

NAG Library Function Document

nag_tsa_cross_corr (g13bcc)

1 Purpose

nag_tsa_cross_corr (g13bcc) calculates cross-correlations between two time series.

2 Specification

```
#include <nag.h>
#include <naggl3.h>

void nag_tsa_cross_corr (const double x[], const double y[], Integer nxy,
    Integer nl, double *s, double *r0, double r[], double *stat,
    NagError *fail)
```

3 Description

Given two series x_1, x_2, \dots, x_n and y_1, y_2, \dots, y_n the function calculates the cross-correlations between x_t and lagged values of y_t :

$$r_{xy}(l) = \frac{\sum_{t=1}^{n-l} (x_t - \bar{x})(y_{t+l} - \bar{y})}{n s_x s_y}, \quad l = 0, 1, \dots, L$$

where

$$\bar{x} = \frac{\sum_{t=1}^n x_t}{n}$$

$$s_x^2 = \frac{\sum_{t=1}^n (x_t - \bar{x})^2}{n}$$

and similarly for y .

The ratio of standard deviations s_y/s_x is also returned, and a portmanteau statistic is calculated:

$$\mathbf{stat} = n \sum_{l=1}^L r_{xy}(l)^2.$$

Provided n is large, L much less than n , and both x_t, y_t are samples of series whose true autocorrelation functions are zero, then, under the null hypothesis that the true cross-correlations between the series are zero, **stat** has a χ^2 -distribution with L degrees of freedom. Values of **stat** in the upper tail of this distribution provide evidence against the null hypothesis.

4 References

Box G E P and Jenkins G M (1976) *Time Series Analysis: Forecasting and Control* (Revised Edition) Holden-Day

5 Arguments

- | | | |
|----|---|---------------------|
| 1: | x [nxy] – const double
<i>On entry:</i> the n values of the x series. | <i>Input</i> |
| 2: | y [nxy] – const double
<i>On entry:</i> the n values of the y series. | <i>Input</i> |
| 3: | nxy – Integer
<i>On entry:</i> n , the length of the time series.
<i>Constraint:</i> $\mathbf{nxy} \geq 2$. | <i>Input</i> |
| 4: | nl – Integer
<i>On entry:</i> L , the maximum lag for calculating cross-correlations.
<i>Constraint:</i> $1 \leq \mathbf{nl} < \mathbf{nxy}$. | <i>Input</i> |
| 5: | s – double *
<i>On exit:</i> the ratio of the standard deviation of the y series to the standard deviation of the x series, s_y/s_x . | <i>Output</i> |
| 6: | r0 – double *
<i>On exit:</i> the cross-correlation between the x and y series at lag zero. | <i>Output</i> |
| 7: | r [nl] – double
<i>On exit:</i> r [$l-1$] contains the cross-correlations between the x and y series at lags L , $r_{xy}(l)$, for $l = 1, 2, \dots, L$. | <i>Output</i> |
| 8: | stat – double *
<i>On exit:</i> the statistic for testing for absence of cross-correlation. | <i>Output</i> |
| 9: | fail – NagError *
The NAG error argument (see Section 3.6 in the Essential Introduction). | <i>Input/Output</i> |

6 Error Indicators and Warnings

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_INT

On entry, **nl** = $\langle value \rangle$.
Constraint: $\mathbf{nl} \geq 1$.

On entry, **nxy** = $\langle value \rangle$.
Constraint: $\mathbf{nxy} > 1$.

NE_INT_2

On entry, $\mathbf{nl} \geq \mathbf{nxy}$: **nl** = $\langle value \rangle$ and **nxy** = $\langle value \rangle$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_ZERO_VARIANCE

One or both of the x and y series have zero variance.

7 Accuracy

All computations are believed to be stable.

8 Parallelism and Performance

Not applicable.

9 Further Comments

If $n < 100$, or $L < 10\log(n)$ then the autocorrelations are calculated directly and the time taken by `nag_tsa_cross_corr` (g13bcc) is approximately proportional to nL , otherwise the autocorrelations are calculated by utilizing fast Fourier transforms (FFTs) and the time taken is approximately proportional to $n\log(n)$. If FFTs are used then `nag_tsa_cross_corr` (g13bcc) internally allocates approximately $6n$ real elements.

