On the second level, the interval \([[M/2], M]\) is bisected yielding a third construction point \(t_{(M/2)+([M-(M/2)]/2]}\), and then the interval \([0, [M/2]]\) is bisected yielding a fourth construction point \(t_{(M/4)}\). This process continues until all the times \(t_i\) for \(i = 1, \ldots, M\) have been processed. For example, on the set of discrete times

\[
t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \quad t_6 \quad t_7 \quad t_8 \quad t_9 \quad t_{10} \quad t_{11} \quad t_{12} \quad t_{13}
\]

the Brownian bridge will be constructed in the order

\[
t_{13} \quad t_6 \quad t_9 \quad t_{11} \quad t_7 \quad t_4 \quad t_{12} \quad t_1 \quad t_3 \quad t_5 \quad t_8 \quad t_2
\]

4: **NAGPUBRIDGEORDER_BISECT_DESCEND_ROUNDUP**

Defines a bridge order based on bisection of the *time indices* interval \([0, M]\), proceeding from top to bottom and rounding all calculations up (where necessary).

The interval is processed in levels. The first construction point is \(t_M\). On the first level, \([0, M]\) is bisected so that the second construction point is \(t_{(M/2)}\). On the second level, the interval \([[M/2], M]\) is bisected yielding a third construction point \(t_{(M/2)+([M-(M/2)]/2]}\), and then the interval \([0, [M/2]]\) is bisected yielding a fourth construction point \(t_{(M/4)}\). This process continues until all the times \(t_i\) for \(i = 1, \ldots, M\) have been processed. For example, on the set of discrete times

\[
t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \quad t_6 \quad t_7 \quad t_8 \quad t_9 \quad t_{10} \quad t_{11} \quad t_{12} \quad t_{13}
\]

the Brownian bridge will be constructed in the order

\[
t_{13} \quad t_7 \quad t_{10} \quad t_4 \quad t_{12} \quad t_9 \quad t_6 \quad t_2 \quad t_{11} \quad t_8 \quad t_5 \quad t_3 \quad t_1
\]
NAG Numerical Routines for GPUs Data Type Document

NagGpuQuasiGen

1 Purpose

Identifies the base quasi-random number generators available in the library. A specific generator is selected by passing one of the symbols given below to the generator initialization functions such as naggpuQuasiRandInitA.

2 Specification

```c
#include <nag_gpu.h>
enum NagGpuQuasiGen {
    NAGGPUQUASIGEN_SOBOL = 300
};
```

3 Description

Low discrepancy (quasi-random) sequences are used in numerical integration, simulation and optimization. Like pseudorandom numbers they are uniformly distributed, but they are not statistically independent. Quasi-random sequences are designed to give a more even distribution in multidimensional space (uniformity), and are often more efficient than pseudorandom numbers in multidimensional Monte Carlo methods.

Let \( x_1, x_2, \ldots, x_N \) be a sequence of \( d \)-dimensional points in the unit cube \( I^d = [0, 1]^d \). Let \( G \) be a subset of \( I^d \) and define the counting function \( S_N(G) \) as the number of \( d \)-dimensional points \( x^i \in G \). For each point \( x = (x_1, x_2, \ldots, x_d) \in I^d \), let \( G_x \) be the rectangular \( d \)-dimensional region

\[
G_x = [0, x_1] \times [0, x_2] \times \cdots \times [0, x_d]
\]

with volume \( x_1 \cdot x_2 \cdot \cdots \cdot x_d = \prod_{i=1}^{d} x_i \). Then one measure of the uniformity of the points \( x_1^1, x_2^2, \ldots, x_N^N \) is the so-called star discrepancy:

\[
D_N^*(x_1^1, x_2^2, \ldots, x_N^N) = \sup_{x \in P^d} \left| S_N(G_x) - N \prod_{i=1}^{d} x_i \right|
\]

which satisfies the inequality

\[
D_N^*(x_1^1, x_2^2, \ldots, x_N^N) \leq C_d (\log N)^d + O\left((\log N)^{d-1}\right) \quad \text{for all} \quad N \geq 2.
\]

The principal aim in the construction of low-discrepancy sequences is to find sequences of points in \( I^d \) with a bound of this form where the constant \( C_d \) is as small as possible.

