# NAG Library Function Document <br> <br> nag_dpbstf (f08ufc) 

 <br> <br> nag_dpbstf (f08ufc)}

## 1 Purpose

nag_dpbstf (f08ufc) computes a split Cholesky factorization of a real symmetric positive definite band matrix.

## 2 Specification

```
#include <nag.h>
#include <nagf08.h>
void nag_dpbstf (Nag_OrderType order, Nag_UploType uplo, Integer n,
    Integer kb, double bb[], Integer pdbb, NagError *fail)
```


## 3 Description

nag_dpbstf (f08ufc) computes a split Cholesky factorization of a real symmetric positive definite band matrix $B$. It is designed to be used in conjunction with nag_dsbgst (f08uec).

The factorization has the form $B=S^{\mathrm{T}} S$, where $S$ is a band matrix of the same bandwidth as $B$ and the following structure: $S$ is upper triangular in the first $(n+k) / 2$ rows, and transposed - hence, lower triangular - in the remaining rows. For example, if $n=9$ and $k=2$, then

$$
S=\left(\begin{array}{lllllllll}
s_{11} & s_{12} & s_{13} & & & & & & \\
& s_{22} & s_{23} & s_{24} & & & & & \\
& & s_{33} & s_{34} & s_{35} & & & & \\
& & & s_{44} & s_{45} & & & & \\
& & & & s_{55} & & & & \\
& & & s_{64} & s_{65} & s_{66} & & & \\
& & & & s_{75} & s_{76} & s_{77} & & \\
& & & & & s_{86} & s_{87} & s_{88} & \\
& & & & & & s_{97} & s_{98} & s_{99}
\end{array}\right)
$$

## 4 References

None.

## 5 Arguments

1: order - Nag_OrderType
Input
On entry: the order argument specifies the two-dimensional storage scheme being used, i.e., rowmajor ordering or column-major ordering. C language defined storage is specified by order $=$ Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: order $=$ Nag_RowMajor or Nag_ColMajor.

2: uplo - Nag_UploType
Input
On entry: indicates whether the upper or lower triangular part of $B$ is stored.
uplo $=$ Nag_Upper
The upper triangular part of $B$ is stored.
uplo $=$ Nag_Lower
The lower triangular part of $B$ is stored.
Constraint: uplo $=$ Nag_Upper or Nag_Lower.
$\mathbf{n}$ - Integer
Input
On entry: $n$, the order of the matrix $B$.
Constraint: $\mathbf{n} \geq 0$.

4: $\quad \mathbf{k b}$ - Integer
On entry: if uplo $=$ Nag_Upper, the number of superdiagonals, $k_{b}$, of the matrix $B$.
If uplo $=$ Nag_Lower, the number of subdiagonals, $k_{b}$, of the matrix $B$.
Constraint: $\mathbf{k b} \geq 0$.

5: $\quad \mathbf{b b}[\operatorname{dim}]$ - double
Input/Output
Note: the dimension, dim, of the array bb must be at least $\max (1, \mathbf{p d b b} \times \mathbf{n})$.
On entry: the $n$ by $n$ symmetric positive definite band matrix $B$.
This is stored as a notional two-dimensional array with row elements or column elements stored contiguously. The storage of elements of $B_{i j}$, depends on the order and uplo arguments as follows:

```
if order \(=\) 'Nag_ColMajor' and uplo \(=\) 'Nag_Upper',
                \(B_{i j}\) is stored in \(\mathbf{b b}\left[k_{b}+i-j+(j-1) \times \mathbf{p d b b}\right]\), for \(j=1, \ldots, n\) and
                \(i=\max \left(1, j-k_{b}\right), \ldots, j ;\)
if order \(=\) 'Nag_ColMajor' and uplo \(=\) 'Nag_Lower',
    \(B_{i j}\) is stored in \(\mathbf{b b}[i-j+(j-1) \times \mathbf{p d b b}]\), for \(j=1, \ldots, n\) and
    \(i=j, \ldots, \min \left(n, j+k_{b}\right)\);
if order \(=\) 'Nag_RowMajor' and uplo \(=\) 'Nag_Upper',
    \(B_{i j}\) is stored in \(\mathbf{b b}[j-i+(i-1) \times \mathbf{p d b b}], \quad\) for \(i=1, \ldots, n\) and
    \(j=i, \ldots, \min \left(n, i+k_{b}\right)\);
if order \(=\) 'Nag_RowMajor' and uplo \(=\) 'Nag_Lower',
        \(B_{i j}\) is stored in \(\mathbf{b b}\left[k_{b}+j-i+(i-1) \times \mathbf{p d b b}\right]\), for \(i=1, \ldots, n\) and
        \(j=\max \left(1, i-k_{b}\right), \ldots, i\).
```

On exit: $B$ is overwritten by the elements of its split Cholesky factor $S$.
6: $\quad$ pdbb - Integer
Input
On entry: the stride separating row or column elements (depending on the value of order) of the matrix $B$ in the array $\mathbf{b b}$.
Constraint: $\mathbf{p d b b} \geq \mathbf{k b}+1$.
7: $\quad$ fail - NagError *
Input/Output
The NAG error argument (see Section 3.6 in the Essential Introduction).

## 6 Error Indicators and Warnings

## NE_BAD_PARAM

On entry, argument $\langle$ value $\rangle$ had an illegal value.

## NE_INT

On entry, $\mathbf{k b}=\langle$ value $\rangle$.
Constraint: $\mathbf{k b} \geq 0$.

On entry, $\mathbf{n}=\langle$ value $\rangle$.
Constraint: $\mathbf{n} \geq 0$.
On entry, pdbb $=\langle$ value $\rangle$.
Constraint: pdbb $>0$.

## NE_INT_2

On entry, $\mathbf{p d b b}=\langle$ value $\rangle$ and $\mathbf{k b}=\langle$ value $\rangle$.
Constraint: $\mathbf{p d b b} \geq \mathbf{k b}+1$.

## NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

## NE_POS_DEF

The factorization could not be completed, because the updated element $b(\langle$ value $\rangle,\langle$ value $\rangle)$ would be the square root of a negative number. Hence $B$ is not positive definite. This may indicate an error in forming the matrix $B$.

## $7 \quad$ Accuracy

The computed factor $S$ is the exact factor of a perturbed matrix $(B+E)$, where

$$
|E| \leq c(k+1) \epsilon\left|S^{\mathrm{T}}\right||S|
$$

$c(k+1)$ is a modest linear function of $k+1$, and $\epsilon$ is the machine precision. It follows that $\left|e_{i j}\right| \leq c(k+1) \epsilon \sqrt{\left(b_{i i} b_{j j}\right)}$.

## 8 Parallelism and Performance

nag_dpbstf (f08ufc) is not threaded by NAG in any implementation.
nag_dpbstf (f08ufc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The total number of floating-point operations is approximately $n(k+1)^{2}$, assuming $n \gg k$.
A call to nag_dpbstf (f08ufc) may be followed by a call to nag_dsbgst (f08uec) to solve the generalized eigenproblem $A z=\lambda B z$, where $A$ and $B$ are banded and $B$ is positive definite.
The complex analogue of this function is nag_zpbstf (f08utc).

## 10 Example

See Section 10 in nag_dsbgst (f08uec).