10 Example

This example reads two time series of length 20. It calculates and prints the cross-correlations up to lag 15 for the first series leading the second series and then for the second series leading the first series.

10.1 Program Text

```
/* nag_tsa_cross_corr (g13bcc) Example Program.
 *
 * Copyright 2002 Numerical Algorithms Group.
 *
 * Mark 7, 2002.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg13.h>

int main(void)
{
    /* Scalars */
    double r0xy, r0yx, statxy, statyx, sxy, syx;
    Integer exit_status, i, nl, nxy;
    NagError fail;

    /* Arrays */
    double *rxy = 0, *ryx = 0, *x = 0, *y = 0;

    INIT_FAIL(fail);

    exit_status = 0;

    printf("nag_tsa_cross_corr (g13bcc) Example Program Results\n");

    /* Skip heading in data file */
    scanf("%*[\n] ");

    /* Read series length and number of lags */
```

```

scanf("%ld%ld%*[\n] ", &nxy, &nl);

/* Allocate memory */
if (!(rxy = NAG_ALLOC(nl, double)) ||
    !(ryx = NAG_ALLOC(nl, double)) ||
    !(x = NAG_ALLOC(nxy, double)) ||
    !(y = NAG_ALLOC(nxy, double)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read series */
for (i = 1; i <= nxy; ++i)
    scanf("%lf", &x[i-1]);
scanf("%*[\n] ");

for (i = 1; i <= nxy; ++i)
    scanf("%lf", &y[i-1]);
scanf("%*[\n] ");

/* Call routine to calculate cross correlations between X and Y */
/* nag_tsa_cross_corr (g13bcc).
 * Multivariate time series, cross-correlations
 */
nag_tsa_cross_corr(x, y, nxy, nl, &sxy, &r0xy, rxy, &statxy, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_tsa_cross_corr (g13bcc), 1st call.\n%s\n",
           fail.message);
    exit_status = 1;
    goto END;
}

/* Call routine to calculate cross correlations between Y and X */
/* nag_tsa_cross_corr (g13bcc), see above. */
nag_tsa_cross_corr(y, x, nxy, nl, &syx, &r0yx, ryx, &statyx, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_tsa_cross_corr (g13bcc), 2nd call.\n%s\n",
           fail.message);
    exit_status = 1;
    goto END;
}

printf("\n");
printf("
                                Between          Between\n");
printf("                                X and Y          Y and X\n");
printf("\n");
printf("Standard deviation ratio%10.4f%15.4f\n", sxy, syx);
printf("\n");
printf("Cross correlation at lag\n");
printf("                                0");
printf("%10.4f%15.4f\n", r0xy, r0yx);
for (i = 1; i <= nl; ++i)
    printf("                                %4ld%10.4f%15.4f\n", i, rxy[i-1],
           ryx[i-1]);
printf("\n");
printf("Test statistic                                %10.4f%15.4f\n", statxy, statyx);

END:
NAG_FREE(rxy);
NAG_FREE(ryx);
NAG_FREE(x);
NAG_FREE(y);

return exit_status;
}

```

10.2 Program Data

```
nag_tsa_cross_corr (g13bcc) Example Program Data
      20      15
      0.02  0.05  0.08  0.03 -0.05  0.11 -0.01 -0.08 -0.08 -0.11
     -0.18 -0.19 -0.09  0.03  0.10  0.15 -0.14  0.07  0.09  0.16
      3.18  3.21  3.26  3.25  3.08  3.01  3.06  3.17  3.12  3.04
      3.26  3.45  3.33  3.70  3.31  3.81  3.33  2.96  3.28  3.10
```

10.3 Program Results

```
nag_tsa_cross_corr (g13bcc) Example Program Results
```

	Between X and Y	Between Y and X
Standard deviation ratio	2.0053	0.4987
Cross correlation at lag		
0	0.0568	0.0568
1	0.0438	-0.0151
2	-0.3762	0.3955
3	-0.4864	0.3417
4	-0.6294	0.5486
5	-0.3871	0.2291
6	-0.1690	0.3190
7	-0.0678	0.1980
8	0.0962	0.0438
9	0.0788	-0.1428
10	0.2910	-0.1376
11	0.0950	-0.0387
12	0.0547	-0.0380
13	0.1855	-0.1551
14	0.0243	-0.1536
15	0.0034	-0.0696
Test statistic	22.1269	17.2917