4 References


5 Symbols

1: **NAGGPUQUASIGEN_SOBO**

Identifies the Sobol’ quasi-random generator. The implementation follows that given in Bratley and Fox (1988). The generator can produce sequences in up to 50000 dimensions. For the first 21201 dimensions, the direction numbers proposed by Joe and Kuo (2008) and Joe and Kuo (2003) are used. Dimensions higher than 21201 follow a proposal by Jaëckel (2002) and use a pseudorandom number generator to create random direction numbers.
1 Purpose

Identifies the orientation of output from the quasi-random number generators. If the generator output is viewed as a matrix, these options effectively allow one to transpose the matrix.

2 Specification

```c
#include <nag_gpu.h>
enum NagGpuQuasiOrient {
      NAGGPUQUASIORIENT_DIMVALS_CONSEC = 500,
      NAGGPUQUASIORIENT_DIMVALS_SCATT
};
```

3 Description

Quasi-random sequences are made up of one or more multidimensional points, with each point composed of several one dimensional values. Fix some integer $d \geq 1$ and consider a $d$-dimensional quasi-random sequence $x^0, x^1, \ldots$ so that each point $x^j = (x^j_1, x^j_2, \ldots, x^j_d)$ is composed of $d$ one dimensional values. When stored in memory, these points can typically be laid out in two ways: either $d$-dimensional points are stored one after the other, so that the one dimensional values of each point are laid out consecutively; or dimensions can be grouped across all the $d$-dimensional points so that the one dimensional values of any given $d$-dimensional point are scattered.

The symbols below select which memory layout is used by the quasi-random generators.

4 References

None.

5 Symbols

1: **NAGGPUQUASIORIENT_DIMVALS_CONSEC**

Specifies that one dimensional values from a $d$-dimensional point are laid out consecutively in memory. Consider $N$ points from a $d$-dimensional quasi-random sequence stored in a linear array $A$. The $j$-th dimension $x^j_n$ of the $n$-th point will be stored at location $A[n * d + (j - 1)]$ for every $0 \leq n < N$ and $1 \leq j \leq d$.

2: **NAGGPUQUASIORIENT_DIMVALS_SCATT**

Specifies that dimensions are grouped across all the $d$-dimensional points. All points from dimension 1 will be stored first, followed by all points from dimension 2, and so on. Consider $N$ points from a $d$-dimensional quasi-random sequence stored in a linear array $A$. The $j$-th dimension $x^j_n$ of the $n$-th point will be stored at location $A[N * (j - 1) + n]$ for every $0 \leq n < N$ and $1 \leq j \leq d$. 

---

Mark 0.6
1 Purpose
Identifies the base pseudorandom number generators that are available. A specific generator is selected by passing one of the symbols given below to the generator initialization functions such as naggpuRandInitA.

2 Specification
#include <nag_gpu.h>
enum NagGpuRandGen {
    NAGGPURANDGEN_MRG32K3A = 200,
    NAGGPURANDGEN_MT19937
};

3 Description
The various pseudorandom number generators that are available are listed in the Random Number Generators Chapter Introduction together with any relevant implementation details.

4 References

5 Symbols
1: NAGGPURANDGEN_MRG32K3A
Identifies L'Ecuyer's multiple recursive generator MRG32k3a discussed in L'Ecuyer (1999).

2: NAGGPURANDGEN_MT19937
Identifies the pseudorandom number generator MT19937 developed in Matsumoto and Nishimura (1998).
NAG Numerical Routines for GPUs Data Type Document

**NagGpuRandOrder**

1 Purpose

Identifies an output ordering for the variates produced by the pseudorandom number generators.

2 Specification

```c
#include <nag_gpu.h>
enum NagGpuRandOrder {
    NAGGPURANDORDER_OPTIMAL = 600,
    NAGGPURANDORDER_CONSISTENT
}
```

3 Description

3.1 Permutations

Pseudorandom number generators are called ‘pseudo’ random since the numbers they produce are not truly random. Indeed, there is a strict deterministic algorithm for computing the next number from one or more preceding numbers, and so the numbers are not independent in the true sense of the word. The art of constructing a good generator lies in choosing this deterministic algorithm such that, for most practical applications where the generator is to be used, the numbers are ‘sufficiently random’ in the sense that the application cannot determine the difference between the pseudorandom numbers and truly random numbers.

