NAG Library Function Document

nag_fft_2d_complex (c06fuc)

1 Purpose

nag_fft_2d_complex (c06fuc) computes the two-dimensional discrete Fourier transform of a bivariate sequence of complex data values.

2 Specification

```
#include <nag.h>
#include <nagc06.h>
```

```
void nag_fft_2d_complex (Integer m, Integer n, double x[], double y[],
     const double trigm[], const double trign[], NagError *fail)
```

3 Description

nag_fft_2d_complex (c06fuc) computes the two-dimensional discrete Fourier transform of a bivariate sequence of complex data values $z_{j_1 j_2}$, where $j_1 = 0, 1, \ldots, m-1$, $j_2 = 0, 1, \ldots, n-1$.

The discrete Fourier transform is here defined by

$$\hat{z}_{k_1 k_2} = \frac{1}{\sqrt{mn}} \sum_{j_1=0}^{m-1} \sum_{j_2=0}^{n-1} z_{j_1 j_2} \exp\left(-2\pi i\left(\frac{j_1 k_1}{m} + \frac{j_2 k_2}{n}\right)\right)$$

for $k_1 = 0, 1, \ldots, m-1$; $k_2 = 0, 1, \ldots, n-1$.

(Note the scale factor of $1/\sqrt{mn}$ in this definition.)

The first call of nag_fft_2d_complex (c06fuc) must be preceded by calls to nag_fft_init_trig (c06gzc) to initialize the **trigm** and **trign** arrays with trigonometric coefficients according to the value of **m** and **n** respectively.

To compute the inverse discrete Fourier transform, defined with $\exp(+2\pi i(\ldots))$ in the above formula instead of $\exp(-2\pi i(\ldots))$, this function should be preceded and followed by calls of nag_conjugate_complex (c06gcc) to form the complex conjugates of the data values and the transform.

This function calls nag_fft_multiple_complex (c06frc) to perform multiple one-dimensional discrete Fourier transforms by the fast Fourier transform algorithm in Brigham (1974).

4 References

Brigham E O (1974) *The Fast Fourier Transform* Prentice–Hall

Temperton C (1983) Self-sorting mixed-radix fast Fourier transforms *J. Comput. Phys.* **52** 1–23

5 Arguments

1: **m** – Integer *Input*

On entry: the number of rows, m, of the bivariate data sequence.

Constraint: $\mathbf{m} \geq 1$.

2: **n** – Integer *Input*

On entry: the number of columns, n, of the bivariate data sequence.

Constraint: $\mathbf{n} \geq 1$.

3: **x**[**m** × **n**] – double *Input/Output*
4: **y**[**m** × **n**] – double *Input/Output*

On entry: the real and imaginary parts of the complex data values must be stored in arrays **x** and **y** respectively. Each row of the data must be stored consecutively; hence if the real parts of z_{j_1,j_2} are denoted by x_{j_1,j_2}, for $j_1 = 0, 1, \ldots, m-1$, $j_2 = 0, 1, \ldots, n-1$, then the mn elements of **x** must contain the values

$$x_{0,0}, x_{0,1}, \ldots, x_{0,n-1}, x_{1,0}, x_{1,1}, \ldots, x_{1,n-1}, \ldots, x_{m-1,0}, x_{m-1,1}, \ldots, x_{m-1,n-1}.$$

The imaginary parts must be ordered similarly in **y**.

On exit: the real and imaginary parts respectively of the corresponding elements of the computed transform.

5: **trigm**[2 × **m**] – const double *Input*
6: **trign**[2 × **n**] – const double *Input*

On entry: **trigm** and **trign** must contain trigonometric coefficients as returned by calls of nag_fft_init_trig (c06gzc). nag_fft_2d_complex (c06fuc) performs a simple check to ensure that both arrays have been initialized and that they are compatible with **m** and **n**. If $m = n$ the same array may be supplied for **trigm** and **trign**.

7: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_C06_NOT_TRIG

Value of **m** and **trigm** array are incompatible or **trigm** array not initialized.

