Adjoint Algorithmic Differentiation (AAD) supports smoothing through extensible adjoint of AAD. The operator-overloading tool dco/c++ produces a regularization effect similar to bump-continuities. Smoothing the indicator function acts up to machine precision and does not cap

Adjoint Algorithmic Differentiation

- \( y = f(x) \) with \( f: \mathbb{R}^n \to \mathbb{R}^m \) efficiently compute \( \partial f(x)/\partial x \)

**Uniform Distribution (Call Spread)**

\[
\begin{align*}
\partial^0 f [g > 0] & \approx \int_{-\infty}^{\infty} \mathbb{I}[g > 0] \cdot \frac{1}{z^2} \exp \left(-\frac{z^2}{2}\right) \, dz \\
\end{align*}
\]

**Normal Distribution**

\[
\begin{align*}
\partial^0 f [g > 0] & \approx \int_{-\infty}^{\infty} \mathbb{I}[g > 0] \cdot \frac{1}{z^2 \sqrt{2\pi}} \exp \left(-\frac{z^2}{2}\right) \, dz \\
\end{align*}
\]

**Barrier Option Monte Carlo (Case Study)**

Payoff for discretized path given in stochastic INF

\[
P(S_t, K, B, r, \delta) = \sum_{i=1}^{n} \mathbb{I}[B_i > 0 \land S_i - N_0 > 0 \land S_i - K > 0 \land (S_i - K)]
\]

with \( S_i = S_i(t_i, \sigma_i, X_i) \).

**Euler-Maruyama Path with dco/c++**

- \( \text{dco::ga1s<double>::global_tape->register_variable(sigma);} \)
- \( \text{dco::ga1s<double>::global_tape->interpret_adjoint();} \)

**Nearest Correlation Matrix (Case Study)**

- \( \text{P}\) of eigenvalues, \( \text{Q}\) of correlation matrix that is positive definite

\[
\left[ \begin{array}{cccc}
Q_{1,1} & \cdots & \cdots & Q_{1,n} \\
\vdots & \ddots & \vdots & \vdots \\
Q_{n,1} & \cdots & \cdots & Q_{n,n}
\end{array} \right]
\]

**Conclusions**

- Monte Carlo sensitivities for discontinuous payoffs
- Smoothing of auxiliary functions and implicit function theorem
- Local sensitivity enable bandwidth calibration
- dco/c++ supports extensible adjoint code patterns for regularization

Contact Information:
LuG Informatics 12, STCE
RWTH Aachen University
D-52056 Aachen, Germany
Phone: +49 (0)241 80 29224
Email: hueser@stce.rwth-aachen.de