Pseudorandom generators produce numbers in a long sequence which we denote \( X_0, X_1, X_2, X_3, \ldots \). For good generators, most applications will perform equally well if given the sequence \( X_1, X_0, X_3, X_2, \ldots \), or the sequence \( X_{10}, X_{15}, X_{20}, X_{25}, \ldots \), as they would if given the original sequence \( X_0, X_1, X_2, \ldots \). The application cannot distinguish between the three sequences, and will produce more or less the same result. For good generators therefore, the output order can be permuted (somewhat) without adversely affecting the majority of applications which use the generator.

Clearly some constraints should be placed on the types of permutations allowed: a permutation which rearranges all the pseudorandom numbers to be in increasing order will no doubt have a serious impact on the application. However, there should be a reasonably large class of benign permutations which do not adversely affect most applications. Such permutations would probably result in a slightly different numerical answer, but this should be in line with the variability inherent in using a (pseudo)random number generator in the first place. For example, initializing a generator with two different seeds and computing two Monte Carlo integrals of a given function will yield two different answers: the difference should be in line with the standard deviation of the Monte Carlo integral estimate.

3.2 Performance Implications

For computer implementations of the generator, it is not always efficient to output the numbers in the original order \( X_0, X_1, X_2, \ldots \). This is often the case for GPUs. Due to the design of the hardware, it may be faster to output the numbers in some permuted order rather than trying to maintain the original order. This *performance-optimal* order is dependent on the launch configuration of the CUDA kernel, so that launching with different numbers of threads and blocks could result in different permutations of the output values. The types of permutations are typically of the form

\[
X_0, X_n, X_{2n}, \ldots, X_{Tn}, \\
X_1, X_{n+1}, X_{2n+1}, \ldots, X_{Tn+1}, \\
\ldots, \\
X_{n-1}, X_{2n-1}, X_{3n-1}, \ldots, X_{(T+1)n-1}, X_{(T+1)n}, X_{(T+2)n}, \ldots
\]
for some given integers $n$ and $T$. (The table-style layout above is simply to ease presentation: the rows should be concatenated one after another to obtain the layout in memory.)

### 3.3 Statistical Properties and Correctness

For many applications such a permutation will be benign and should not affect interpretations of the results. However it should be emphasized that permuting the output of a generator in effect creates a new pseudorandom generator, the statistical properties of which are not necessarily known. Some users may be uncomfortable with this, and may prefer to maintain the original ordering of the generator on the grounds of statistical correctness.

In other cases, a GPU implementation of an application may have to be compared to a reference CPU implementation, with the aim of obtaining the same results (to some appropriate tolerance) from both. In this case the order in which the pseudorandom numbers are used becomes important: for example for some interest rate models, constructing a sample path from the sequence $X_0, X_1, X_2, X_3, \ldots$ will yield a different path than if the sequence $X_1, X_0, X_3, X_2, \ldots$ were used. In this case it may be easier to maintain the original ordering of the sequence simply so that numbers can be read back and used in the correct order.

### 3.4 Implementation

The GPU pseudorandom number generators can output numbers either in an ordering aimed at peak performance, or in the original order specified by the generator algorithm. The performance ordering is referred to as *optimal ordering*, while the original generator order is referred to as *consistent ordering*, and the choices are indicated to the generator functions by passing one of the symbols below. Much work has gone into optimising the performance of the consistent order generators, and in many cases it is comparable to that of the original order versions. The double precision functions are typically worst affected.

Both the consistent and optimal order generators are based on parameterized GPU kernels. The parameters are called *tuning parameters*. They control how the compute load is distributed across the GPU and hence control the performance. The consistent order generators will produce output in the same order regardless of the tuning parameter values. For the optimal order generators, the values of the tuning parameters affect the output ordering. The tuning parameters are contained in the NagGpuRandTune structure: please see the documentation there for further information.

By default, the library will choose tuning parameters based on distribution, precision, base generator, problem size (number of random values required) and GPU architecture, with the aim of maximizing performance. For optimal ordering therefore, when allowing the library to select default tuning parameters (by passing a null NagGpuRandTune pointer to the generator functions such as naggpuRandUniformA), the following behaviour can be expected: when generating $n$ points from a given distribution and base generator, the output ordering may vary between one GPU and another, between single and double precision on the same GPU, and the ordering for $n$ points may be different from the ordering for $n + 1$ points for the same precision on the same GPU. By contrast, the consistent order generators will always maintain the same ordering, even though the library may select different default tuning parameters based on distribution, precision, base generator, problem size and GPU, in order to maximize performance.

