Confluent Hypergeometric Function
\( _1F_1(a; b; x) \) (S22BA, S22BB)

The routines S22BA and S22BB, new at Mark 24, provide the functionality to calculate the confluent hypergeometric function \( _1F_1(a; b; x) \), also known as Kummer’s function \( M(a, b, x) \). This has a wide variety of applications, including CIR processes and pricing Asian options. Many special functions are also expressible as special cases of \( _1F_1 \), including the incomplete gamma function, Bessel functions and Laguerre polynomials.

\( M(a, b, x) \) is one of the independent solutions to the differential equation,
\[
x \frac{d^2M(a, b, x)}{dx^2} + (b - x) \frac{dM(a, b, x)}{dx} - aM(a, b, x), \tag{1}
\]
and can be defined via the power series,
\[
M(a, b, x) = \sum_{j=0}^{\infty} \frac{(a)_j x^j}{(b)_j j!}, \tag{2}
\]
where \( (\alpha)_j = 1(\alpha) (\alpha + 1) \ldots (\alpha + j - 1) \) is the rising Pockhammer function of \( \alpha \in \{a, b\} \).

S22BA returns the value \( M \) directly given the values \( a, b \) and \( x \). S22BB returns the solution in the form \( M(a, b, x) = m_f \times 2^{m_s} \). \( M(a, b, x) \) rapidly exceeds standard precision limits for even moderate values of the parameters \( a, b \) and \( x \) \( (\sim O(100)) \), and as such the availability of the fractional component \( m_f \) and scale \( m_s \) allows for meaningful results to be returned over much greater ranges. Figure 1 shows \( M(-150 \leq a \leq 150, -150 < b < 150, x = 25) \), plotted as \( \frac{M}{|M|} \log_2(|M| + 1) \) to emphasize the highly oscillatory nature and scale of the function.

S22BB also accepts the parameters \( a \) and \( b \) as integral and decimal fractional components to increase the accuracy in the floating point calculations. This can provide a significant improvement to the solution when small
perturbations to integral values are required. For example, consider the solutions for $M(-199.999999, -400.00001, 600)$. S22BA gives $M(a, b, x) = -0.1320802726327450 \times 10^{295}$, whereas S22BB gives $M(a_{ni} + a_{dr}, b_{ni} + b_{dr}, x) = -0.1320803101722191 \times 10^{295}$, where the nearest integer and decimal remainder components are $a_{ni} = -200, a_{dr} = 10^{-6}, b_{ni} = -200$ and $b_{dr} = -10^{-6}$. The relative differences is $O(10^{-7})$. 

Figure 1: $M(a, b, x)$ for $a \in [-150, 150], b \in (-150, 150]$ and $x = 25$. 