Interest Rate Models: An ALM Perspective

Ser-Huang Poon
Manchester Business School

NAG Quant Day London 2008
Interest Rate Models: An ALM Perspective (with NAG implementation)

Ser-Huang Poon
Manchester Business School

Full paper: http://www.personal.mbs.ac.uk/ser-huang-poon/

Joint work with Eric Guan, Bing Gan and Aisha Khan
All postgraduate students at MBS
MSc QFFE (Quantitative Finance and Financial Engineering) Industry Linked Project
QFFE Course Structure

Semester I: 3 months
- Asset Pricing Theory
- Derivative Securities
- Stochastic Calculus
- VBA, C++

Optional units
- Portfolio Investment (Barra)
- Macro and International Investment
- Martingale Process

Semester II: 3 months
- Financial Econometrics
- Interest Rate & Credit Derivatives

Optional units
- Credit Risk Modelling & Management
- Computational Finance
- Mathematical Modelling of Finance
- Real Options

Summer Dissertation: June-July, Preparation starts from Spring Submission of thesis in first week of September
Project Remit

- Proposed by Group ALM of ABN AMRO in Amsterdam
- How sensitive is model choice to ALM outcome
  - Consider switching from BK to HW
  - Question if they should move to more sophisticated model: LMM or SMM
- No access to bank’s confidential data
- Four contesting models:
  - Hull-White (HW)
  - Black-Karasinski (BK)
  - Libor Market Model (LMM) and
  - Swap Market Model (SMM)
- Test 20 annual hedging performance of 10x1 Bermudan swaption of the single factor version of the four contesting models on EURO and USD.
Different Types of Interest Rates

- **Short rate**
  \[ dr = (\theta(t) - ar)dt + \sigma dz \]

- **Forward rate**
  \[ dL_i(t) = \mu_i(t)L_i(t)dt + \sigma_i(t)^T L_i(t)dW(t) \]
  \[ 0 \leq t \leq T_i, \quad i = 1, 2, \ldots, N \]

- **Libor rate**

- **Swap rate**
  - Fixed strike in exchange for a series of floating rates
  - Payer vs. Receiver

[No arbitrage condition must hold for all interest rates and all interest rate products]
# Cash Flows and Rates

<table>
<thead>
<tr>
<th></th>
<th>( t = 0 )</th>
<th>( t = 1 )</th>
<th>( t = 2 )</th>
<th>( \ldots )</th>
<th>( t = t )</th>
<th>( \ldots )</th>
<th>( t = T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_0 )</td>
<td>( f_0(0) )</td>
<td>( f_1(0) )</td>
<td>( f_2(0) )</td>
<td>( \vdots )</td>
<td>( f_T(0) )</td>
<td>( f_T(1) )</td>
<td>( f_T(2) )</td>
</tr>
<tr>
<td>( r_1 )</td>
<td>( f_1(1) )</td>
<td>( f_1(1) )</td>
<td>( f_1(1) )</td>
<td>( \vdots )</td>
<td>( f_{T+1}(0) )</td>
<td>( f_{T+1}(1) )</td>
<td>( f_{T+1}(2) )</td>
</tr>
<tr>
<td>( r_2 )</td>
<td>( f_2(2) )</td>
<td>( f_2(2) )</td>
<td>( f_2(2) )</td>
<td>( \vdots )</td>
<td>( f_{T+2}(0) )</td>
<td>( f_{T+2}(1) )</td>
<td>( f_{T+2}(2) )</td>
</tr>
<tr>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \vdots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( r_T )</td>
<td>( r_T )</td>
<td>( r_T )</td>
<td>( r_T )</td>
<td>( \vdots )</td>
<td>( r_T )</td>
<td>( r_T )</td>
<td>( r_T )</td>
</tr>
</tbody>
</table>

\( S_0, T(0) \) | \( S_1, T(0) \) | \( S_1, T(1) \) | \( S_1, T(2) \) | \( \vdots \) | \( S_{T-1, T}(0) \) | \( S_{T-1, T}(1) \) | \( S_{T-1, T}(2) \) | \( \ldots \) | \( S_{T-1, T}(t) \) | \( \ldots \) | \( S_{T-1, T}(T) \)
Plain Vanilla Products

• Yield curve, Y(T)
• Bond prices
• Forward rate agreement (FRA)
• Caplet, floorlet, cap, floor
• Swap, forward starting swap
• Swaption: European vs. Bermudan
Price quotation in Black Vol

- Only for European (ATM) products and under the terminal measure
- Bloomberg gives implied vol for Gaussian model also

\[
PS_{\text{swaption}}_{t,T} = \delta \sum_{j=n+1}^{N+1} B_{t,T_j} \left[ S(t, T_j, T_N) N(d_1) - kN(d_2) \right]
\]
Bermudan Swaption

- Constant maturity swap vs. Fixed tail swap
- Fixed tail swap resembles home mortgage with prepayment feature
- Fixed tail swap can be hedged using 10x1 co-terminal European swaptions
Choice of Calibrated Instruments