Tuning parameters can be specified directly to the generator functions (see NagGpuRandTune). For consistent order, these parameters will affect performance only, but for optimal order they may affect performance and output order. User supplied tuning parameters will override any default library choices: for optimal order generators, the associated output ordering will thus be maintained across different platforms.

It is worth noting that for optimal order generators, fixing an ordering by specifying a set of tuning parameters could also be useful when comparing GPU and CPU applications to ensure that numbers are read back in a certain order. For example, one could launch an optimal order MRG32k3a generator with $T$ threads and $B$ blocks and then write a kernel with $T$ threads and $B$ blocks which reads the numbers back so that each CUDA thread reads a segment $X_i, X_{i+1}, X_{i+2}, X_{i+3}, \ldots$ from the original generator sequence. This could help achieve coalesced memory transfers.
3.5 Ordering and Non-Uniform Distributions

The notion of ‘original order of the generator’ only exists for the uniform distribution, since pseudorandom generators are only designed to produce uniform pseudorandom numbers. There are several methods of transforming uniform numbers into other distributions. Where such a transform requires only a single uniform random number (e.g. though the inverse CDF method), the notion of ‘original order’ is unambiguous and means that each uniform random number is replaced by its transformed value.

However where more than one uniform number is required (e.g. through a rejection method), the notion is less clear. In this case the documentation for the relevant generator function may state how the uniform numbers were used and what the output ordering is, but if these relationships are too involved the information will be omitted. In all cases, the output from consistent order GPU generators will agree with the output from their serial CPU counterparts.

4 References

None.

5 Symbols

1: NAGGPUTANDORDER_OPTIMAL
   Specifies that the output order may be permuted to improve performance.

2: NAGGPUTANDORDER_CONSISTENT
   Specifies that output must be in the order specified by the generator algorithm. The output ordering will remain unchanged (consistent) across different GPU architectures, problem sizes and launch configurations. This is the ordering used by all the serial CPU generator functions such as nagCPURandUniformA.
1 Purpose
Identifies the types of bit-scrambling which can be applied to the quasi-random number generators. A specific type of scrambling is selected by passing one of the symbols given below to the generator initialization functions such as naggpuQuasiRandInitA.

2 Specification
#include <nag_gpu.h>
enum NagGpuScramTypes {
   NAGGPUSCRAMTYPES_NONE = 400,
   NAGGPUSCRAMTYPES_OWEN,
   NAGGPUSCRAMTYPES_FAURE_TEZUKA,
   NAGGPUSCRAMTYPES_OWEN_FAURE_TEZUKA
}

3 Description
Scrambled quasi-random sequences are an extension of standard quasi-random sequences. They attempt to eliminate the bias inherent in a quasi-random sequence while retaining its low-discrepancy properties. The use of a scrambled sequence allows error estimation of Monte Carlo results by performing a number of iterates (i.e. computing several Monte Carlo integrations of the same function) and then computing the variance of the sample of different results. Scrambling can also improve the high dimensional performance of a quasi-random generator by destroying systematic dependencies between dimensions (e.g. as is sometimes observed in two or three dimensional projections). These dependencies are often caused by poor choices of initialization data.

This implementation of scrambled quasi-random sequences is based on TOMS Algorithm 823 and details can be found in the accompanying paper, Hong and Hickernell (2003). Three methods of scrambling are supplied; the first is a restricted form of Owen’s scrambling (Owen (1995)), the second is based on the method of Faure and Tezuka (2000) and the third method is a combination the first two.

4 References

5 Symbols
1: NAGGPUSCRAMTYPES_NONE
   No scrambling is to be performed.
2: **NAGGUSCRAMTYPES_OWEN**
The restricted Owen's scrambling described in Hong and Hickernell (2003) is to be performed.

3: **NAGGUSCRAMTYPES_FAURE_TEZUKA**
The Faure-Tezuka scrambling described in Hong and Hickernell (2003) is to be performed.

4: **NAGGUSCRAMTYPES_OWEN_FAURE_TEZUKA**
The combined Owen-Faure-Tezuka scrambling described in Hong and Hickernell (2003) is to be performed.
1 Purpose
Identifies the origin of the parameters used to launch a GPU kernel.