Value of **n** and **trign** array are incompatible or **trign** array not initialized.

NE_INT_ARG_LT

On entry, **m** = ⟨*value*⟩.
Constraint: **m** ≥ 1.

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 1.

7 Accuracy

Some indication of accuracy can be obtained by performing a subsequent inverse transform and comparing the results with the original sequence (in exact arithmetic they would be identical).

8 Further Comments

The time taken is approximately proportional to $mn\log(mn)$, but also depends on the factorization of the individual dimensions m and n. The function is somewhat faster than average if their only prime factors are 2, 3 or 5; and fastest of all if they are powers of 2; it is particularly slow if m or n is a large prime, or has large prime factors.

9 Example

This program reads in a bivariate sequence of complex data values and prints the two-dimensional Fourier transform. It then performs an inverse transform and prints the sequence so obtained, which may be compared to the original data values.

9.1 Program Text

```
/* nag_fft_2d_complex (c06fuc) Example Program.
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 * Mark 8 revised, 2004.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagc06.h>

int main(void)
{
  Integer  exit_status = 0, i, j, m, n;
  NagError fail;
  double   *trigm = 0, *trign = 0, *x = 0, *y = 0;

  INIT_FAIL(fail);

  printf("nag_fft_2d_complex (c06fuc) Example Program Results\n");
  /* Skip heading in data file */
  scanf("%*[^\n]");
  while (scanf("%ld%ld", &m, &n) != EOF)
    {
      if (m*n >= 1)
        {
          if (!(trigm = NAG_ALLOC(2*m, double)) ||
              !(trign = NAG_ALLOC(2*n, double)) ||
              !(x = NAG_ALLOC(m*n, double)) ||
              !(y = NAG_ALLOC(m*n, double)))
            {
              printf("Allocation failure\n");
              exit_status = -1;
              goto END;
            }
        }
      else
        {
          printf("Invalid m or n.\n");
          exit_status = 1;
          return exit_status;
        }
      printf("\n\nm = %2ld  n = %2ld\n", m, n);
      /* Read in complex data and print out. */
      for (j = 0; j < m; ++j)
        {
          for (i = 0; i < n; ++i)
            scanf("%lf", &x[j*n + i]);
          for (i = 0; i < n; ++i)
            scanf("%lf", &y[j*n + i]);
        }
      printf("\nOriginal data values\n\n");
      for (j = 0; j < m; ++j)
        {
          printf("Real");
          for (i = 0; i < n; ++i)
            printf("%10.4f%s", x[j*n + i],
                   (i%6 == 5 && i != n-1?"\n     ":""));
          printf("\nImag");
```

```
    for (i = 0; i < n; ++i)
      printf("%10.4f%s", y[j*n + i],
              (i%6 == 5 && i != n-1?"\n     ":""));
    printf("\n\n");
  }
/* Initialize trig arrays */
/* nag_fft_init_trig (c06gzc).
 * Initialization function for other c06 functions
 */
nag_fft_init_trig(m, trigm, &fail);
if (fail.code != NE_NOERROR)
  {
    printf("Error from nag_fft_init_trig (c06gzc).\n%s\n",
            fail.message);
    exit_status = 1;
    goto END;
  }

/* nag_fft_init_trig (c06gzc), see above. */
nag_fft_init_trig(n, trign, &fail);
if (fail.code != NE_NOERROR)
  {
    printf("Error from nag_fft_init_trig (c06gzc).\n%s\n",
            fail.message);
    exit_status = 1;
    goto END;
  }

/* Compute transform */
/* nag_fft_2d_complex (c06fuc).
 * Two-dimensional complex discrete Fourier transform
 */
nag_fft_2d_complex(m, n, x, y, trigm, trign, &fail);
if (fail.code != NE_NOERROR)
  {
    printf("Error from nag_fft_2d_complex (c06fuc).\n%s\n",
            fail.message);
    exit_status = 1;
    goto END;
  }

printf("\nComponents of discrete Fourier transforms\n\n");
for (j = 0; j < m; ++j)
  {
    printf("Real");
    for (i = 0; i < n; ++i)
      printf("%10.4f%s", x[j*n + i],
              (i%6 == 5 && i != n-1?"\n     ":""));
    printf("\nImag");
    for (i = 0; i < n; ++i)
      printf("%10.4f%s", y[j*n + i],
              (i%6 == 5 && i != n-1?"\n     ":""));
    printf("\n\n");
  }
/* Compute inverse transform */
/* nag_conjugate_complex (c06gcc).
 * Complex conjugate of complex sequence
 */
nag_conjugate_complex(m*n, y, &fail);
if (fail.code != NE_NOERROR)
  {
    printf("Error from nag_conjugate_complex (c06gcc).\n%s\n",
            fail.message);
    exit_status = 1;
    goto END;
  }

/* nag_fft_2d_complex (c06fuc), see above. */
nag_fft_2d_complex(m, n, x, y, trigm, trign, &fail);
if (fail.code != NE_NOERROR)
  {
```

```
          printf("Error from nag_fft_2d_complex (c06fuc).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }

      /* nag_conjugate_complex (c06gcc), see above. */
      nag_conjugate_complex(m*n, y, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_conjugate_complex (c06gcc).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }

      printf("\nOriginal data as restored by inverse transform\n\n");
      for (j = 0; j < m; ++j)
        {
          printf("Real");
          for (i = 0; i < n; ++i)
            printf("%10.4f%s", x[j*n + i],
                    (i%6 == 5 && i != n-1?"\n     ":""));
          printf("\nImag");
          for (i = 0; i < n; ++i)
            printf("%10.4f%s", y[j*n + i],
                    (i%6 == 5 && i != n-1?"\n     ":""));
          printf("\n\n");
        }
 END:
      if (trigm) NAG_FREE(trigm);
      if (trign) NAG_FREE(trign);
      if (x) NAG_FREE(x);
      if (y) NAG_FREE(y);
    }
  return exit_status;
}
```

