Multi-Regime Analysis

Applications to Fixed Income
This research has been done in collaboration with my friend, Thierry F. Bollier, who was the first to recognize the relevance of MRPA to financial markets and to explore its applications.

Working Papers:
Bollier (2009)
Bollier, Hedges and Schwartz (2010)
Bollier and Hipes (2011)
Latent Factor Models Are Nice

- Appropriate for strongly correlated assets with nearly uncorrelated idiosyncratic noise
- Familiar and useful model for market participants and econometricians
- E.g., stock market "industry factors"
- Equivalent to PCA when the noise is uniform

\[ \tilde{r} = \mu + W\tilde{y} + \sigma\tilde{e} \]
PC1: a “level” shift of the Libor curve – almost parallel move of all rates over 2 years

PC2: a “slope” shift – a flattening of the 2y-10y and 2-30y slope

PC3: a “curvature” shift – a tightening of the belly (2 to 7 years) versus the wings

Litterman and Scheinkman (1991)
Nice, But *Unrealistic*

- \( \hat{y} \sim N(0, I) \quad \hat{\varepsilon} \sim N(0, I) \)
- Assumptions of zero means and unit variances for the factors are harmless
- Assumption of *normal* factors is unrealistic
- Nevertheless it’s simple & entrenched
- Empirical distribution of a single financial variable
  - 6M LIBOR & A Hedge Fund Index
Gaussian Mixtures

Single Gaussian

Four Gaussian Mixture

Better Fit to Center
Gaussian Mixtures – Left Hand Tail

Notice the Scale

Single Gaussian

Four Gaussian Mixture

Better Fit to Tail
A SINGLE ASSET EXAMPLE (BOLLIER, ET. AL. 2010)

As shown in the distribution of the CS/Tremont HF index (April 1994 to January 2010), realized returns do not display the symmetry of a standard Gaussian distribution. Standard central metrics like mean and volatility cannot capture this complexity, which suggests that observed returns may be generated by two or more overlapping distributions each with distinct mean and volatility.

In this univariate example, MRPA has identified three underlying ‘basis’ distributions: Quiet, High Volatility (“HiVol”) and Crisis. The composite or ‘Mixture’ distribution (shown in the solid yellow line in Graphs 2 and 3 below) is the weighted average of the basis distributions using mixing probabilities: 70% for Quiet, 25% for HiVol, and 5% for Crisis.
The proposition that markets display regime changes is intuitive.

The combination of Gaussian linear factor models with multiple regimes yields a tractable mixture model for returns.

Providing greater distributional accuracy.

Incorporating correlations, fat tails and skew.
Fix a time interval: daily changes in USD swap rates

During each interval the market is resident in a specific regime where a single factor model is operational

The regimes are hidden, but ex post, we will estimate their posterior probability from ML

Latent variable models and the EM algorithm have a Bayesian flavor, including the concepts of priors, Bayesian likelihood, and posteriors
MRPA in Symbols

- Sequence of market (observed) returns
  \[ \{r_1, r_2, \ldots, r_T\}, \quad r_t \in \mathbb{R}^N \]

- Sequence of regimes
  \[ \tilde{x}(t) \in \{1, 2, \ldots, M\} \]

- Factor model for regime, \( m \)
  \[ \tilde{r}_m = \mu_m + W_m \tilde{y} + \sigma_m \tilde{\epsilon} \]
  \[ \tilde{y} \sim N(0, I) \]
MEANS AND PRINCIPAL COMPONENTS

Sample Means

PC1 in 4 and 3 Regime MRPA
PRINCIPAL COMPONENTS (CONT’D)

PC2 in 4 and 3 Regime MRPA

PC3 in 4 and 3 Regime MRPA
Regime Switching Dynamics

• How does past regime-residence history influence next period's regime draw?

• Vague Prior Hypothesis:

• IID regime switching: \( \tilde{x}(t) \sim \pi_m \)

• Analogous to the vague prior hypothesis in Bayesian statistics

• Motivation: In practice, priors have little influence on posteriors, so it's a weak assumption (Bartholomew & Knott, 1994)
The EM Iteration

Log-likelihood

\[ \mathcal{L} = \sum_{t=1}^{T} \ln \left\{ \sum_{m=1}^{M} \pi_m \cdot p(r_t|m) \right\} \]

Bayes’ Theorem

\[ p(m|r_t) = \frac{p(r_t|m) \pi_m}{p(r_t)} \]

Update priors

\[ \hat{\pi}_m = \frac{1}{T} \sum_{t=1}^{T} p(m|r_t) \]

Update means

\[ \hat{\mu}_m = \frac{\sum_{t=1}^{T} p(m|r_t) r_t}{T \hat{\pi}_m} \]
Soft Clustering

ML inference assigns a *posterior* probability to each period.