2 Specification
#include <nag_gpu.h>

enum NagGpuTuneOrigin {
    NAGGPUTUNEORIGIN_NA = 100,
    NAGGPUTUNEORIGIN_DEFAULT,
    NAGGPUTUNEORIGIN_USER,
    NAGGPUTUNEORIGIN_AUTO
};

3 Description
Several of the GPU functions in the library are based on parameterized kernels. Default values for these parameters are stored in the library, but users are allowed to override these and specify different launch configurations. This can improve performance since the optimal parameter values are highly dependent on the type of graphics card that is being used. For more information on performance tuning, consult the documentation for any of the tuning structures, e.g. NagGpuRandTune.

The symbols below identify where a set of tuning parameters originated and distinguish default values stored in the library from values specified by the user.

4 References
None.

5 Symbols
1: NAGGPUTUNEORIGIN_NA
   The function does not use tuning parameters.

2: NAGGPUTUNEORIGIN_DEFAULT
   The tuning parameters are default values used by the library.

3: NAGGPUTUNEORIGIN_USER
   Tuning parameters were supplied by the user.

4: NAGGPUTUNEORIGIN_AUTO
   This value is reserved and is currently unused.
Chapter Introduction

Error Handling Functions

naggpuErrorCheckPrintExit
naggpuErrorCopyMsg

List of Structures

NagGpuError
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Error Handling

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   3.1 Error Handling Functions ................................. 2
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1 Scope of the Chapter
This chapter contains functions and structures for reporting errors.

2 Library Error Handling
All error handling is implemented through the NagGpuError structure which contains information relating to errors that occur at runtime. This structure does not need to be initialized and a single instance can be used across multiple library functions calls. Note that passing a NULL NagGpuError pointer to a library function will result in undefined behaviour, possibly a segmentation fault.

On exit from a library function, a non-zero value of NagGpuError.code indicates that some error occurred, and the documentation for that function should be consulted for the causes of the error. The documentation will list the possible errors, the causes, and the corresponding values of NagGpuError.code.

Alternatively, a null terminated ANSI C string describing the error can be obtained at runtime. On exit from a function, the msgLength member of NagGpuError will return the length (including null terminator) of the error message. A buffer of that size should then be allocated and passed, together with the NagGpuError structure, to naggpuErrorCopyMsg. Please see the naggpuErrorCopyMsg documentation for further details.

The function naggpuErrorCheckPrintExit can be called to check whether a NagGpuError structure contains an error code. If so, it obtains the associated error message by calling naggpuErrorCopyMsg and prints it to cerr before terminating the program. The function is provided as a convenience for those users for whom such a style of error handling is appropriate.

Functions which interact with the GPU can generate CUDA errors. Any CUDA errors that these functions encounter will be indicated in the NagGpuError structure and will also be returned to the user. On entry, these functions will check the CUDA runtime error status, and if any prior error is encountered, it will be indicated to the user and the function will return. The CUDA error checking will not clear the CUDA runtime error status: this is equivalent to calling cudaPeekAtLastError() and not cudaGetLastError() in the CUDA runtime library. Clearing the CUDA error status is left to the user. Note that CUDA errors returned by library functions may relate to previous calls to the library, another GPU library, the CUDA runtime library, or the user’s own GPU code. Please see NVIDIA CUDA (2011) for detailed information on the CUDA runtime error system, and how to trap CUDA errors correctly when executing code asynchronously on the GPU.

3 Functionality Index
3.1 Error Handling Functions
Error handling,
check errors and if any, print message and close program ......................... naggpuErrorCheckPrintExit
retrieve an error message ................................................................. naggpuErrorCopyMsg

4 References
1 Purpose
naggpuErrorCheckPrintExit checks whether a NagGpuError structure contains an error code and if so, prints the associated error message to cerr and terminates the program.

2 Specification
#include <nag_gpu.h>
extern "C"
void naggpuErrorCheckPrintExit(const NagGpuError *error)

3 Description
This function checks whether NagGpuError.code is zero and if not, it calls naggpuErrorCopyMsg to get the error message, prints it to cerr and then terminates the program. Please see the NagGpuError documentation for further details on error handling. The following code snippet illustrates a typical use of this function:

```c
#include <nag_gpu.h>

void myFunc()
{
    NagGpuError err;
    // ... user code
    naggpuCallToSomeNagFunc(..., &err);
    naggpuErrorCheckPrintExit(&err);
    // ... further calls to GPU library
}
```

Note: If a NULL NagGpuError pointer is passed to error, naggpuErrorCheckPrintExit will print a warning message to cerr and will then terminate the program.

