

NAG Library Function Document

nag_complex_log_gamma (s14agc)

1 Purpose

nag_complex_log_gamma (s14agc) returns the value of the logarithm of the gamma function $\ln \Gamma(z)$ for complex z .

2 Specification

```
#include <nag.h>
#include <nags.h>
```

```
Complex nag_complex_log_gamma (Complex z, NagError *fail)
```

3 Description

nag_complex_log_gamma (s14agc) evaluates an approximation to the logarithm of the gamma function $\ln \Gamma(z)$ defined for $\text{Re}(z) > 0$ by

$$\ln \Gamma(z) = \ln \int_0^{\infty} e^{-t} t^{z-1} dt$$

where $z = x + iy$ is complex. It is extended to the rest of the complex plane by analytic continuation unless $y = 0$, in which case z is real and each of the points $z = 0, -1, -2, \dots$ is a singularity and a branch point.

nag_complex_log_gamma (s14agc) is based on the method proposed by Kölbig (1972) in which the value of $\ln \Gamma(z)$ is computed in the different regions of the z plane by means of the formulae

$$\begin{aligned} \ln \Gamma(z) &= \left(z - \frac{1}{2}\right) \ln z - z + \frac{1}{2} \ln 2\pi + z \sum_{k=1}^K \frac{B_{2k}}{2k(2k-1)} z^{-2k} + R_K(z) && \text{if } x \geq x_0 \geq 0, \\ &= \ln \Gamma(z+n) - \ln \prod_{\nu=0}^{n-1} (z+\nu) && \text{if } x_0 > x \geq 0, \\ &= \ln \pi - \ln \Gamma(1-z) - \ln(\sin \pi z) && \text{if } x < 0, \end{aligned}$$

where $n = [x_0] - [x]$, $\{B_{2k}\}$ are Bernoulli numbers (see Abramowitz and Stegun (1972)) and $[x]$ is the largest integer $\leq x$. Note that care is taken to ensure that the imaginary part is computed correctly, and not merely modulo 2π .

The function uses the values $K = 10$ and $x_0 = 7$. The remainder term $R_K(z)$ is discussed in Section 7.

To obtain the value of $\ln \Gamma(z)$ when z is real and positive, nag_log_gamma (s14abc) can be used.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Kölbig K S (1972) Programs for computing the logarithm of the gamma function, and the digamma function, for complex arguments *Comp. Phys. Comm.* **4** 221–226

5 Arguments

- 1: **z** – Complex *Input*
On entry: the argument z of the function.
Constraint: **z.re** must not be ‘too close’ (see Section 6) to a non-positive integer when **z.im** = 0.0.
- 2: **fail** – NagError * *Input/Output*
 The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_TOO_CLOSE_INTEGER

On entry, **z.re** is ‘too close’ to a non-positive integer when **z.im** = 0.0: **z.re** = $\langle value \rangle$, **nint(z.re)** = $\langle value \rangle$.

7 Accuracy

The remainder term $R_K(z)$ satisfies the following error bound:

$$\begin{aligned} |R_K(z)| &\leq \frac{|B_{2K}|}{|(2K-1)!} z^{1-2K} \\ &\leq \frac{|B_{2K}|}{|(2K-1)!} x^{1-2K} \text{ if } x \geq 0. \end{aligned}$$

Thus $|R_{10}(7)| < 2.5 \times 10^{-15}$ and hence in theory the function is capable of achieving an accuracy of approximately 15 significant digits.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example evaluates the logarithm of the gamma function $\ln \Gamma(z)$ at $z = -1.5 + 2.5i$, and prints the results.

10.1 Program Text

```
/* nag_complex_log_gamma (s14agc) Example Program.
 *
 * Copyright 2002 Numerical Algorithms Group.
 *
 * Mark 7, 2002.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
```

```

#include <nags.h>

int main(void)
{
    Integer  exit_status = 0;
    Complex  y, z;
    NagError fail;

    INIT_FAIL(fail);

    /* Skip heading in data file */
    scanf("%*[\n]");
    printf("nag_complex_log_gamma (s14agc) Example Program Results\n");
    printf("      z                ln(Gamma(z))\n");
    while (scanf(" (%lf,%lf)%*[\n] ", &z.re, &z.im) != EOF)
    {
        /* nag_complex_log_gamma (s14agc).
         * Logarithm of the Gamma function ln Gamma(z)
         */
        y = nag_complex_log_gamma(z, &fail);
        if (fail.code == NE_NOERROR)
            printf("(%.1f,%.1f)  (%13.4e,%13.4e)\n", z.re, z.im, y.re,
                y.im);
        else
        {
            printf("Error from nag_complex_log_gamma (s14agc).\n%s\n",
                fail.message);
            exit_status = 1;
            goto END;
        }
    }

    END:

    return exit_status;
}

```

10.2 Program Data

```

nag_complex_log_gamma (s14agc) Example Program Data
(-1.5, 2.5) : Value of z

```

10.3 Program Results

```

nag_complex_log_gamma (s14agc) Example Program Results
      z                ln(Gamma(z))
(-1.5, 2.5)  (-5.0140e+00, -4.0718e+00)

```
