Inhomogeneous Time Series

The G13 chapter of the NAG library contains a wide range of techniques for investigating and modelling the statistical structure of univariate and multivariate time series, that is, a series of observations collected at different points in time. In this discussion we will denote a generic univariate time series as a sequence of pairs of values (z_i, t_i), for $i = 1, 2, \dots$, where the z's represent an observed scalar value and the t's the time that the value was observed.

Prior to Mark 24, all of the methods included in G13 required the time series to be homogeneous, that is, the sampling times had to be equally spaced, with $t_i - t_{i-1} = \delta$ for all i and some value δ. In many real world applications this assumption does not hold and so the observed series is inhomogeneous. Standard time series analysis techniques cannot be used on an inhomogeneous series without first preprocessing the series to construct an artificial homogeneous series by, for example, resampling the series at regular intervals. At Mark 24 we have introduced a series of operators, suggested by Zumbach and Müller [1], that can be used to extract robust information directly from an inhomogeneous time series. In this context, robust information means that the results should be essentially independent of minor changes to the sampling mechanism used when collecting the data, for example, changing a number of time stamps or adding or removing a few observations.

The basic operator available for inhomogeneous time series is the exponential moving average (EMA). This operator has a single parameter, τ, and is an average operator with an exponentially decaying kernel given by:

$$\frac{e^{-t/\tau}}{\tau}.$$

which gives rise to the following iterative formula:

$$\mathrm{EMA}[\tau; z](t_i) = \mu \mathrm{EMA}[\tau; z](t_{i-1}) + (\nu - \mu) z_{i-1} + (1 - \nu) z_i$$

where

$$\mu = e^{-\alpha} \qquad \text{and} \qquad \alpha = \frac{t_i - t_{i-1}}{\tau}.$$

and the value of ν depends on the method of interpolation chosen. Given the EMA, a number of other operators can be defined, including:

i. ***m*-Iterated Exponential Moving Average**, defined as

$$\mathrm{EMA}[\tau, m; z] = \mathrm{EMA}[\tau; \mathrm{EMA}[\tau, m-1; z]] \text{ where } \mathrm{EMA}[\tau, 1; z] = \mathrm{EMA}[\tau; z].$$

ii. **Moving Average (MA)**, defined as

$$\mathrm{MA}[\tau, m_1, m_2; z](t_i) = \frac{1}{m_2 - m_1 + 1} \sum_{j=m_1}^{m_2} \mathrm{EMA}[\hat{\tau}, j; z](t_i) \text{ where } \hat{\tau} = \frac{2\tau}{m_2 + m_1}$$

iii. **Moving Standard Deviation (MSD)**, defined as

$$\mathrm{MSD}(\tau, m, p; z) = \mathrm{MA}[\tau, 1, m; |z - \mathrm{MA}[\tau, 1, m; z]|^p]^{1/p}$$

iv. **Differential (Δ)**, defined as

$$\Delta[\tau,\alpha,\beta,\gamma;z] = \gamma(\text{EMA}[\alpha\tau,1;z] + \text{EMA}[\alpha\tau,2;z] - 2\text{EMA}[\alpha\beta\tau,4;z])$$

v. **Volatility**, defined as

$$\text{Volatility}[\tau,\tau',m,p;z] = \text{MA}\left[\tau/2,1,m;|\Delta[\tau';z]|^p\right]^{1/p}$$

All of the above operators, and others, can be obtained by calling one or more of the three inhomogeneous time series routines introduced at Mark 24; G13MEF, G13MFF and G13MGF.

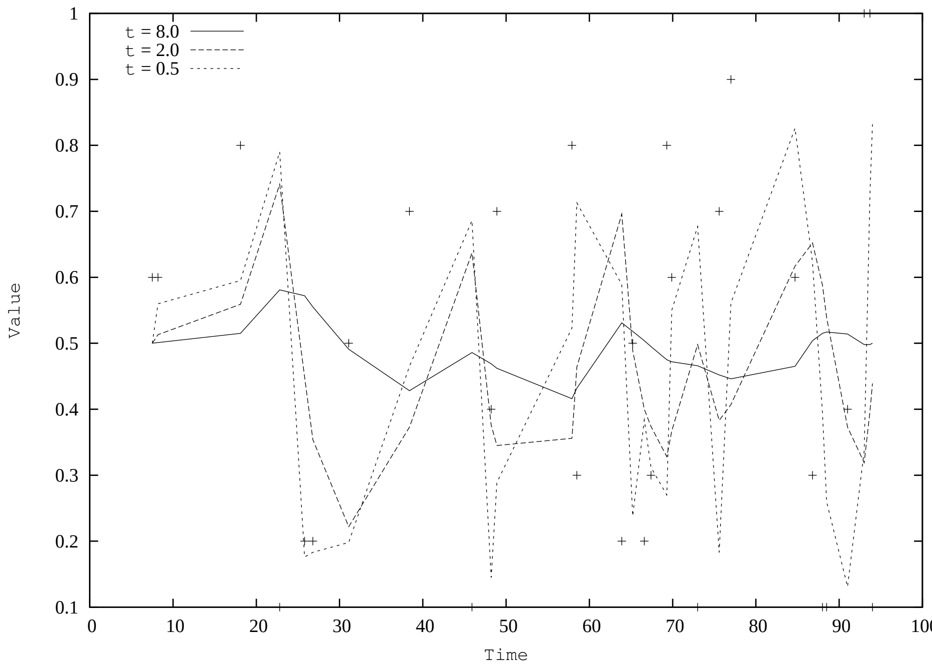

Figure 1: Simulated inhomogeneous times series and the corresponding EMA$[\tau, 2; y]$ showing the effect of altering the decay rate parameter τ

References

[1] G O Zumbach and U A Müller. Operators on inhomogeneous time series. *International Journal of Theoretical and Applied Finance*, 4(1):147–178, 2001.