

NAG Library Routine Document

F07BDF (DGBTRF)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F07BDF (DGBTRF) computes the LU factorization of a real m by n band matrix.

2 Specification

```
SUBROUTINE F07BDF (M, N, KL, KU, AB, LDAB, IPIV, INFO)
INTEGER          M, N, KL, KU, LDAB, IPIV(min(M,N)), INFO
REAL (KIND=nag_wp) AB(LDAB,*)
```

The routine may be called by its LAPACK name *dgbrtf*.

3 Description

F07BDF (DGBTRF) forms the LU factorization of a real m by n band matrix A using partial pivoting, with row interchanges. Usually $m = n$, and then, if A has k_l nonzero subdiagonals and k_u nonzero superdiagonals, the factorization has the form $A = PLU$, where P is a permutation matrix, L is a lower triangular matrix with unit diagonal elements and at most k_l nonzero elements in each column, and U is an upper triangular band matrix with $k_l + k_u$ superdiagonals.

Note that L is not a band matrix, but the nonzero elements of L can be stored in the same space as the subdiagonal elements of A . U is a band matrix but with k_l additional superdiagonals compared with A . These additional superdiagonals are created by the row interchanges.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

- | | | |
|----|---|--------------|
| 1: | M – INTEGER | <i>Input</i> |
| | <i>On entry:</i> m , the number of rows of the matrix A . | |
| | <i>Constraint:</i> $M \geq 0$. | |
| 2: | N – INTEGER | <i>Input</i> |
| | <i>On entry:</i> n , the number of columns of the matrix A . | |
| | <i>Constraint:</i> $N \geq 0$. | |
| 3: | KL – INTEGER | <i>Input</i> |
| | <i>On entry:</i> k_l , the number of subdiagonals within the band of the matrix A . | |
| | <i>Constraint:</i> $KL \geq 0$. | |

- 4: KU – INTEGER *Input*
On entry: k_u , the number of superdiagonals within the band of the matrix A .
Constraint: $KU \geq 0$.
- 5: AB(LDAB,*) – REAL (KIND=nag_wp) array *Input/Output*
Note: the second dimension of the array AB must be at least $\max(1, N)$.
On entry: the m by n matrix A .
 The matrix is stored in rows $k_l + 1$ to $2k_l + k_u + 1$; the first k_l rows need not be set, more precisely, the element A_{ij} must be stored in

$$AB(k_l + k_u + 1 + i - j, j) = A_{ij} \quad \text{for } \max(1, j - k_u) \leq i \leq \min(m, j + k_l).$$
 See Section 8 in F07BAF (DGBSV) for further details.
On exit: if $INFO \geq 0$, AB is overwritten by details of the factorization.
 The upper triangular band matrix U , with $k_l + k_u$ superdiagonals, is stored in rows 1 to $k_l + k_u + 1$ of the array, and the multipliers used to form the matrix L are stored in rows $k_l + k_u + 2$ to $2k_l + k_u + 1$.
- 6: LDAB – INTEGER *Input*
On entry: the first dimension of the array AB as declared in the (sub)program from which F07BDF (DGBTRF) is called.
Constraint: $LDAB \geq 2 \times KL + KU + 1$.
- 7: IPIV(min(M,N)) – INTEGER array *Output*
On exit: the pivot indices that define the permutation matrix. At the i th step, if $IPIV(i) > i$ then row i of the matrix A was interchanged with row $IPIV(i)$, for $i = 1, 2, \dots, \min(m, n)$. $IPIV(i) \leq i$ indicates that, at the i th step, a row interchange was not required.
- 8: INFO – INTEGER *Output*
On exit: $INFO = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If $INFO = -i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If $INFO = i$, $U(i, i)$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Accuracy

The computed factors L and U are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(k)\epsilon P|L||U|,$$

$c(k)$ is a modest linear function of $k = k_l + k_u + 1$, and ϵ is the *machine precision*. This assumes $k \ll \min(m, n)$.

8 Further Comments

The total number of floating point operations varies between approximately $2nk_l(k_u + 1)$ and $2nk_l(k_l + k_u + 1)$, depending on the interchanges, assuming $m = n \gg k_l$ and $n \gg k_u$.

A call to F07BDF (DGBTRF) may be followed by calls to the routines:

F07BEF (DGBTRS) to solve $AX = B$ or $A^T X = B$;

F07BGF (DGBCON) to estimate the condition number of A .

The complex analogue of this routine is F07BRF (ZGBTRF).

9 Example

This example computes the LU factorization of the matrix A , where

$$A = \begin{pmatrix} -0.23 & 2.54 & -3.66 & 0.00 \\ -6.98 & 2.46 & -2.73 & -2.13 \\ 0.00 & 2.56 & 2.46 & 4.07 \\ 0.00 & 0.00 & -4.78 & -3.82 \end{pmatrix}.$$

Here A is treated as a band matrix with one subdiagonal and two superdiagonals.

9.1 Program Text

```

Program f07bdfe

!      F07BDF Example Program Text
!
!      Mark 24 Release. NAG Copyright 2012.
!
!      .. Use Statements ..
Use nag_library, Only: dgbtrf, nag_wp, x04cef
!      .. Implicit None Statement ..
Implicit None
!      .. Parameters ..
Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
Integer                    :: i, ifail, info, j, k, kl, ku, ldab, &
                           m, n
!      .. Local Arrays ..
Real (Kind=nag_wp), Allocatable :: ab(:, :)
Integer, Allocatable          :: ipiv(:)
!      .. Intrinsic Procedures ..
Intrinsic                    :: max, min
!      .. Executable Statements ..
Write (nout,*) 'F07BDF Example Program Results'
!      Skip heading in data file
Read (nin,*)
Read (nin,*) m, n, kl, ku
ldab = 2*kl + ku + 1
Allocate (ab(ldab,n),ipiv(n))

!      Read A from data file

k = kl + ku + 1
Read (nin,*)((ab(k+i-j,j),j=max(i-kl,1),min(i+ku,n)),i=1,m)

!      Factorize A
!      The NAG name equivalent of dgbtrf is f07bdf
Call dgbtrf(m,n,kl,ku,ab,ldab,ipiv,info)

!      Print details of factorization

Write (nout,*)
Flush (nout)

```

```

!      ifail: behaviour on error exit
!           =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
      ifail = 0
      Call x04cef(m,n,kl,kl+ku,ab,ldab,'Details of factorization',ifail)

!      Print pivot indices

      Write (nout,*)
      Write (nout,*) 'IPIV'
      Write (nout,99999) ipiv(1:min(m,n))

      If (info/=0) Write (nout,*) 'The factor U is singular'

99999 Format ((3X,7I11))
      End Program f07bdfe

```

9.2 Program Data

F07BDF Example Program Data

```

  4  4  1  2           :Values of M, N, KL and KU
-0.23  2.54 -3.66
-6.98  2.46 -2.73 -2.13
          2.56  2.46  4.07
          -4.78 -3.82   :End of matrix A

```

9.3 Program Results

F07BDF Example Program Results

Details of factorization

	1	2	3	4
1	-6.9800	2.4600	-2.7300	-2.1300
2	0.0330	2.5600	2.4600	4.0700
3		0.9605	-5.9329	-3.8391
4			0.8057	-0.7269

IPIV

	2	3	3	4
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