- Guided by no-arbitrage or hedging trades
- Rates fixed using bond prices or FRA
- Volatility fixed using caplet or European swaptions
- The choice of calibrated instruments determines the complexity of optimisation procedure and the time needed to reach convergence.
Short Rate Trinomial Tree

\[ dr = (\theta(t) - ar)dt + \sigma dz \]

- Time step is 0.1 year, 11year=110 time steps
- \( \theta(t) \) calibrated to bond prices
- Mean reversion rate \( a(t) \rightarrow a \), estimated from time series data
- \( \sigma(t) \) follows 3-point interpolation
Simulate LMM and SMM under $T(n+1)$ measure

\[
\frac{dL_i(t)}{L_i(t)} = - \sum_{j=i+1}^{N} \frac{\delta_j L_j(t) \sigma_{ij}(t)}{1 + \delta_j L_j(t)} dt + \sigma_i(t) \cdot dW_{Q_{n+1}}^T(t)
\]

\[
dS_{n,M}(t) = S_{n,M}(t)\sigma_{n,M}(t) dW_t^M + \text{drift}
\]

\[
\mu_n = \sum_{i=n}^{M-2} \left( \tau_{i+1} \frac{C_{i+1,M}}{C_{M-1,M}} \rho_{n,i+1} S_{n,M} \sigma_{n,M} S_{i+1,M} \sigma_{i+1,M} \prod_{j=n+1}^{i} (1 + \tau_j S_{j,M}) \right)
\]
LMM & SMM Simulations

- For each path

\[
\begin{align*}
0 & \quad 1 & \quad 2 & \quad \ldots & \quad t & \quad \ldots & \quad T \\
L_T(0) & \quad L_T(1) & \quad L_T(2) & \quad \ldots & \quad L_T(t) & \quad \ldots & \quad L_T(T) \\
L_{2,T}(0) & \quad L_{2,T}(1) & \quad L_{2,T}(2) \\
L_{1,T}(0) & \quad L_{1,T}(1) \\
L_{0,T}(0)
\end{align*}
\]

- Delta $\delta = 1$ yr, 10,000 simulations, single factor model
- Time step $\delta$ is not crucial if predictor-corrector method is used to correct the drift
- Use NAG random number generator g05ddc
Approximation for early exercise decision

Path 1  \( \text{BSwn}_1(0) \) \( \cdots \) \( \text{BSwn}_1(T-1) \) \( \text{BSwn}_1(T) \)
2  \( \text{BSwn}_2(0) \) \( \cdots \) \( \text{BSwn}_2(T-1) \) \( \text{BSwn}_2(T) \)
\vdots
10,000  \( \text{BSwn}_{10,000}(0) \) \( \cdots \) \( \text{BSwn}_{10,000}(T-1) \) \( \text{BSwn}_{10,000}(T) \)

\[
\begin{align*}
E_0[\tilde{\text{BSwn}}(1)] \ ? \\
\max(\tilde{C}_T, E) \leftarrow (S_1 - X)^+ \\
C_0 \leftarrow \max(\tilde{C}_T, E) \leftarrow (S_2 - X)^+ \\
\end{align*}
\]

\[
E_T[\tilde{\text{BSwn}}(T)] \ ?
\]

Longstaff-Schwartz (2001) approx for lower bound

\[
E_T[\tilde{Z}_{t+1}] = a + bS_t + c(S_t)^2
\]

- Call regression routine \texttt{g02dac}
Calibration

1. Convert Black Vol into European swaption prices using \texttt{g01eac} (probs for std normal distribution).

2. Build short rate HW, estimate \( \sigma_0, \sigma_3 \) and \( \sigma_{11} \) by minimising RMSPE of 10 co-terminal European swaptions using NAG \texttt{e04unc} (solving nonlinear least square using sequential QP).

3. Use the calibrated model to calculate 10x1 Bermudan swaption prices by stepping backward through the tree.

4. Build short rate BK; repeat steps 2-3 for lognormal rate.
Calibration (cont’d)

5. LMM calibrated to European swaption

Use Rebonato’s approximation of swaption vol

\[
\left( \nu_{n,M}^{LFM} \right)^2 = \sum_{i,j=\alpha+1}^{\beta} \frac{\omega_i(0) \omega_j(0) L_i(0) L_j(0) \rho_{ij}}{T_n \left[ S_{n,M}(0) \right]^2} \int_0^{T_n} \sigma_i(t) \sigma_j(t) \, dt
\]

where \( \nu_{n,M}^{LFM} \) is a proxy for Black volatility \( \nu_{n,M}(T_n) \).

\[
\sigma_k = \phi_k \times \varphi_k(a, b, c, d) \\
\varphi_k(a, b, c, d) = \left[ a(T_k - t) + b \right] e^{-c(T_k - t)} + d
\]

Call Nag e04unc to minimise vol errors for 10 rates instead of minimising 10 pricing errors using 10,000 simulation for each loop.
Calibration (cont’d)

(Cont’d LMM)
- 10,000 simulation (g05ddc) to calculate European swaption prices,
- 10 regressions (each with 10,000 observations, g02dac) to derive Bermudan prices.

