Risk modelling, portfolio optimisation and performance backtest

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NAG Quant Day – London 2008
21 February 2008
Backtest – **does the skill justify the fees?**

- What are the principal drivers of *OUT*-performance?
- How is performance affected by practicalities?
- Do I invest according to my selection (macro and micro)?
- What affects my performance – alpha or beta?
- Am I too risk avert?
- Is my constraint too tight?
- Ultimately: Have I been good or lucky?
Introduction

Contents

♦ Performance factor backtest (almost like a style)
♦ Optimisation – problems and solutions
♦ Brief introduction into the PAS risk model
♦ Risk and return attribution
♦ Optimised backtest

Publications:

• Optimisation with Alphas, D. Jessop, et al. UBS – March 2005
• European return attribution – Nick Nelson et al. March 2006
Sales to EV as performance factor – sector neutral

Equally Weighted
Monthly re-balancing

Source UBS
Sales to EV as performance factor – sector bias

Equally Weighted - Monthly re-balancing

Source UBS
Rebalance impact

Equally weighted, quarterly re-balance

Source UBS
Weighting impact

Market Cap Weighted – monthly rebalancing

Source UBS
Optimisation: Problem definition and constraints

♦ Maximise objective function

\[ U = \sum_{i} \alpha_i (w_i - b_i) - \lambda \sum_{i,j} (w_i - b_i) \Omega_{ij} (w_j - b_j) \]

subject to constraints of

1. Long only portfolio – positive portfolio weights
2. Limited turnover (transaction costs)
3. Lower and upper limits of holdings
4. Maximum number of stocks
5. Minimum and maximum exposure to industry, country, style and market risk
Why is optimisation hard?

♦ Optimisation would be easy except for the integer restrictions we put on the solution
  – The most common of these is of the form “give me a 100 stock portfolio from the S&P 500”
  – This could be generalised to limit the number of assets, including options etc. in a fund.
  – There can be other integer constraints – e.g. limit the number of trades, threshold constraints

♦ It is impossible to solve these problems exactly for any reasonable size problem: in choosing 100 stocks from 500 there are around $2 \times 10^{107}$ possible combinations of stocks (if there are no constraints).
The efficient frontier with discrete constraints

Optimisation in practice

The optimisation algorithm – Mixed Integer Programming

– uses the factor risk model

– adds discrete decision variables to represent integer constraints

– splits the risk aversion parameter $\lambda$ into systematic and specific, thus allowing separate control of active and passive risk

– uses penalty function to soften the boundaries of holding and turnover constraints
Backtest with optimisation

- Read in initial portfolio (benchmark is a possible choice for a start)
- Read in candidate set and user alphas
- If user alphas are to be combined with other views use mixed estimation capability for outperformance forecast – was not used in the present example
- Estimate risk model for the time of portfolio construction, based on 60 month stock and factor return data (prior to the portfolio date).
- Pass to the optimiser program risk information (stock specific risk, factor covariance matrix, stock betas).
- Set up boundaries of holdings, turnover and exposure
- Run the optimisation
- Calculate optimised portfolio return, tracking error, turnover, MCAR’s …
- Use solution portfolio as initial for the next rebalancing
The mathematics of intuition — *the matrix of returns*

- Asset returns vary with time and across market.
- Algebraic representation – a matrix – an array of values each identified by a time and cross-sectional label
- Investment history is an analogous array of portfolio weights selected by an investor. A skilful investor has selected the right assets at the right times, thus covering the returns with a winning array of portfolio weights.
- Time-axis continuity and its definite direction of causality create an asymmetry in the way asset returns are analysed in time and the cross-sectional direction. This asymmetry is further enhanced by human disposition to treat historical sample as an adequate representation of ensemble of probabilities.

Is the mean of last sixty monthly returns a better estimate of the next month’s return than the mean of the last month’s returns of all the stocks in the market?
The mathematics of intuition – *in search of simplicity*

♦ Linear model: small change in factors results in small change in stock returns

\[
\begin{pmatrix}
r_{11} & \cdots & r_{1N} \\
r_{21} & \cdots & \vdots \\
\vdots & \ddots & \vdots \\
r_{T1} & \cdots & r_{TN}
\end{pmatrix}
\begin{pmatrix}
f_{11} & \cdots & f_{1K} \\
f_{21} & \cdots & \vdots \\
\vdots & \ddots & \vdots \\
f_{T1} & \cdots & f_{TK}
\end{pmatrix}
\begin{pmatrix}
\beta_{11} & \cdots & \beta_{1N} \\
\beta_{21} & \cdots & \vdots \\
\vdots & \ddots & \vdots \\
\beta_{K1} & \cdots & \beta_{KN}
\end{pmatrix}
\begin{pmatrix}
\varepsilon_{11} & \varepsilon_{12} & \cdots & \varepsilon_{1N} \\
\varepsilon_{21} & \cdots & \vdots \\
\vdots & \ddots & \vdots \\
\varepsilon_{T1} & \cdots & \varepsilon_{TN}
\end{pmatrix}
\]

