Two-stage spline approximation to scattered data

The NAG Library’s capabilities for surface fitting have been extended at Mark 24.

Our Curve and Surface Fitting Chapter now includes a routine for computing a spline approximation to a set of scattered data. Typical real-world input (from, say, geosciences, data mining, or medical imaging) can contain millions of points: this routine is designed to easily accommodate large data sets. For sufficiently-dense input data consisting of $n$ points, $n$-linear complexity and memory usage can be attained.

The new fitting routine uses least-squares polynomials to compute local spline approximations over a uniform triangulation of the approximation domain. These approximations are then $C^1$ smoothed to form the final spline. Two evaluators, for a vector and a mesh, have been provided for computing values of the spline.

The Franke function

$$f(x, y) = 0.75 \exp \left( - \left( \frac{(9x - 2)^2 + (9y - 2)^2}{4} \right) \right) +$$

$$0.75 \exp \left( - \frac{(9x + 1)^2}{49} - \frac{(9y + 1)}{10} \right) +$$

$$0.5 \exp \left( - \left( \frac{(9x - 7)^2 + (9y - 3)^2}{4} \right) \right) -$$

$$0.2 \exp \left( - \frac{(9x - 4)^2 - (9y - 7)^2}{} \right)$$

is widely used for demonstrating surface-fitting methods. Figure 1 shows the surface generated by the new routine when given a random set of input $(x, y)$ points on the unit square.

Figure 1: Calculation and Evaluation of Bivariate Spline Fit from Scattered Data using Two-Stage Approximation