9.2 Program Data

```
nag_fft_2d_complex (c06fuc) Example Program Data
3  5
     1.000     0.999     0.987     0.936     0.802
     0.000    -0.040    -0.159    -0.352    -0.597
     0.994     0.989     0.963     0.891     0.731
    -0.111    -0.151    -0.268    -0.454    -0.682
     0.903     0.885     0.823     0.694     0.467
    -0.430    -0.466    -0.568    -0.720    -0.884
```

9.3 Program Results

```
nag_fft_2d_complex (c06fuc) Example Program Results


m =  3  n =  5

Original data values

Real    1.0000    0.9990    0.9870    0.9360    0.8020
Imag    0.0000   -0.0400   -0.1590   -0.3520   -0.5970

Real    0.9940    0.9890    0.9630    0.8910    0.7310
Imag   -0.1110   -0.1510   -0.2680   -0.4540   -0.6820

Real    0.9030    0.8850    0.8230    0.6940    0.4670
Imag   -0.4300   -0.4660   -0.5680   -0.7200   -0.8840


Components of discrete Fourier transforms
```

```
Real    3.3731    0.4814    0.2507    0.0543   -0.4194
Imag   -1.5187   -0.0907    0.1776    0.3188    0.4145

Real    0.4565    0.0549    0.0093   -0.0217   -0.0759
Imag    0.1368    0.0317    0.0389    0.0356    0.0045

Real   -0.1705   -0.0375   -0.0423   -0.0377   -0.0022
Imag    0.4927    0.0584    0.0082   -0.0255   -0.0829


Original data as restored by inverse transform

Real    1.0000    0.9990    0.9870    0.9360    0.8020
Imag   -0.0000   -0.0400   -0.1590   -0.3520   -0.5970

Real    0.9940    0.9890    0.9630    0.8910    0.7310
Imag   -0.1110   -0.1510   -0.2680   -0.4540   -0.6820

Real    0.9030    0.8850    0.8230    0.6940    0.4670
Imag   -0.4300   -0.4660   -0.5680   -0.7200   -0.8840
```