

The Quadratic Eigenvalue Problem

Vibration problems, for example those that occur in a structure such as a bridge, are often modelled by the generalized eigenvalue problem

$$Kx = \lambda Mx,$$

where K is the stiffness matrix and M is the mass matrix. When damping effects are also included, the problem becomes a quadratic eigenvalue problem (QEP)

$$(\lambda^2 M + \lambda C + K)x = 0,$$

where C is the damping matrix. Other applications of the QEP include the dynamical analysis of mechanical systems in acoustics, linear stability of flows in fluid mechanics, electrical circuit simulation and in modelling microelectronic mechanical systems. The excellent review paper by Tisseur and Meerbergen [2001] describes many of the applications of the quadratic eigenvalue problem.

It is important therefore to be able to solve such problems efficiently and reliably. To solve the QEP, it is usual to linearize the problem and solve the resulting generalized eigenvalue problem. For example, the so called first companion linearization is given by

$$\begin{pmatrix} C & K \\ -I & 0 \end{pmatrix} - \lambda \begin{pmatrix} -M & 0 \\ 0 & -I \end{pmatrix},$$

which gives a generalized eigenvalue problem of twice the dimension of the QEP, from which the eigenvalues and eigenvectors of the QEP can be readily computed. A poor choice of linearization can lead to a generalized problem with poor numerical properties, as can poor scaling. The paper Hammarling et al. [2013] describes the detailed considerations needed to produce a good algorithm for this problem.

The group of Professors Nick Higham and Françoise Tisseur, both NAG members, have been very active in research into the QEP and more general polynomial eigenvalue problems and we are proud to have had the opportunity to collaborate with the group. We are pleased to announce the fruits of this collaboration between Manchester and NAG. Routines f02jcc (nag_eigen_real_gen_quad) for real matrices and f02jqc (nag_eigen_complex_gen_quad) for complex matrices are available in our Mark 24 C library. Fortran and MATLAB versions are in preparation. Please contact us if you need either of these versions; we may be able to assist.

References

- S. Hammarling, C. J. Munro, and F. Tisseur. An algorithm for the complete solution of quadratic eigenvalue problems. *ACM Trans. Math. Software*, 39(3):18:1–18:19, 2013. (<http://eprints.ma.man.ac.uk/1815/>).
- F. Tisseur and K. Meerbergen. The quadratic eigenvalue problem. *SIAM Review*, 43:235–286, 2001.