♦ Factors can be regarded as aggregated asset returns. In what follows, we argue, however, that the widely available market indices do not represent adequately risk factors.
The mathematics of intuition — time series vs cross sectional

♦ Time series regression

\[
\begin{pmatrix}
  r_{1,p} \\ r_{2,p} \\ \vdots \\ r_{T,p}
\end{pmatrix}
= \begin{pmatrix}
  1 \\ 1 \\ \vdots \\ 1
\end{pmatrix}
+ \begin{pmatrix}
  \alpha_p \\ \beta_{1,p} \\ \vdots \\ \beta_{N,p}
\end{pmatrix}
\begin{pmatrix}
  f_{1,1} \\ f_{2,1} \\ \vdots \\ f_{T,1}
\end{pmatrix}
+ \begin{pmatrix}
  \beta_{1,p} \\ \beta_{2,p} \\ \vdots \\ \beta_{N,p}
\end{pmatrix}
\begin{pmatrix}
  f_{1,2} \\ f_{2,2} \\ \vdots \\ f_{T,2}
\end{pmatrix}
+ \cdots
\begin{pmatrix}
  \beta_{1,p} \\ \beta_{2,p} \\ \vdots \\ \beta_{N,p}
\end{pmatrix}
\begin{pmatrix}
  f_{1,K} \\ f_{2,K} \\ \vdots \\ f_{T,K}
\end{pmatrix}
\]

• Presumed factors
• Large number (NxK) of estimated parameters – betas, N regressions
• Smaller out of sample errors

♦ Cross sectional regression

\[
\begin{pmatrix}
  r_{t,1} \\ \vdots \\ r_{t,N}
\end{pmatrix}
= \begin{pmatrix}
  \alpha_1 \\ \vdots \\ \alpha_N
\end{pmatrix}
+ \begin{pmatrix}
  \beta_{1,1} \\ \vdots \\ \beta_{1,N}
\end{pmatrix}
\begin{pmatrix}
  f_{t,1}
\end{pmatrix}
+ \begin{pmatrix}
  \beta_{2,1} \\ \vdots \\ \beta_{2,S}
\end{pmatrix}
\begin{pmatrix}
  f_{t,2}
\end{pmatrix}
+ \cdots
\begin{pmatrix}
  \beta_{K,1} \\ \vdots \\ \beta_{K,N}
\end{pmatrix}
\begin{pmatrix}
  f_{t,K}
\end{pmatrix}
\]

• Presumed loadings (betas)
• Smaller number (TxK) of estimated parameters – factor returns, T regressions
• Larger out of sample errors
• Quicker response to changes in constituents.

♦ Matrix algebra offers a third method of regression, based on Principal Component Analysis (PCA) or similar techniques. This method does not presume risk factors or loadings, it uses only the returns' matrix.
Limitations common to all three regression methods

All three methods, different as they appear, require (approximate at least) normality of the return distribution.

All three regressions do not take into account possible change of the loading during the sampled period.

Unproven (and sometimes plainly non-existent) adequacy of historical sampling is a shortcoming of all three methods.
Estimating UBS Country Sector Model

– The objective – time series regression for reliable risk attribution.
– It requires a set of reliable factor returns

✦ Be aware of the shortcomings of the readily available market cap weighted indices:
  • possible spurious correlations with local markets (e.g. UK oil companies, Pharmaceuticals and Banks in Switzerland, Nokia in Finland, until not long ago Nortel in Canada)
  • business conducted across market borders and yet multinationals are listed somewhere
  • Industry composition bias – practically all markets. Bias is created by dominant constituents and dominant markets. It is not an artifact, but a reflection of economic reality. Needs to be accounted for in controlled manner.

**In our model we use** *Global Market, 10 Global Sector and 8 Local Market factors*

✦ We estimate the factors in an iterative procedure; each iteration a two-step constrained least square regression
  ♦ Cross sectional estimate of factor returns
  ♦ Time series estimate of loadings

**NAG chapters used:** e04, f06, g02, m01
Estimated factors and ICB indices ex-Global Market

Oil and Gas and Technology – estimated *Global Sector risk* factor is close to the ICB index of largely global industries. Financials – the estimated *Global Sector* factor does not include sector index gains due to local effects (e.g. interest rates). US – the estimated *Local Market* factor distinguishes between the performance of US and the global slowing down.