It is the probability of residing in regime, $m$, during a given period, after observing the return for that period.

The *posterior* probabilities provide a soft clustering of the periods into regimes.

Typical of multi-regime (latent class) models.
POSTERIOR PROBABILITY: 4 REGIMES / 5 FACTORS

USD Libor Swaps - November 1997 to July 2010

<table>
<thead>
<tr>
<th></th>
<th>HiVol1</th>
<th>HiVol2</th>
<th>Quiet</th>
<th>Crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Priors</td>
<td>38.0%</td>
<td>7.1%</td>
<td>53.4%</td>
<td>1.4%</td>
</tr>
<tr>
<td>Vol from PCs</td>
<td>103 bps</td>
<td>158 bps</td>
<td>53 bps</td>
<td>220 bps</td>
</tr>
<tr>
<td>Vol from Residuals</td>
<td>0.3 bps</td>
<td>0.4 bps</td>
<td>0.3 bps</td>
<td>0.7 bps</td>
</tr>
</tbody>
</table>

Posterior mixing probabilities time series
Mixture of 4 regimes

Dates, for period: 1997-11-06 to 2010-07

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MONTE CARLO SIMULATION TEST

Generated returns using 4 regimes, 8 factors and a noise term.
Conditionally Stationary

- Market dynamics are highly non-stationary
- In MRPA, the non-stationary aspect has been isolated in the regime-switching process
- We cannot easily *anticipate* a crisis, because the underlying regime-process is complex, but we can easily recognize one when we are in it
- In other words, we make no attempt here to forecast regimes, only to characterize them
- Within a regime, *is* there evidence that the market returns are stationary?
Evidence: Sub-sample robustness

Mixing Frequency

<table>
<thead>
<tr>
<th>5 Factors</th>
<th>HiVol1</th>
<th>HiVol2</th>
<th>Quiet</th>
<th>Crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>11/1997 to 07/2010</td>
<td>38.0%</td>
<td>7.1%</td>
<td>53.4%</td>
<td>1.4%</td>
</tr>
<tr>
<td>11/1997 to 07/2004</td>
<td>45.4%</td>
<td>7.0%</td>
<td>46.5%</td>
<td>1.2%</td>
</tr>
<tr>
<td>07/2004 to 07/2010</td>
<td>32.1%</td>
<td>6.8%</td>
<td>58.9%</td>
<td>2.2%</td>
</tr>
</tbody>
</table>

Volatility from PCs (Bps)

<table>
<thead>
<tr>
<th>5 Factors</th>
<th>HiVol1</th>
<th>HiVol2</th>
<th>Quiet</th>
<th>Crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>11/1997 to 07/2010</td>
<td>103</td>
<td>158</td>
<td>53</td>
<td>220</td>
</tr>
<tr>
<td>11/1997 to 07/2004</td>
<td>97</td>
<td>157</td>
<td>52</td>
<td>128</td>
</tr>
<tr>
<td>07/2004 to 07/2010</td>
<td>109</td>
<td>151</td>
<td>54</td>
<td>239</td>
</tr>
</tbody>
</table>
Market history does repeat itself, only the “when” is hard. The benefit is that the factor models can be estimated using a full history. Richer collection of market events, better confidence intervals. Contrast with the common practice of sampling from rolling 2Y windows.
A NEW DIMENSION: TAIL RISK BY REGIME

95% Conditional VaR or Expected Tail Loss – Bps – 10 Day Horizon

<table>
<thead>
<tr>
<th></th>
<th>HiVol1</th>
<th>HiVol2</th>
<th>Quiet</th>
<th>Crisis</th>
<th>PCA – full sample</th>
<th>PCA - rolling 2y</th>
<th>MRPA - full sample</th>
<th>MRPA - rolling 2y</th>
</tr>
</thead>
<tbody>
<tr>
<td>USS6M</td>
<td>-22</td>
<td>-60</td>
<td>-6</td>
<td>-180</td>
<td>-26</td>
<td>-42</td>
<td>-41</td>
<td>-49</td>
</tr>
<tr>
<td>FLY.3M-2-10</td>
<td>-35</td>
<td>-121</td>
<td>-18</td>
<td>-439</td>
<td>-57</td>
<td>-97</td>
<td>-88</td>
<td>-111</td>
</tr>
<tr>
<td>FLY.2-5-10</td>
<td>-11</td>
<td>-18</td>
<td>-6</td>
<td>-37</td>
<td>-11</td>
<td>-10</td>
<td>-13</td>
<td>-14</td>
</tr>
<tr>
<td>FLY.7-10-20</td>
<td>-3</td>
<td>-6</td>
<td>-2</td>
<td>-9</td>
<td>-5</td>
<td>-5</td>
<td>-3</td>
<td>-5</td>
</tr>
<tr>
<td>FLY.2-10-20</td>
<td>-6</td>
<td>-10</td>
<td>-3</td>
<td>-17</td>
<td>-7</td>
<td>-11</td>
<td>-6</td>
<td>-10</td>
</tr>
<tr>
<td>FLY.2-10-30</td>
<td>-10</td>
<td>-15</td>
<td>-6</td>
<td>-25</td>
<td>-10</td>
<td>-15</td>
<td>-9</td>
<td>-14</td>
</tr>
<tr>
<td>CON-2-5-10-30</td>
<td>-7</td>
<td>-10</td>
<td>-4</td>
<td>-15</td>
<td>-7</td>
<td>-7</td>
<td>-6</td>
<td>-8</td>
</tr>
<tr>
<td>CON-5-7-10-20</td>
<td>-2</td>
<td>-3</td>
<td>-2</td>
<td>-7</td>
<td>-6</td>
<td>-3</td>
<td>-2</td>
<td>-3</td>
</tr>
</tbody>
</table>
General Observations