6. SMM; same as step 5 but for SMM. Need additional step to solve simultaneous equations (f07adc LU factorization, and f07aec solution of real sys of linear equations) in order to derive the discount factor.

7. Repeat steps 2-6 for pricing 9x1 swaption.

8. Repeat steps 2-7 20 times through the sample period.

9. Do it once for Euro rate; repeat the whole process for USD interest rate; supposed to calibrate to Brazilian data, but no appropriate market data.
11-Y Bermudan Swaption Prices

11Y Bermudan Swaption Prices (EUR)

11Y Bermudan Swaption Prices (USD)
10-Y Bermudan Swaption Prices

10-Y Bermudan Swaption Prices (EUR)

10-Y Bermudan Swaption Prices (USD)
RMSE (Pricing Error) by Date

EUR

USD

NAG Quant Day London 2008
RMSE (Pricing Error) by Contract

EUR

USD

[Graphs showing RMSE by contract for EUR and USD with categories HW, BK, SMM, LMM]
Yield Curves

Maturity

Yield (%)
Use NAG g03aac to carry out PCA analysis
Delta Hedging

- “Bumping” the yield curve by perturbing the first three PCA factors by ± mean absolute change.

- Assume no change in interest rate volatility.

- Calculate price sensitivities (delta) of 10x1 Bermudan swaption, and the hedge instruments (swaps, 1-, 5- and 11-year)

- Form a delta hedged portfolio at t i.e. Σ delta=0. Use NAG e04ccc (unconstrained min using simplex algorithm).

- Unwind at t+1 and use price information of 9x1 Bermudan swaption and swaps (0-, 4- and 10-year) to calculate portfolio value at t+1.

- Calculate P&L; good model should give P&L=0.
P&L Analysis

- nag_opt_simplex (e04ccc) is used
- RMSE of Hedging P&L

<table>
<thead>
<tr>
<th></th>
<th>HW</th>
<th>BK</th>
<th>SMM</th>
<th>LMM</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUR</td>
<td>1.486%</td>
<td>1.751%</td>
<td>1.886%</td>
<td>1.752%</td>
</tr>
<tr>
<td>USD</td>
<td>5.544%</td>
<td>5.484%</td>
<td>5.701%</td>
<td>7.754%</td>
</tr>
</tbody>
</table>
P&L of Hedging Results

EUR

USD

-0.01
0.00
0.01
0.02
0.03
0.04
0.05

Feb/06 May/06 Aug/06 Nov/06 Feb/07 May/07 Aug/07

-0.02
-0.01
0.00
0.01
0.02
0.03
0.04

Feb/06 May/06 Aug/06 Nov/06 Feb/07 May/07 Aug/07
Summary

- Pricing and hedging results are similar for all four models.

- HK~BK, and LMM~SMM

- For 1-factor pricing and long term risk management, there is little to choose between them. One should focus more on setting scenarios.

- Given 1-factor finding above, for ALM purpose, HW seems best; fast and better convergence property when compared with BK.
NAG Visual Studio C++

• Visual C++ Express 2005 - CLDLL07XL (version 8 is now available)

One time link for all current projects when you start working in the C++ studio
• Started Visual Studio, click on Tools -> Options; Expand 'Projects and Solutions' and click on VC++ Directories;
• Change the 'Show directories for:' box so that it refers to include files and add the following line
  C:\Program Files\Numerical Algorithms Group\CLDLL074X\include
• Change the 'Show directories for:' box to show library files and add the following line
  C:\Program Files\Numerical Algorithms Group\CLDLL074X\include
• Finally change the box to show Executable files and add the line
  C:\Program Files\Numerical Algorithms Group\CLDLL074X\include
• Then click on OK. Do this once for all projects.
C++ (Cont’d)

To compile:
• Right click on the project name and click on 'properties'.
• Expand 'Configuration Properties' and then 'Linker'.
• Now click on 'Input'.
• The resulting window will contain an empty 'additional dependencies' line. Add `nagc.lib` to this. You should now be able to compile.

Calling C-Nag library from C++ code
• The best way to start is to download the example programme from NAG website, and modify the programme to suit your application.
QFFE Industry Linked Project

- Recognition of the needs of financial practitioners. The course is structured to
  - 1) better prepare the students for life in market roles, and
  - 2) investigate real world issues through the dissertation.

- Invitation for small, self-contained industry relevant research topics
- Students are available for internship from September to December.
- Contributors include:
  - Alliance and Leicester,
  - Standard Bank
  - Morgan Stanley
  - ABN AMRO
  - Shell
  - Numerical Algorithms Group
  - Riskmetrics