Source: UBS
We impose a set of fairly intuitive constraints:

- market cap weighted sum of Global Market betas = 1 (World’s beta with Global Market =1)
- analogous constraints for industry and regional constituents (e.g. US equity beta with respect to US Local Market =1)
- market cap weighted sum of alpha (free terms) = 0 (no self-outperformance)
- market cap weighted sum of excess Global Sector returns=market cap weighted sum of excess Local Market returns=0
- excess Global Sectors returns, excess Local Markets returns orthogonal to Global Market returns
Risk Model for Portfolio Analysis System (PAS)

The global country-sector model is used in the Portfolio Analysis System (PAS). Upon establishing the factor (Global Market, Global Sector and Local Market) returns time series, stock loadings with respect to these factors are estimated.

$$Var(P) = \sum_{i,j \in P} w_i w_j \sum_{qp} \beta_{q,i} \beta_{p,j} Var(f)_{qp} + \Delta$$

$w_i$ - weight of the stock $i$ in a portfolio, benchmark or active

$Var(f)_{qp}$ - factor covariance matrix

$\Delta$ - share of the variance unexplained by the model

$$MCAR_i = \frac{\partial(T.E.)}{\partial w_i} = \frac{\sum_{j} \Omega_{ij} (w_j - w_{Bj})}{\sqrt{\left(\sum_{j'j} (w_{j'} - w_{Bj'}) \Omega_{j'j} (w_j - w_{Bj})\right)}}$$
Sectors, Countries, Styles

\[
\begin{align*}
\text{Var}(P)_{\text{Sector}} &= \sum_{ij \in P} \left[ w_i w_j \sum_{S,S'} \beta_{i,S} \beta_{j,S'} \Phi_{S,S'} \right] \\
\text{Var}(P)_{\text{Country}} &= \sum_{ij \in P} \left[ w_i w_j \sum_{C,C'} \beta_{i,C} \beta_{j,C'} \Phi_{C,C'} \right] \\
\text{Var}(P)_{\text{Style}} &= \sum_{ij \in P} \left[ w_i w_j \sum_{F,F'} \beta_{i,F} \beta_{j,F'} \Phi_{F,F'} \right] \\
\text{Var}(P) &= \text{Var}(P)_{\text{Factor}} + \text{Var}(P)_{\text{Specific}} \\
\text{Var}(P)_{\text{MKT}} &= \text{Var}(P)_{\text{Factor}}
\end{align*}
\]
Return Attribution - Germany

Source UBS
Return Attribution - Sweden

Source UBS
Risk attribution – Sweden, growing impact of Europe

Source UBS
## Optimisation result (a fraction)

<table>
<thead>
<tr>
<th>Sedol</th>
<th>Portfolio</th>
<th>Benchmark</th>
<th>Initial</th>
<th>Solution</th>
<th>Active</th>
<th>Alpha</th>
<th>Beta</th>
<th>wrmcar</th>
<th>Buy</th>
<th>Sell</th>
<th>AvTradVolMln</th>
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<td>0</td>
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<td>2026242</td>
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<td>100</td>
<td>77.83524</td>
</tr>
</tbody>
</table>

Source: UBS
Portfolio performance – US, Dividend Yield alpha

Source: UBS
Portfolio performance – US, Earnings Yield alpha

Source: UBS
Portfolio performance – US, ROE alpha

Source: UBS
Portfolio tracking errors – US, Dividend Yield alpha

Source: UBS
Portfolio tracking errors – US, Earnings Yield alpha

Source: UBS
Turnover – US, Dividend Yield alpha

Source: UBS
Turnover – US, Earnings Yield alpha

Source: UBS
Portfolio Alpha Ratio Optimised to Top 50 Alpha

Does the fund follow the alpha? – correlate weights with alpha
or
measure fund’s alpha ratio of top 50 alpha stock portfolio

Source: UBS
Earnings Yield Alpha – high risk gains in performance

Source: UBS
Earnings Yield Alpha – small count portfolio risk

Source: UBS
High and Low risk implied alpha

solution weights and implied alphas – rank correlation

Max unconstrained Utility  \[ \Rightarrow \alpha_I = \lambda C_{ij} w_j \]

Source: UBS
IR for constrained and unconstrained holdings

IR = active return/TE

Source: UBS
The cost of constraint

Source UBS
Summary

• Initial backtest of performance drivers required for top – bottom selection and alpha generation.
• Rebalancing frequency, industry tilt, weighting scheme, all can affect performance.
• To examine true performance of a fund we need to include risk aversion and holding constraints in the backtest – optimise at each rebalancing.
• Solution analysis: Fundamental statistics – tracking error; information ratios; active position; principal contributors to the tracking error and to outperformance holding.
  