3.1 Synchronization
This function is blocking, but will not force synchronization between the host and the device.

4 References
None.

5 Arguments
1: error – const NagGpuError *  
   Input
   On entry: the structure describing the error. If this is NULL, a warning message is printed to cerr and the program is terminated.
   Constraint: error ≠ NULL.
6 Error Indicators and Warnings

None.

7 Example

There is no example program specifically for this function since it is used by all the other example programs. Please see any of the example programs for usage of this function.
1 Purpose

naggpuErrorCopyMsg copies a character string describing an error message into a buffer allocated by the user.

2 Specification

```c
#include <nag_gpu.h>
extern "C"
void naggpuErrorCopyMsg(char *buff, const NagGpuError *error)
```

3 Description

The NagGpuError structure contains information relating to errors that occur at runtime. It does not need to be initialized and a single instance can be used across multiple library functions calls. On exit from a library function, a non-zero value of NagGpuError::code indicates that some error occurred, and the documentation for that function should be consulted. The documentation will list the possible errors, the causes, and the corresponding values of NagGpuError::code.

Alternatively, a null terminated ANSI C string describing the error can be obtained at runtime. On exit from a library function, the msgLength member of NagGpuError will return the length (including null terminator) of the error message. A buffer of that size should then be allocated and passed, together with the NagGpuError structure, to naggpuErrorCopyMsg.

This function copies the character string describing the error to the user allocated buffer. The following code snippet illustrates a typical use of this function:

```c
#include <nag_gpu.h>
#include <iostream>

void checkError(const NagGpuError *error)
{
    using namespace std;
    if(error->code != 0) {
        char * buff = new char[error->msgLength];
        naggpuErrorCopyMsg(buff, error);
        cout << buff << endl;
        delete[] buff;
        // A simple solution: exit
        exit(-1);
    }
}

doFunc()
{
    NagGpuError err;
    // ... user code
    naggpuCallToSomeNagFunc(..., &err);
    checkError(err);
    // ... further calls to GPU library
}
Note: If a NULL NagGpuError pointer is passed to error, naggpuErrorCopyMsg will print a warning message to cerr and will kill the program. Passing a NULL pointer to buff is safe, but will result in the function returning without doing anything.

3.1 Synchronization
This function is blocking, but will not force synchronization between the host and the device.

4 References
None.

5 Arguments
1: buff[error → msgLength] – char *
   A character buffer into which the message will be copied.
   On exit: contains a null terminated message describing the error.

2: error – const NagGpuError *
   On entry: the structure describing the error. If this is NULL, a warning message is printed to cerr and the program is killed.
   Constraint: error ≠ NULL.

6 Error Indicators and Warnings
None.

7 Example
There is no example program specifically for this function since it is used by all the other example programs. Please see any of the example programs for usage of this function.
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NagGpuError

1 Purpose
NagGpuError is used to indicate errors to the user, and is used in conjunction with naggpuErrorCopyMsg. It must not be modified directly in any way.

2 Specification
#include <nag_gpu.h>
struct NagGpuError {
    int code;
    int msgLength;
    int subCodes[10];
    char param2[256];
    int param3;
    void (*param4)(char*, const NagGpuError*);
};

3 Description
The NagGpuError structure contains information relating to errors that occur at runtime. It does not need to be initialized and a single instance can be used across multiple library functions calls. On exit from a library function, a non-zero value of NagGpuError.code indicates that some error occurred, and the documentation for that function should be consulted. The documentation will list the possible errors, the causes, and the corresponding values of NagGpuError.code.

Alternatively, a null terminated ANSI C string describing the error can be obtained at runtime. On exit from a library function, the msgLength member of NagGpuError will return the length (including null terminator) of the error message. A buffer of that size should then be allocated and passed, together with the NagGpuError structure, to naggpuErrorCopyMsg.

Only the members code and msgLength should be queried by the user - the remaining members are private to the library and should not be modified in any way.

4 References
None.

5 Members
The full structure definition is provided so that the library can be called from languages other than C/C++. The members of this structure not documented below are private to the library and must not be modified in any way.

1: code – int
   The error flag. Zero means no error occurred, while a non-zero value indicates that some error occurred during execution. This value must not be altered directly by the user.

2: msgLength – int
   The length (including null terminator) of the ANSI C character string which describes the error indicated by code. This value must not be altered directly by the user. The string can be obtained by calling naggpuErrorCopyMsg.
3: subCodes[10] – int

Elements of this array may be used by some functions in the library to provide additional information. The documentation for a function will state whether this array is used, and if so what information it contains.