- Over 50% of the sample is characterized by zero mean, low volatility and tight confidence intervals (Quiet regime)
- A few percent of events are violent with large MM moves and wide confidence intervals (Crisis)
- EM estimation procedure is standard, fast and reliable
- Number of regimes is based on heuristics: insufficiently distinct PCs and the onset of multiple, shallow max during the EM iteration
- Portfolio risk computation is very, very fast
Cross-Market MRPA

Combine single-market MRPAs into an effective cross-market model

• Factor loadings are held fixed
• Joint prior distribution is estimated
• Factor correlations are estimated
• Motivation: External markets influence, but do not dominate, the individual-market marginal distributions

Reveals the level of dependence in cross-market regimes, which can be explored with contingency tables
With MxMRPA we can refine our estimate of regime probabilities in one market based on the conditions observed in the other market, with applications in dynamic Gap risk measurement and position margining.
In MxMRPA, the joint prior distribution is estimated from the historical swap and swaption data, permitting the extraction of regime coincidence information.

The probability of each Vol regime conditional on the Swap regime shows strong dependence of the Vol market on the Swap market, as expected.

<table>
<thead>
<tr>
<th>Joint Priors</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swaps</td>
<td>H1</td>
</tr>
<tr>
<td>H1</td>
<td>16.9%</td>
</tr>
<tr>
<td>Qt</td>
<td>14.8%</td>
</tr>
<tr>
<td>Cs</td>
<td>2.7%</td>
</tr>
<tr>
<td></td>
<td>34.4%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Conditional On Swaps</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swaps</td>
<td>H1</td>
</tr>
<tr>
<td></td>
<td>H1</td>
</tr>
<tr>
<td></td>
<td>Qt</td>
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<td></td>
<td>Cs</td>
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<td></td>
<td>34.4%</td>
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MRPA - Swaptions

ATM Mixture Model (MRPA) Distribution (1Y into 5Y EUR Swaption)
In MxMRPA we maintain the quantification of risk within each single market, but combine the regimes into global market regimes, while simultaneously capturing cross market correlations in regimes and their factors.

All single currency portfolios retain the same risk characteristics, whether viewed with a single-market or multi-market perspective.

As in single-regime MRPA, the model is particularly effective at identifying the empirical fat tails in the cross-market portfolio returns associated with financial crisis and high volatility regimes.
Heteroscedastic Noise

- Relax the constraint that every asset has the same level of idiosyncratic noise
- Desirable for heterogeneous asset collections
  - Stocks
  - Commodity Futures
  - CDS
- Initial Results
  - Fewer factors are indicated
  - Retains all the virtues of the original approach
Literature Sampling

- **Brigo, Damiano, Fabio Mercurio, Giuli Sartorelli [2003]**, *Alternative Asset-Price Dynamics and Volatility Smile*, Quantitative Finance 3(3), 173-183
- **Chan, Nicholas, Mila Getmansky, Shane M. Hass, and Andrew W. Lo [2007]**, *Systemic Risk and Hedge Funds in Risks of Financial Institutions*, Mark Carey and Rene Stulz, eds., University of Chicago, 2007
Conclusions

MRPA is a tractable multi-regime model with potential applications to a wide range of financial data sets

MRPA incorporates multi-asset correlations from the beginning

MRPA effectively characterizes regimes despite a lack of *a priori* information about the underlying regime-switching process

MRPA yields accurate distributions of asset and portfolio returns in fixed-income

Tail-risk calculations are fast, accurate and based on large data sets

Hierarchical, cross-market, MRPA models are conveniently assembled from single-market MRPAs

Heterogeneous asset collections are analyzed with heteroscedastic MRPA