  Does the fund follow my strategist recommendation?
  – Did I pick up the stocks with high beta to expected outperforming factors?
  Do I follow my stock selector recommendation?
  – Did I pick up the stocks with high alpha?
  – How is my performance affected by constraints?
• Alpha fund - make sure the return forecasts are represented in the holdings – beware of over-constraining and of excessive risk aversion.
Required Disclosures

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**UBS Investment Research: Global Equity Rating Allocations**

<table>
<thead>
<tr>
<th>UBS 12-Month Rating</th>
<th>Rating Category</th>
<th>Coverage</th>
<th>IB Services</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy</td>
<td>Buy</td>
<td>55%</td>
<td>39%</td>
</tr>
<tr>
<td>Neutral</td>
<td>Hold/Neutral</td>
<td>36%</td>
<td>36%</td>
</tr>
<tr>
<td>Sell</td>
<td>Sell</td>
<td>8%</td>
<td>20%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>UBS Short-Term Rating</th>
<th>Rating Category</th>
<th>Coverage</th>
<th>IB Services</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy</td>
<td>Buy</td>
<td>less than 1%</td>
<td>25%</td>
</tr>
<tr>
<td>Sell</td>
<td>Sell</td>
<td>less than 1%</td>
<td>50%</td>
</tr>
</tbody>
</table>

1: Percentage of companies under coverage globally within the 12-month rating category.
2: Percentage of companies within the 12-month rating category for which investment banking (IB) services were provided within the past 12 months.
3: Percentage of companies under coverage globally within the Short-Term rating category.
4: Percentage of companies within the Short-Term rating category for which investment banking (IB) services were provided within the past 12 months.

Source: UBS. Rating allocations are as of 31 December 2007.

**UBS Investment Research: Global Equity Rating Definitions**

<table>
<thead>
<tr>
<th>UBS 12-Month Rating</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy</td>
<td>FSR is &gt; 6% above the MRA.</td>
</tr>
<tr>
<td>Neutral</td>
<td>FSR is between -6% and 6% of the MRA.</td>
</tr>
<tr>
<td>Sell</td>
<td>FSR is &gt; 6% below the MRA.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>UBS Short-Term Rating</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy</td>
<td>Buy: Stock price expected to rise within three months from the time the rating was assigned because of a specific catalyst or event.</td>
</tr>
<tr>
<td>Sell</td>
<td>Sell: Stock price expected to fall within three months from the time the rating was assigned because of a specific catalyst or event.</td>
</tr>
</tbody>
</table>
Definitions & exceptions

KEY DEFINITIONS

Forecast Stock Return (FSR) is defined as expected percentage price appreciation plus gross dividend yield over the next 12 months.

Market Return Assumption (MRA) is defined as the one-year local market interest rate plus 5% (a proxy for, and not a forecast of, the equity risk premium).

Under Review (UR) Stocks may be flagged as UR by the analyst, indicating that the stock's price target and/or rating are subject to possible change in the near term, usually in response to an event that may affect the investment case or valuation.

Short-Term Ratings reflect the expected near-term (up to three months) performance of the stock and do not reflect any change in the fundamental view or investment case.

EXCEPTIONS AND SPECIAL CASES

US Closed-End Fund ratings and definitions are:
- Buy: Higher stability of principal and higher stability of dividends;
- Neutral: Potential loss of principal, stability of dividend;
- Reduce: High potential for loss of principal and dividend risk.

UK and European Investment Fund ratings and definitions are:
- Buy: Positive on factors such as structure, management, performance record, discount;
- Neutral: Neutral on factors such as structure, management, performance record, discount;
- Reduce: Negative on factors such as structure, management, performance record, discount.

Core Banding Exceptions (CBE): Exceptions to the standard +/-6% bands may be granted by the Investment Review Committee (IRC). Factors considered by the IRC include the stock's volatility and the credit spread of the respective company's debt. As a result, stocks deemed to be very high or low risk may be subject to higher or lower bands as they relate to the rating. When such exceptions apply, they will be identified in the Companies Mentioned or Company Disclosure table in the relevant research piece.